

**ICl&C'97**

**INTERNATIONAL  
CONFERENCE ON  
INFORMATICS  
AND CONTROL**

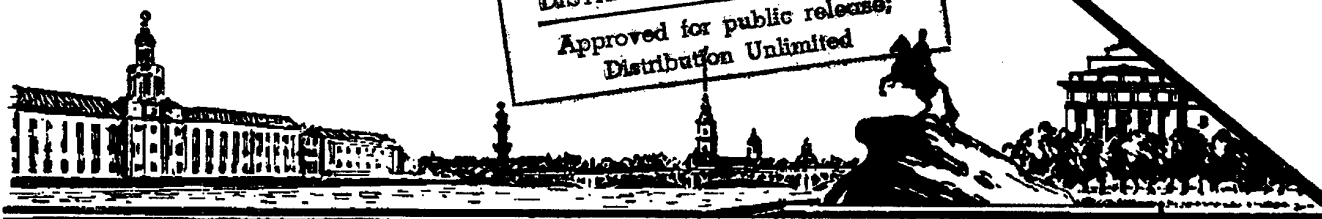
**PROCEEDINGS**

**June 9-13, 1997  
St. Petersburg  
RUSSIA**

**DTIC QUALITY INSPECTED 4**

**19970710 076**

**DISTRIBUTION STATEMENT A**  
Approved for public release;  
Distribution Unlimited



ISBN 5-85733-079-3

PUBLISHED BY ST. PETERSBURG INSTITUTE FOR INFORMATICS AND  
AUTOMATION OF THE RUSSIAN ACADEMY OF SCIENCES (SPIIRAS)

39, 14 Linia, VO  
St. Petersburg, 199178, Russia  
[spiiran@iiias.spb.su](mailto:spiiran@iiias.spb.su)

COVER DESIGNED BY NIKOLAY D. GORSKY

Copyright © SPIIRAS 1997

**REPORT DOCUMENTATION PAGE**

Form Approved OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE 12 June 1997	3. REPORT TYPE AND DATES COVERED Conference Proceedings	
4. TITLE AND SUBTITLE International Conference on Informatics and Control (ICI&C'97) Proceedings			5. FUNDING NUMBERS F6170897W0117	
6. AUTHOR(S) Conference Committee				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) St.Petersburg Institute For Informatics & Automation 39, 14 Line, V.O. St.Petersburg 199178 Russia			8. PERFORMING ORGANIZATION REPORT NUMBER N/A	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) EOARD PSC 802 BOX 14 FPO 09499-0200			10. SPONSORING/MONITORING AGENCY REPORT NUMBER CSP 97-1025	
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.			12b. DISTRIBUTION CODE A	
13. ABSTRACT (Maximum 200 words)  The Final Proceedings for International Conference on Informatics and Control, 9 June 1997 - 13 June 1997  The Topics covered include: Intelligent control systems and information processing; Nueroinformatics and nueronet control; Robots' control and computer integrated technologies; Qualitative techniques for control and information processing tasks; Stochastic systems for information processing in motion control tasks; Safety control in complex systems, Transport Control (vehicles, vessels, aircraft, etc.); Satellite remote sensing in control and control related applications; Telecommunications: basics.				
14. SUBJECT TERMS  Control, Control System, Advanced Computing, Computers, Electronic Devices, Instrumentation, Optimization, Signal Processing, Human Factors			15. NUMBER OF PAGES 849-1354	
			16. PRICE CODE N/A	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL	

**International Conference on Informatics and Control  
(ICI&C'97)**

# **PROCEEDINGS**

**VOLUME 3 OF 3**

**St. Petersburg, Russia**

# TABLE OF CONTENTS

## CHAPTER XI. INFORMATICS AND CONTROL IN BIOLOGY

AND MEDICINE .....	851
MEDICAL-ECONOMICAL MONITORING SYSTEM IN ST.PETERSBURG	
<i>Krasilnikov I. A.</i> .....	853
HEARTH FIBRILLATION AND PARALLEL SUPERCOMPUTERS	
<i>Kogan B. Y., Karplus W. J., Chudin E. E.</i> .....	857
MYOKINEMETRIC CONTROL OF A PROSTHETIC PREHENSOR FROM RESIDUAL FOREARM MUSCULATURE	
<i>Bowker P., Heath G. H.</i> .....	866
COMMON SYSTEM OF INFORMATION MAINTENANCE OF INFECTIOUS EPIDEMIOLOGICAL SERVICE AND MANAGEMENT OF EPIDEMIOLOGICAL PROCESS IN ST.PETERSBURG	
<i>Taits B. M.</i> .....	871
A TELEMETERING SYSTEM FOR NONINVASIVE MEASUREMENT AND INTELLIGIBLE-PICTORIAL IMAGING OF THE CARDIAC ELECTROPHYSIOLOGICAL FUNCTION	
<i>Titomir L. I., Blatov I. V., Trunov V. G., Aidu E. A. I., Barinova N. E.</i> .....	874
HIGH-RESOLUTION ELECTROCARDIOGRAPHY IN THE PROGNOSIS OF DEVELOPMENT AND EVALUATION OF PREVENTION EFFICIENCY OF LIFE THREATENING ARRHYTHMIA	
<i>Ivanov G. G., Titomir L. I., Dvornikov V. E., Eleuov A. U., Wambo S.</i> .....	881
ABOUT THE SOLUTION OF THE REVERSE ELECTROCARDIOGRAPHIC PROBLEM	
<i>Kuteeva G. A.</i> .....	885
CONTROL SIGNALS FOR BIOLOGY PROCESSES	
<i>Polonnikov R. I.</i> .....	890
REGISTRATION AND PROCESSING OF THE DIGITAL INFORMATION IN BIOTECHNICAL SYSTEM BY COMPUTER ENCEFALOGRAPHS	
<i>Polonnikov R. I., Kobrin V. P.</i> .....	896
ADAPTIVE BIOFEEDBACK CONTROL IN BIOLOGICAL SYSTEMS	
<i>Suvorov N. B.</i> .....	905
MATHEMATICAL MODELS OF TWO BIOSYSTEMS	
<i>Blinkova N. V., Voloshinova T. V., Ershov B. A., Pasynkov V. E.</i> .....	910
INFORMATION IN MEDICAL SYSTEMS AND ITS USE FOR DISEASED DIAGNOSING AND PREVENTING	
<i>Musijchuk Yu. I.</i> .....	917

KNOWLEDGE-BASED EVOKED POTENTIAL PROCESSING <i>Strakhovich E. V, Ampilova N.</i> .....	922
THE UNIFICATION OF THE INFORMATION AT PREPARATION OF THE REPORTS ABOUT HEALTH OF THE POPULATION <i>Krasilnicov I. A., Musijchuk Yu. I.</i> .....	926
<b>CHAPTER XII. ADAPTIVE CONTROL</b> .....	<b>931</b>
ADAPTIVE FEEDFORWARD COMPENSATION OF STRUCTURAL VIBRATION IN SATELLITE-BASED LASER COMMUNICATION SYSTEMS <i>Busch T. E., Nicholson D. J., Scormin V. A.</i> .....	933
A REVISED BLIND EQUALIZATION ALGORITHM ADAPTIVE FOR MPSK MODULATION <i>Yu Hongchuan, Kang Nansheng, Fang Tinjian.</i> .....	938
OUTPUT FEEDBACK ADAPTIVE COMPENASATOR OF EXTERNAL DISTURBANCES FOR NONLINEAR SYSTEMS <i>Nikiforov V. O., Pogromsky A. Yu.</i> .....	943
MAJORIZING FUNCTIONS APPROACH IN ADAPTIVE CONTROL DESIGN OF NONLINEAR SYSTEMS <i>Putov V.V., Polushin I.G.</i> .....	951
ADAPTIVE CONTROL OF NONLINEAR BUSINESS CYCLE MODELS <i>Andrievsky B.R., Dymkin E.G., Fradkov A. L.</i> .....	958
A SUCCESSIVE DAMPING METHOD FOR CONTROL PROBLEMS BY INCOMPLETE FEEDBACK. <i>Kirin N. E., Kirin A. N.</i> .....	966
CONTROL OF OSCILLATIONS IN ELECTROMECHANICAL SYSTEM <i>Blekhman I. I., Nagibina O. L., Tomchina O. P., Yakimova K. S.</i> .....	972
OPTIMAL UNIVERSAL REGULATORS IN THE PROBLEM OF DAMPING OF FORCED OSCILLATIONS <i>Shiriaev A. S.</i> .....	980
PROJECTIVE OBSERVABILITY OF DYNAMICAL SYSTEMS <i>Bolotin Yu. V., Morgunova S. N.</i> .....	983
MODELING, CONTROL AND LABORATORY EXPERIMENTS WITH OSCILLATORY MECHANICAL SYSTEM <i>Konoplev V. A., Konjukhov A.P.</i> .....	990
STOCHASTIC CONTROL MINIMAX PROBLEM <i>Fomin V. N.</i> .....	997
STRUCTURE AND METHODS OF DYNAMICAL DECISION- MAKING AND STOCHASTIC OPTIMAL CONTROL <i>Baranov V. V., Salyga V. I.</i> .....	1002

ROBUSTNESS MEASURES IN DISCRETE-TIME SYSTEMS WITH MULTIPLE: PARAMETERS; NONLINEARITIES, AND DELAYS <i>Eslami M.</i> .....	1008
MULTIDIMENSIONAL FREQUENCY ROBUST CONTROL SYSTEMS <i>Akunov T.A., Akunova A., Ushakov A. V.</i> .....	1022
THE ROBUSTNESS ANALYSIS FOR DYNAMIC SYSTEMS WITH INTERVAL PARAMETRIC UNCERTAINTY: FREQUENCY DOMAIN TECHNIQUE DEVELOPMENT <i>Akhmedjanov F. M., Gusev Yu. M., Krymsky V. G.</i> .....	1032
PARAMETRICAL SYNTHESIS OF THE CONTROL SYSTEM IN THE PROCESS OF THE IMITATIVE TESTS <i>Degtyarjov O. V., Evstifeyev V. V.</i> .....	1042
IDENTIFICATION OF CRITICAL PHENOMENA IN ENERGETIC SYSTEMS <i>Gugushvili A., Gomareli N., Dalakishvili G.</i> .....	1047
CONFLICT-FREE IDENTIFICATION IN ADAPTIVE SYSTEMS <i>Goosev S. A., Popov O. S.</i> .....	1052
IDENTIFICATION ALGORITHM OF TEMPERATURE-HUMIDITY MODES MATHEMATICAL MODEL PARAMETERS OF ARCHIVAL AND BOOKDEPOSITORY ROOMS <i>Baskakov L. V., Diduk G. A., Zigalenko E. N.</i> .....	1059
IDENTIFICATION OF THE LINEAR TIME-INVARIANT DYNAMICAL SYSTEMS IN CONDITIONS OF INCOMPLETE INFORMATION ACCORDING TO ITS STATE <i>Volobueva O. P.</i> .....	1064
<b>CHAPTER XIII. SAFETY CONTROL IN COMPLEX SYSTEMS</b> .....	<b>1067</b>
LOGIC-PROBABILISTIC THEORY OF SAFETY OF COMPLEX SYSTEMS <i>Ryabinin I. A.</i> .....	1069
A DIAGNOSTIC ENGINE FOR POWER TRANSMISSION NETWORKS <i>Baroni P., Lamperti G., Pogliano P., Tornielli G., Zanella M.</i> .....	1076
DEVELOPMENT OF ALGORITHMICAL PROVISION FOR THE CALCULATION OF SYSTEMS RELIABILITY AND SAFETY ON THE BASIS OF TUPLE ALGEBRA <i>Kulik B.</i> .....	1087
THE BINARY DYNAMIC SYSTEMS SAFETY CONTROL <i>Greck B. V., Tomashevsky L. G.</i> .....	1094
THE ELECTROSTATIC FIELD PARAMETERS ASSOCIATED WITH OIL TANKS OPERATIONS <i>Galka V.L.</i> .....	1100

NEW AREAS AND TASKS OF SIMULATION AND ANALYSES OF RISK BY LOGIC-PROBABILISTIC METHOD <i>Solojentssev E. D.</i> .....	1109
SAFETY AND STABILITY SECURING OF AUTOMATIC CONTROL SYSTEMS AT PARAMETERS VARIATIONS <i>Petrov Ju. P.</i> .....	1118
TECHNICAL DIAGNOSTICS ASSOCIATED WITH TANK FILLING OPERATIONS <i>Galka V. L.</i> .....	1122
THE PROBLEM OF THE SUSTAINABLE DEVELOPMENT <i>Ignatiev M.</i> .....	1134
COMPLEX METHODOLOGY OF SAFETY REGULATION – SYSTEM OF DYNAMIC BARRIERS (SOB) PREVENTING THE DEVELOPMENT OF EMERGENCY TRANSIENTS AT NUCLEAR POWER PLANTS <i>Stebenev A.S., Eperin A. P., Khoudiakov M. M., Rakitin I. D.</i> .....	1142
KNOWLEDGE REPRESENTATION IN DIAGNOSING TASKS OF THE EXPERTISE AREAS WITH STRUCTURED SUBSPACE <i>Miasnikova G.</i> .....	1149
<b>CHAPTER XIV. SIMULATION OF PROCESSES IN OCEAN</b> .....	<b>1157</b>
OBJECT-ORIENTED MODELING THE OPERATIONS RESEARCH PROBLEMS <i>Popovich V. V., Rodionov A. A.</i> .....	1159
THEORY AND PRACTICE OF AUTOMATED STRUCTURAL-LOGICAL SIMULATION OF SYSTEMS <i>Mozhaev A. S.</i> .....	1169
COMPUTER TECHNOLOGY OF SEISMIC DATA PROCESSING WITH THE PURPOSE OF DETECTION OF OBJECTS WALKING AND MOVING ALONG THE EARTH SURFACE <i>Pahomov A. A.</i> .....	1178
DIGITAL INFORMATION SYSTEMS AND SOME PROBLEMS OF APPLICATION OF ALGORITHMS OF DATA PROCESSING AND DECISION MAKING CHARACTERISTIC OF A MAN-INTERPRETER <i>Pahomov A. A.</i> .....	1181
COMPUTER SIMULATION OF MULTIPLICATION SEARCH TASKS <i>Popovich V. V., Tretiakov A. G.</i> .....	1184
MODELING FORMATION PROCESSES OF INTERMITTENT TURBULENCE IN THE OCEAN <i>Rodionov A., Papenko H.</i> .....	1189
AUTOSYSTEM MARKETING RESEARCH: FORMATION AND UTILIZATION <i>Azarenko S. A., Habibulin R. K., Chetvertakov M. M.</i> .....	1194

APPLICATION OF MARINE HYDROGRAPHIC INFORMATION SYSTEMS  
FOR RESOURCE AND ECOLOGICAL PROBLEMS SOLUTION

*Mit'ko A., Mit'ko V., Hvostov A.* ..... 1198

**CHAPTER XV. ROBOT CONTROL** ..... 1203

ASTATISM IN NONLINEAR CONTROL SYSTEMS WITH APPLICATION TO  
ROBOTICS

*Pervozvanski A. A., Freidovich L. B.* ..... 1205

COMPARATIVE STUDY OF METHODS FOR ENERGY -OPTIMAL GAIT  
GENERATION FOR BIPED ROBOTS

*Roussel L., Canudas-de-Wit C., Goswami A.* ..... 1213

AUTONOMOUS MICROROBOTS

*Rembold U., Fatikow S.* ..... 1223

ROBOT – LIKE SYSTEM FOR PHYSICAL IMITATION OF  
WEIGHTLESSNESS ON EARTH

*Kulakov F. M., Naumov V. B., Smirnov E. N., Evlampiev A. A.,  
Shaluha V. A., Neokesariyskiy V. N.* ..... 1231

PECULIARITIES OF CONTROL BY ROBOTIC SYSTEMS USING ALLOYS  
WITH SHAPE MEMORY EFFECT

*Panteleev S. V., Pervishko A. F., Priadko A. I., Semenov S. I., Viahhi I. E.* ..... 1237

DYNAMICS OF A CO-OPERATING ROBOTIC FIXTURE FOR SUPPORTING  
AUTOMATIC DEBURRING TASKS

*Michelini R. C., Acaccia G. M., Callegari M., Molfino R. M., Razzoli R. P.* ..... 1244

A DISTRIBUTED CONTROL ARCHITECTURE FOR AUTONOMOUS ROBOT  
SYSTEMS

*Laengle T., Lueth T. C., Rembold U., Woern H.* ..... 1255

PROBLEMS OF SENSING AND SPATIAL MOTION CONTROL IN ACTIVE  
ROBOTICS

*Miroshnik I., Boltunov G., Shiegin V.* ..... 1267

A HIERARCHICAL NAVIGATION SYSTEM FOR AN AUTONOMOUS  
MOBILE ROBOT

*Betourne A., Fefe A., Nicosia S., Valigi P.* ..... 1275

ROBUST HYBRID FORCE/POSITION CONTROL OF ROBOTS IN  
CONDITIONS OF UNCERTAINTY

*Burdakov S. F., Paramonov L. V.* ..... 1283

ADAPTIVE CONTROL WITH PRECISE IDENTIFICATION OF UNKNOWN  
PARAMETERS OF DYNAMIC MODEL FOR ROBOT MANIPULATORS

*Altbregin A. G., Korolev V. A.* ..... 1290

TIME-MINIMAL ROBOT CONTROL ALONG KNOWN PATH <i>Kulakov F. M., Naumov V. B.</i> .....	1297
COOPERATION IN HUMAN-ROBOT-TEAMS <i>Laengle T., Hoeniger T., Rembold U., Woern H.</i> .....	1306
UNIFIED SOFTWARE INTENDED FOR ROBOT WITH TECHNICAL VISION SYSTEM AND ARTIFICIAL INTELLIGENCE ELEMENTS <i>Arkhipov V. D., Vartanov V. L., Makarychev V. P.</i> .....	1315
THE MATHEMATICAL MODEL AND THE NUMERICAL ROBOT SIMULATION IN CAD PACKAGE CAEIR <i>Solovjev A. E.</i> .....	1324
MULTILINK MECHANISM AS A CONSTRUCTIONAL BASE FOR INFORMATION SUPPORT OF A MAN-MACHINE ASSEMBLY COMPLEX <i>Daurov S, Ovsyannikov E., Strobel G.</i> .....	1329
<b>LATE PAPERS</b> .....	<b>1335</b>
THE BASIC INFORMATIONAL CONTENTS LEVELS IN DIRECT AND INVERSE PROBLEMS OF SATELLITE ECOLOGY <i>Smokty O. I.</i> .....	1337
AN AUTOMATED COMPLEX OF THE CONTROL AND FORCASTING THE EXTREME SITUATIONS IN AN ENVIRONMENTAL MONITORING SYSTEM <i>Lenchuk S., Kulganov V., Shvilin Yu., Nesterov S., Mescheryakov E., Sorokin L.</i> .....	1345
<b>AUTHORS INDEX</b> .....	<b>1349</b>

**CHAPTER XI:**

***INFORMATICS AND CONTROL IN BIOLOGY AND  
MEDICINE***

THIS CHAPTER INCLUDES PAPERS

PRESENTED AT THE CONFERENCE SESSION:

INFORMATICS AND CONTROL IN BIOLOGY AND MEDICINE

Organized by: *Prof. Revold I. Polonnikov,*  
*Prof. Igor A. Krasilnikov.*

# MEDICAL-ECONOMICAL MONITORING SYSTEM IN ST PETERSBURG

Igor A. Krasilnikov

St.Petersburg Administration Public Health Committee

30, Shkapina Str., 198095, St.Petersburg, Russia

e-mail: igorkras@infopro.spb.su

**Abstract.** The information of medical-economical monitoring can be provided for definition of needs in financial resources and for the development of system of established tariffs for polyclinic services. The practical questions of the project of medical -economical monitoring considered in this message.

**Keywords.** Medical insurance, medical-economical monitoring, out-patient services, clinical-statistical group, tariff, medical standard.

City public health now has to transfer the direct budget financing of out-patient departments to the payment through the Territory Fund of Compulsory Medical Insurance (CMI) and private insurance companies for the particular services, rendered by the organization.

The costs for different services in polyclinics have to be determined to implement such structure changes. The existing state medical-economical statistics does not allow to carry out similar operations. The information necessary for account of costs of some out-patient services can be received indirectly and by sample investigations. In particular, the sources of information could be some polyclinic and department computer databases and also the original manuscripts - histories of cases. However, the databases were frequently projected for completely other purposes, and now scientific development of their analysis is required.

The actual costs of some out-patient services can't be proved and on its basis estimation of real needs of the city public health financing can't be achieved without realization of this work. According to available data, the budget of 1997 provides 30% of public health needs. Its ratification in the present volume will inevitably result to system shock and sharp deterioration of the city dwellers health and increase of mortality.

The project of establishment of public health actual costs should be aimed at the creation of medical-economical monitoring model. On the basis of monitoring information can be provided for definition of needs in financial resources and for the development of system of established tariffs for polyclinic services. That will allow to proceed to financing of out-patient departments through the Territory Fund and private insurance companies.

The realization of the project will allow to receive in short terms the information on structure of demand on out-patient services in city regions and in age-sex groups. A system of dynamics of demand and cost of medical services according to real inquiries and opportunities of the population can be constructed on the basis of this information. A practical consequence of the project will be the review of city public health budget to prevent the further deterioration of health parameters of the city population.

Expected results:

1. Creation of a system of medical and economical reporting in St. Petersburg compatible to the European standards.
2. Creation of an infrastructure of information system of medical-economical monitoring for introduction of the new reporting.
3. Distribution of developed monitoring model and appropriate information technology to other large cities of North-West region of Russian Federation.

The basic problem of the program development is a scientific substantiation of information base volume for realization of accounts of tariffs costs of out-patient services. The work on collection and processing of available medical-economical data should be carried out to establish real expenses on various services in out-patient organizations and whole public health of the city.

The plan of realization of research

1. Medical standards developed at a level of Russian Federation and at some public health branches should be the basis. As a rule, available standards were developed for stationary treatment and require adaptation for out-patient conditions.
2. All cases of out-patient treatment are coded according to International Classification of Diseases (ICD-9) under the principal diagnoses (the reason of reference for medical help)
3. Being based on ICD-9 codes a grouping of completed cases of out-patient treatment is achieved, and so-called clinical-statistical groups (CSG) are formed.
4. Two types of parameters are defined for each CSG:
  - real (on the basis of analysis of polyclinic databases during 1-3 years)
  - theoretical (on the basis of expert estimations, given by chief specialists of the Public Health Committee)
5. The lists of diagnostic procedures (laboratory diagnostics, instrumental diagnostics, experts consultations) and medical procedures (drug treatment, physical and active methods of treatment, operations) are defined as for real, and for theoretical types of CSG parameters, with the indication of frequency of each procedure.
6. The procedures are detailed up to a level, at which it is possible to give a general (averaged) estimation of procedure's cost.
7. The real frequency of procedure is defined by analysis of out-patient databases.
8. The theoretical frequency of procedures is defined by experts with the indication of minimum and optimum frequency on each procedure.
9. Real terms of treatment (duration of completed out-patient treatment case) and received results (outcomes) and also theoretically desirable (probable) ones are defined for each CSG.
10. The cost of fulfillment is accounted for each procedure (this can be done for separate medical-prophylactic organization (MPO), groups of one-type MPO, or for all city MPO).
11. All above described data are inclosed in a relational database.

Thus, each completed case of treatment gets in specific CSG depending on a range. Data for fixed period (month, quarter, the year) can be analyzed and the procedure cost is known. An estimation of real cost of medical help can be given as for separate CSG, and for the entire

out-patient service (or for its separate part, for example, included in the Territory Program of the Compulsory Medical Insurance). These data can be compared to volumes of means that are theoretically necessary for maintenance of an acceptable level of medical service (minimum and optimum levels in accordance with experts estimations, included in the system). They can also be the basis for formation of public health budget.

Analysis the data of polyclinics, serving more than 10% of city population (more than 500000 persons) in various and different types of regions (central, peripheral) will make the results of this research representative.

The results of work include the description of used methodical approaches, database system management (DBSM) on magnetic carriers and database users guide. They can be used at all levels of public health (MPO, region, Public Health Committee, Territory fund of CMI) for an estimation of an economic efficiency of financing - comparing really received results (outcomes of treatment) at the real financing level with theoretically possible results in case of increase of financing to an optimum level. However, it can't be excluded that quite acceptable results can be received at a minimum financing level, and on the other hand, the increase of financing will not probably result in improvement of the efficiency of medical help.

It is necessary to ensure the permanent functioning of the system and maintenance of actual databases of typical regions for observation of the situation in the city.

The developed system will allow to improve public health management on the basis of medical-economical monitoring.

Revealing of establishments with personal reporting and analysis of person databases of the city out-patient services is necessary for information maintenance of this work. During the preliminary researches presence of such bases was established in two city regions: central with 150000 population and «sleeping» one with 500000 population. The time scope is about 3 years. Presence of similar databases in some medical organizations is established in three more regions.

Taking into account the fact that the information on existence of similar databases is not always accessible, as it is not compulsory to represent it, it is necessary to carry out an objective interrogation of out-patient departments to maintain the representative of initial data.

At the following stage it is necessary to select databases which relevantly display structure of demand on out-patient services and to develop a form of representation of data suitable for purposes of present research and the methods of its analysis. Creation of software is also required, which in particular will allow to organize integration and data processing, taken from bases created in various spheres and with various structures.

The fulfilled work will allow at the following stage to develop temporary tariffs on out-patient services, that will differ depending on group of particular establishment (personnel, medical technical resources equipment and areas). The data of passport of medical establishment of the city license committee should be used at this stage. The mechanism of the control for tariffs validity and their regular updating should be developed simultaneously. Constant medical-economical monitoring will form the basis for constant control and updating of tariffs for out-patient departments.

On the basis of monitoring data it is possible to establish the methods to attribute medical organization (and/or particular services rendered by organization) to a determined tariff

group, using a sample of case reports data, book-keeping data on organization equipment, organization budget use data, age-sex parameters of the served population. The tariffs should be expressed in MSR (minimal salary rate).

It is desirable to ensure applicability of methods to the conditions of North-West regions of Russia. On the final stage of the project the following should be formulated:

- Medical-economical monitoring model, on the basis of which it is possible to receive information for definition of needs in financial resources.
- The offers in creation of medical-economical monitoring system
- The offers in transition to reporting compatible to European standards
- The offers on development of system of established tariffs on polyclinic services, that will allow to proceed to financing of out-patient city departments through the Territory Fund and private insurance companies.

# HEART FIBRILLATION AND PARALLEL SUPERCOMPUTERS

B. Y. Kogan, W. J. Karplus, E. E. Chudin

Computer Science Department, University of California, Los Angeles. USA

e-mail: kogan@cs.ucla.edu

**Abstract.** The Luo and Rudy III cardiac cell mathematical model is implemented on the parallel supercomputer CRAY - T3D. The splitting algorithm combined with variable time step and an explicit method of integration provide reasonable solution times and almost perfect scaling for rectilinear wave propagation. The computer simulation makes it possible to observe new phenomena: the break-up of spiral waves caused by intracellular calcium dynamics and the non-uniformity of the calcium distribution in space during the onset of the spiral wave.

**Key Words** • simulation on parallel computers • intracellular calcium effect • heart fibrillation

**Introduction.** It is generally recognized that the leading cause of sudden cardiac death is ventricular fibrillation very often preceded by ventricular tachycardia. Both represent a severe violation of heart rhythmical activity due to distortions of excitation wave propagation along the heart muscle. The latter can not be understood from the point of view of a single cell. Disorder of cardiac tissue conduction for electrical excitation propagation is caused directly by distortion of cell electrical couplings and indirectly by changes in cells functioning.

Electrophysiological experiments on living tissue with multielectrode mapping or voltage-sensitive dyes do not lead to discovery of the mechanisms of these phenomena and give only a rough picture of wave propagation.

The sophisticated mathematical models of excitation propagation along the heart muscle (based on the latest clamp-experiment data and cardiac tissue investigations) are comprised of nonlinear parabolic partial differential equations of the reaction-diffusion type. Complex nonlinearities, a diversity of the time constants (introducing stiffness), and the necessity to get solution for 2D and 3D of complex configuration makes analytical solution impossible, even after the rough simplifications of the model.

The computer simulation approach is the most promising, but for sequential computers this problem is computationally intractable. Indeed, the simplest FitzHugh-Nagumo cell model [1], compiled in C and running on a Pentium 100 PC, runs approximately in real time (cardiac cycle per sec.). The minimum model necessary to simulate tachycardia would consist of 128x128 of such cell models and would require 16,000 times more time. The overall computational time will significantly grow when we simulate several cardiac cycles and use more sophisticated cell models. Therefore, to make computer simulation feasible it is necessary to transfer the simulation to massively parallel computers and select the appropriate numerical algorithms.

In this paper we present some results of a wave propagation study on parallel supercomputers (CM, T3D) and discuss the details of the problem implementation.

**General formulation of mathematical model.** The majority of the modern mathematical models of action potential (AP) generation and propagation [2,3,4,5,6] are synthesized on the basis of the Hodgkin-Huxley formalism and can be represented in the following general form:

$$\begin{aligned}
\frac{\partial V}{\partial t} &= \frac{1}{C} F(V, \vec{U}) + D \Delta V + \frac{1}{C} I_{st}(t) & \dim \vec{U} &= N \\
\frac{\partial \vec{U}}{\partial t} &= \vec{f}(V, \vec{U}, \vec{\mu}, t) & \dim \vec{\mu} &= M \\
\vec{\mu} &= \vec{\mu}(t)
\end{aligned} \tag{1}$$

Here:

V is cell membrane potential in [mV]  
t is time in [msec]  
D is diffusion coefficient in [cm<sup>2</sup>/sec]  
Δ is the Laplacian

F(V,  $\vec{U}$ ) is a nonlinear function which represents the sum of ionic currents in [μA/cm<sup>2</sup>]

I<sub>st</sub>(t) is the stimulus in [μA/cm<sup>2</sup>]

C is membrane capacity in [μF/cm<sup>2</sup>]

$\vec{U}$  describes the dynamic of ionic channels permeability (gate variables) and ionic concentrations in intracellular compartments

$\vec{\mu}$  is the vector of parameters, which varies with time.

These equations are solved for the following initial and boundary conditions: V( $\vec{r}$ , 0),

$\vec{U}(\vec{r}, 0)$  and

$\frac{\partial V}{\partial n} = 0$ ,  $\vec{r}$  - space coordinates  $\vec{n}$  - direction of the normal to the boundary G.

When D = 0 (no diffusion) system (1) reduces to the so-called point model which characterizes the behavior of single cell.

The simplest second-order FitzHugh-Nagumo point model has only one generalized gate variable (N=1 and M=0 in (1)) and three nonlinear functions of V, while the most sophisticated thirteenth-order Luo and Rudy III model [6] includes 9 equations for ionic gate variables and 3 for [Ca<sup>2+</sup>]<sub>i</sub> in intracellular compartments (N=12 in (1)) and 33 nonlinear functions.

The response of the Luo-Rudy III point model to over-threshold stimulus shown in Fig. 1 demonstrates the shape of the AP and time course of the gate variable j. The cell recovery processes: the changing of gate variables and calcium concentration in time after completion of AP are shown on Fig. 2.

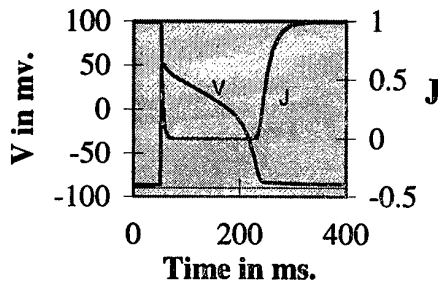


Fig. 1. AP and gate variable j after long DI. APD equals to 183 ms. measured at -72 mv. J variable recovers 70 ms. after completion of AP and affects dV/dt of the next AP.

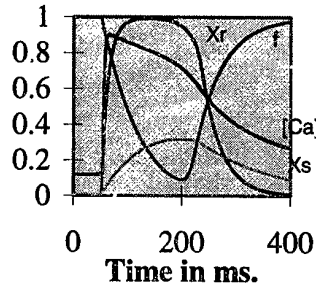


Fig. 2. Time course of gate variables f, Xr, Xs and [Ca<sup>2+</sup>]<sub>i</sub>. Recovery time of gate variables (100 - 400 ms.) is shorter than the one of [Ca<sup>2+</sup>]<sub>i</sub>.

APD restitution curve for isolated cell is given below on Fig. 3.

The increased complexity of cell mathematical models reflects the necessity to reproduce correctly the cell recovery processes and intracellular  $[Ca^{2+}]_i$  dynamics which control the contractility of separate heart muscle fibers and produces the feedback on AP. All these factors play a crucial role in excitation wave propagation and especially in the initiation and maintenance of spiral waves and their break-up.

**Massively parallel computer systems.** The only way to make computer simulations feasible under the conditions of growing complexity of the mathematical models is to transfer the simulation from sequential computers to massively parallel ones and to select the most efficient numerical algorithm for the given problem. Parallel computers are divided onto two basic types: single instruction multiple data (SIMD) and multiple instruction multiple data (MIMD).

**The original SIMD massively parallel computer** architecture was used in CM-2 computer introduced in 1987 by Thinking Machine Corporation [7]. A fully expanded CM-2 consists of 64K processors and provides two types of interprocessors communication. One of them requires minimum time to interchange the data between the neighboring processors on a 2D grid. Each processing element contains a 1-bit arithmetic and logic unit and 64 K bits of local memory. In addition each processing element has four 1-bit flag registers, an I/O interface, and shares an optional floating point accelerator.

During execution, the front-end computer(s) where all programs are stored sends a stream of high-level instructions to sequencers, which in turn convert these to microcode nano-instructions. The latter are broadcast to individual processing elements and control sequentially their timing and operations. So, the processor memory contains only data and immediate computations, and does not contain code for execution.

As shown in [8], the architecture of CM-2 is very suitable for simulating excitation propagation in 2D cardiac tissue. In the course of the simulation on the CM-2 of the entire system (1) with FH-N second order point model it was shown that the shape of the APD restitution curve affects the changes in length and velocity of propagated waves, the onset of stationary and non-stationary propagation of spiral waves, and site specific induction of spiral waves in response to premature stimulation [9]. All these results were obtained using the method of lines in combination with the Ashour-Hanna numerical integration method [10].

The simulation on CM-2 also proved that geometry and boundary properties of narrow paths created by scars after myocardial infarction play a crucial role in the possible onset of reentry [11]. The comparative analysis of the existing second-order simplified models [12] based on simulation data obtained with CM-2 showed that all these models are simplified versions of old Noble model [2]. Therefore they do not take into consideration the restitution of maximum speed of cell depolarization and membrane current, related to changes of intracellular  $[Ca^{2+}]_i$ . The latter are presented in novel physiological models [3,4] and play an important role in propagation phenomena. Some improvements of the simplified models were realized in [13] where a third-order simplified point model was proposed. This model retains almost all wave propagation properties incorporated in the above mentioned physiological models. In the course of simulation on CM-2 with a 128x128 grid of these simplified cell models, it was found [14] that the spontaneous break-up of spiral wave front resembles the shape of AP propagation during ventricular fibrillation.

The further use of the Connection Machine CM-2 is restricted by the following drawbacks:

1. The limited number of processing elements effectively limits the size of simulation grid and corresponding cardiac tissue. The use of virtual processors does not help.
2. The small amount of local processor memory do not allows the use of advance numerical technique(including functions tabulation).
3. The comparatively low computational speed of processing elements makes long term simulation difficult.

These drawbacks, in a view of the recent tremendous improvements of sequential computers performance, make the further simulation of our problem on the CM-2 computer impractical. At the same time the power of the modern sequential computers is not enough to simulate the waves propagation in 3D space and in 2D with the Luo and Rudy III [6] sophisticated point model. Therefore it seems reasonable to transfer the simulation of heart problems to MIMD parallel computers.

**MIMD parallel computers** combine the advantages of sequential and parallel computations but require high speed of data interchange between the processing elements and an appropriate numerical algorithm. Parallel supercomputers of the MIMD type now are presented on the market by IBM (SP-1, SP-2) and Cray Research, Inc. (Cray T3D and Cray T3E). They are scaleable computers consisting of a number of processing elements with distributed memory, host computer(s), and a special interprocessor communication network. All these component are physically located in one cabinet.

Let us shortly consider the characteristics of the T3D massively parallel computer which was used for simulation of heart fibrillation. It consists of 256 processing elements (PEs) front-ended by a host system of three Cray Y-MP computers with 512 Mbytes of memory. All PEs of the T3D are interconnected by 3-D torus communication network capable of transferring data between two nodes with a speed over 140 Mbytes/sec. These PEs are 150 MHz version of the DEC EV4 Alpha microprocessor with additional high speed networking hardware added by Cray Research Inc. The peak performance of each PE is 150 MFlops. The Cray T3D memory is physically distributed between PEs, so that each of them has 64 Mbytes of local memory. The total memory of 256 PEs is 16,384 Gbytes. The memory is globally addressable. Each PE also has a small (1024 of 64 bits words) high-speed, random-access memory (so-called cache memory) that temporarily stores frequently or recently accessed local data,

The T3D operating system (OS) is distributed with a microkernel on each PE. A microkernel OS takes less than 4Mbytes of PEs memory and provides only a small set of basic OS functions. The basic OS (unicos8.0.3) runs on a host Cray Y-MP computer. This OS handles most of the system calls generated by jobs running on the T3D, performs scheduling and resource allocation for these jobs, and distributes the disc resources. All communication with the T3D is performed via Cray Y-MP.

**Implementation on T3D.** The objective of the simulation is to find the distribution of membrane potential in time and space for a 2D model of cardiac tissue. For this purpose the Luo-Rudy III point model (for details see original publication [6]) was used in system (1). The cardiac tissue, assigned to be a uniform isotropic syncytium (continuous medium) is approximated by a grid of 256 x 256 nodes connected by coupling resistors. The splitting operator method [15] is used to allow variable time step integration. According to this method, the integration of (1) is split into two parts: integration of the diffusion equation

$$\frac{\partial V}{\partial t} = D\Delta V \quad (2)$$

and integration of the point model equations.

$$\begin{aligned} \frac{\partial V}{\partial t} &= \frac{1}{C} F(V, \bar{U}) + \frac{1}{C} I_{st}(t) \\ \frac{\partial \bar{U}}{\partial t} &= \bar{f}(V, \bar{U}, \bar{\mu}, t) \\ \bar{\mu} &= \bar{\mu}(t) \end{aligned} \quad (3)$$

Equations (2) and (3) are solved in the following consecutive time cycles. Equation (2) is solved during time step  $\Delta t/2$  with a given initial and boundary conditions. Then equations (3) are solved with given initial conditions for the variable  $\bar{U}$ . For the variable  $V$ , the initial conditions are obtained from the solution of (2) at the end of time  $\Delta t/2$ . Equations (3) are stiff ODEs. Therefore, the integration with a variable time step is used to decrease the overall simulation time. We used  $\Delta t_1 = 0.05\Delta t$  to integrate the stiff part of the equations. Then the values of  $V$ , obtained at the end of  $\Delta t$ , are used as initial conditions for next integration step of diffusion equation (2), during another time step  $\Delta t/2$ . This completes the first cycle. Subsequently the initial conditions for the first solution of equation (2) are taken from the solution at the end of the previous cycle.

The grid is divided between "n" available parallel processors so that each processor is solving equations (2) and (3) for  $256 \times 256 / n$  nodes. The geometrical apportionment of the grid nodes to the processors can be accomplished in different ways. The optimum one, results in the equal computational load for all PEs and a minimum exchange of data between the PEs. We used the division on parallel strips. The data of the Laplacian calculations are exchanged between all processors twice during each time step  $\Delta t$ .

The explicit Euler numerical integration algorithm is used for equation (2) with fixed time step equal  $\Delta t/2 = 0.1\text{ms}$ . The well known implicit integration methods (such as alternative direction) are difficult to implement in parallel computations. Equations (3) are solved by explicit Euler method with the exception of the equations for the gate variables with time constants  $\tau_k \sim \Delta t_1$ . Particularly, in the Luo-Rudy III model, only one time constant is encountered, which value has the order of the chosen smallest time step  $\Delta t_1$ . This time constant is associated with the gating mechanism of the sodium channel. The solution for these gate variable is obtained by the so-called hybrid method proposed in [16] and [17] and analyzed in [18]. For this application the hybrid method provides better stability than Euler method.

**Spiral waves propagation in the 2D model of cardiac tissue.** The goal of this simulation is to find the effect of intracellular calcium concentration dynamics on spiral waves propagation in the model of 2D isotropic cardiac tissue. For this purpose the tissue model is represented as a grid of  $256 \times 256$  nodes. Each node of the grid is a Luo-Rudy III model of one or patch of cardiac cells, connected to the four neighbors by coupling resistance.

The efficiency of parallel simulation on T3D is illustrated in Fig. 4 for the case of rectilinear front propagation and the chosen algorithm. Fig. 4 shows that computation time is almost inversely proportional to the number of parallel PEs. This dependence is changed for more complicated wave propagation fronts where the execution time increases significantly with a decrease of PEs number.

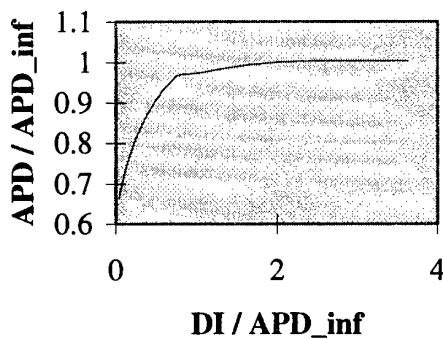


Fig. 3. Normalized restitution curve obtained by S1- S2 protocol. APD<sub>inf</sub> is APD after infinite DI. The slope of this curve is less than 1 except at very short DIs where it is equal to 1.

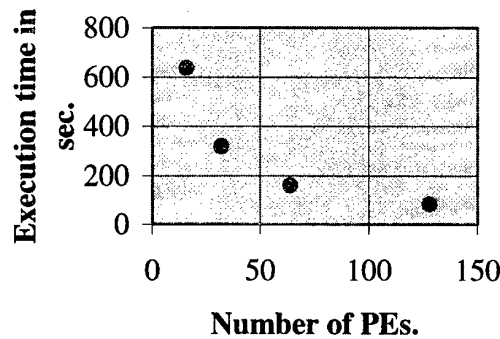


Fig. 4. Execution time versus the number of PEs. The graphic corresponds to 100 ms. of rectilinear wave propagation.

The simulation of rectilinear wave propagation shows that the velocity of wave front is equal to 39 cm/sec which is close to its real physiological value. The spiral wave is initiated by the application of a premature stimulus in the area 10 x 200 nodes before the tail of the previous rectilinear wave front. The stimulus is applied inside the window of vulnerability which occupies 10x256 nodes beginning with the nodes on a tail of preceding wave. The stimuli applied beyond the borders of that window do not produce a propagating wave or create a circular wave. The spiral waves initiated inside the window of vulnerability are unstable.

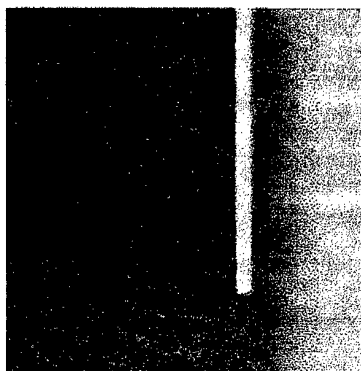
The simulation results are shown in Fig. 5,6. The start of time reading is related to the moment of rectilinear wave initiation. Premature stimulus application (Fig. 5a) leads to the formation of the spiral wave (Fig. 5b). While spiral wave continues to be "stationary", the calcium becomes nonuniformly distributed inside the spiral site (Fig. 5c). That causes the deformation of the spiral wave front during the next 400 msec. of spiral wave rotation (Fig. 6a). As a result of  $[Ca^{2+}]_i$  feedback on AP propagation this deformation increases with the further rotation of the spiral wave, and the break-up of the wave front occurs (Fig. 6b). Subsequently the numerous break-ups occur, which lead to the full disorganization of wave propagation (Fig. 6c). The break-up of the wave front is not observed when the nonuniform  $[Ca^{2+}]_i$  distribution is fixed after some time of spiral wave rotation, and when the L-type calcium channel is blocked. These facts allow us to hypothesize that the wave front break-up is triggered by the  $[Ca^{2+}]_i$  dynamic. This demonstrates a new mechanism of break-up comparing with previously studied in [19] and opens an avenue for reconsideration of the effects of antifibrillatory drugs.

**Conclusions.** The implementation of the latest and most detailed physiological model on the T3D parallel supercomputer allows us to observe the phenomenon of spiral wave break-up caused by intracellular calcium dynamics.

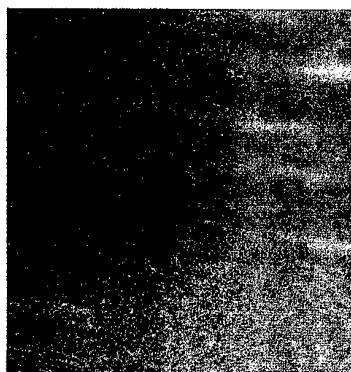
The computer simulation model can be used for testing the effect of various antifibrillatory drugs.

The chosen numerical algorithm provides a computation time inversely proportional to the number of processing elements when a rectilinear wave is propagating. For spiral wave propagation, the computational time is higher than for rectilinear wave, and does not follow inverse proportionality relation. The deviation from that relation is getting smaller for larger numbers of processing elements.

Membrane Potential

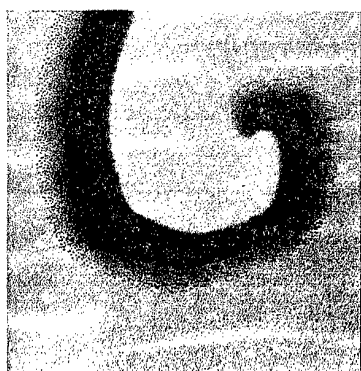


Calcium



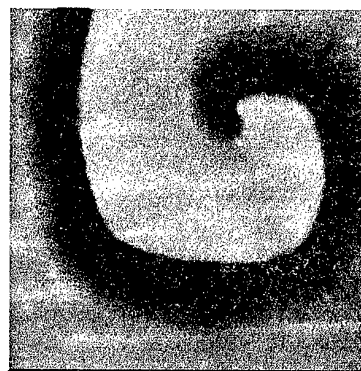
time = 300 ms.

**a**



time = 1100 ms.

**b**



time = 1500 ms.

**c**

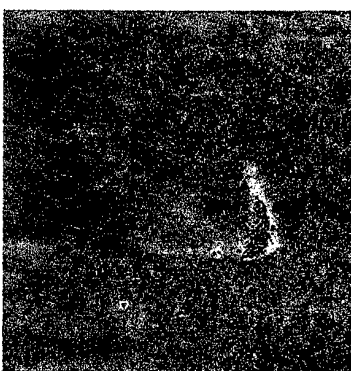
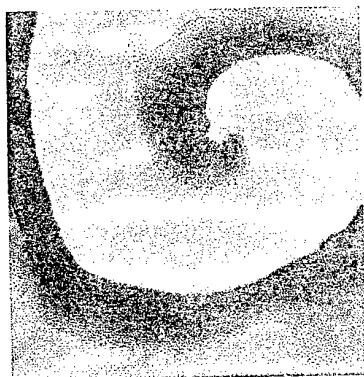


Fig. 5. Spiral wave initiation and development. The following color code was used: for membrane potential  $V$  - blue if  $V < -60\text{mv.}$ , red for the wave front, and shades of green otherwise; for  $[\text{Ca}]$  - blue if  $[\text{Ca}] < 0.6\text{ microM}$ , red if  $[\text{Ca}] > 1.6\text{ microM}$ , shades of green otherwise.

Membrane Potential

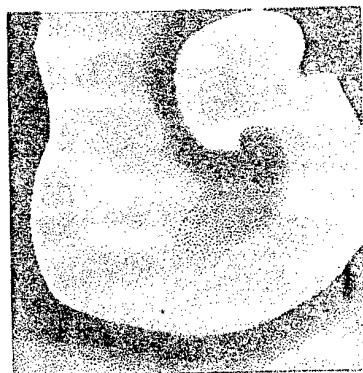


Calcium



time = 1900 ms.

**a**



time = 2300 ms.

**b**



time = 2700 ms.

**c**



Fig. 6. Spiral wave deformation and break-up. Color code is the same as on Fig. 5.

A further decrease of the computation time would require an algorithm which provides a uniform distribution of the computational load among processing elements \*.

## References

- [1] FitzHugh, R. (1969). In H. P. Schwan, ed., *Biological Engineering*, Chapter 1, 1-85. McGraw-Hill, New York.
- [2] Noble, D. (1962). *J. Physiol.*, 160, 317-352.
- [3] Beeler, G. W. and H. Reuter (1977). *J. Physiol.*, 268, 177-210.
- [4] Luo, C. and Y. Rudy (1991). *Circ. Res.*, 68, 1501- 1526.
- [5] Luo, C., Y. Rudy (1994). *Circ. Res.*, 74, : 1071- 1096 , and 1097-1113.
- [6] Zeng, J., K. Laurita, D. Rosenbaum, Y. Rudy (1995). *Circ. Res.*, 77, 140-152.
- [7] Hillis, W. D. (1985). *The Connection Machine*. MIT Press, Cambridge, Mass.,.
- [8] Kogan, B., W. Karplus, A. Pang (1990). *Simulation*, 55, 271-281.
- [9] Kogan, B., W. Karplus, B. Billett, A. Pang, H. Karagueuzian, S.Khan (1991). *Physica D*, 50, 327-340.
- [10] Ashour, S. and O. Hanna (1990) *Computers Chem. Engng*, 14(3), 267-272.
- [11] Kogan, B., W. Karplus, B. Billett, and W. Stevenson (1992). *Physica D*, 59, 275-296.
- [12] Karpoukhin, M., B. Kogan, W. Karplus (1995). In *Proceedings of 28<sup>th</sup> Hawaii Intern. Conference on System Sciences*, eds., L. Hunter, B. Shriver , 5, 112-122.
- [13] Kogan, B., W. Karplus, and M. Karpoukhin (1995). In H. Power and R. Hart, eds., *Computer Simulation in Biomedicine*, 145-154. Computational Mechanics Publications, Southampton Boston.
- [14] Karpoukhin, M. (1996). PhD thesis, Computer Science Dept., UCLA.
- [15] Strang, G. (1969). *SIAM , J., Numerical Analysis*, 5, 506-517.
- [16] Moore, J. W. and F. Ramon (1974). *J. theor. Biol.*, 45, 249-273.
- [17] Rush, S. and H. Larsen (1978). *Trans. On Biomed. Engineering*, 25, 389-392.
- [18] Victorri, B. and A. Vinet, F. A. Roberge, J-P. Drouhard (1985). *Computers and Biomedical Research*, 18, 10-23.
- [19] Karma, A. (1993). *Phys. Rev. Lett.*, 71, 1103-1106.

---

\* We would like to thank Dr. James Weiss and Dr. Alan Garfinkel for their deep interest in this work. The CRAY supercomputer used for this project was provided by NASA offices mission to planet Earth, aeronautics, and space sciences and JPL.

Research in computer simulation of propagation of electrical excitation in the 2D cardiac tissue at UCLA computer science department is supported in part by the NIH Grant SCOR in Sudden Cardiac Death P50 HL52319, and by NASA/Dryden Research Center Grant NCC 2-374.

# MYOKINEMETRIC CONTROL OF A PROSTHETIC PREHENSOR FROM RESIDUAL FOREARM MUSCULATURE

G. H. Heath and P. Bowker  
Department of Rehabilitation,  
University of Salford,  
Salford, M6 6PU, UK.  
Glyn.Heath@ucsalf.ac.uk

**Abstract.** A control system has been developed which allows a prosthetic hand to grasp in synchrony with an anatomical hand, using a sensor which measures the degree of contraction of any superficial muscle group that contributes to anatomical grasping. After a forearm amputation, a significant amount of those muscles used for grasping are frequently preserved and are capable of isotonic contraction. Accurate sensing of residual muscle contraction may allow the prosthetic prehensor to assume the position of its anatomical equivalent. Such a system may prove useful in the control of forearm prostheses by amputees.

**Keywords.** Myokinematic, prosthesis, prehension, control.

**Introduction.** Upper limb prostheses are traditionally controlled either by the relative movement between two body segments or by muscle generated electrical (myoelectric) activity. Both of these control systems suffer from the disadvantage that the controlling physiological signals are not directly related to anatomical joint position. Hence there is a significant degree of conscious effort required to control currently available prosthetic systems. The development of prostheses controlled using signals derived from residual musculature relate to control of the corresponding anatomic joint would be likely to require less conscious effort and thus to provide improved functional benefits to below elbow amputees.

The use of myokinematic activity of residual musculature in the stump, for the control of prosthetic actuators, was first demonstrated in 1949 by Wilms [1]. Unfortunately, these early myokinematic controlled prostheses were not very reliable in their clinical application and with the invention of myoelectric prostheses in 1957 [2], further research into myokinematic control decreased and probably ceased by the mid 1970s. It is proposed however, that developments in digital processing, miniature electric motors and discrete reliable myokinematic transducers now allow the problems of earlier myokinematic controlled prostheses to be overcome.

**Background Theory.** Prostheses controlled by the myokinematic activity of residual muscle use the physiological activity directly associated with the position control of an amputated anatomical joint, to control its prosthetic equivalent. Myokinematic activity of surface muscles can be measured through the skin using a Hall-effect transducer [3], the output of which will be proportional to the degree of linear contraction of the muscle and

hence proportional; to the joint angle [4]. The output of such a transducer, thus indicates the degree of contraction of the muscle and hence the apparent position of the amputated anatomical joint. In addition, proprioceptive feedback relating to with the position of an amputated anatomical joint, originates from proprioceptors present in the residual muscles associated with that joint and not the joint itself. These proprioceptors sense both the length and change in length of the muscle and thus provide information to the brain regarding the position the joint. The preservation of proprioceptive position feedback of amputated joints and the assumed relationship between residual muscles and the amputated joints, implies that myokinematic control could provide an almost subconscious control of prosthetic actuators.

In order to determine whether these principles lead to improved prosthetic control it is necessary to construct a myotonically controlled prosthetic prehensor and initially to test whether this can reliably imitate the actions of an anatomic prehensor.

### ***Position Proportional Control of a prosthetic Prehensor Using Myokinematic Transducers.***

**Method.** A forearm cuff containing a myokinematic transducer placed over the flexor musculature of the fingers, was to control a prosthetic prehensor driven by a geared motor with an encoder. The output from the myokinematic transducer ( $V_{in}$ ), which was taken to be linearly proportional to the degree of finger flexion, was converted to a digital signal which was then compared to the output from an encoder ( $V_{out}$ ), which was proportional to the angular position of the prosthetic prehensor. The error ( $V_{in} - V_{out}$ ) was then processed by the control program, to give a signal ( $V_c$ ) which caused a rotation of the motor of the prehensor by an amount proportional to the error, thus providing the desired grasping position. Feedback from the encoder in the motor ( $V_{out}$ ) provided confirmation to the control program that the prehensor has assumed the desired position. Once the desired position of the prehensor was achieved another signal was taken from the myokinematic transducer (see Fig.1).

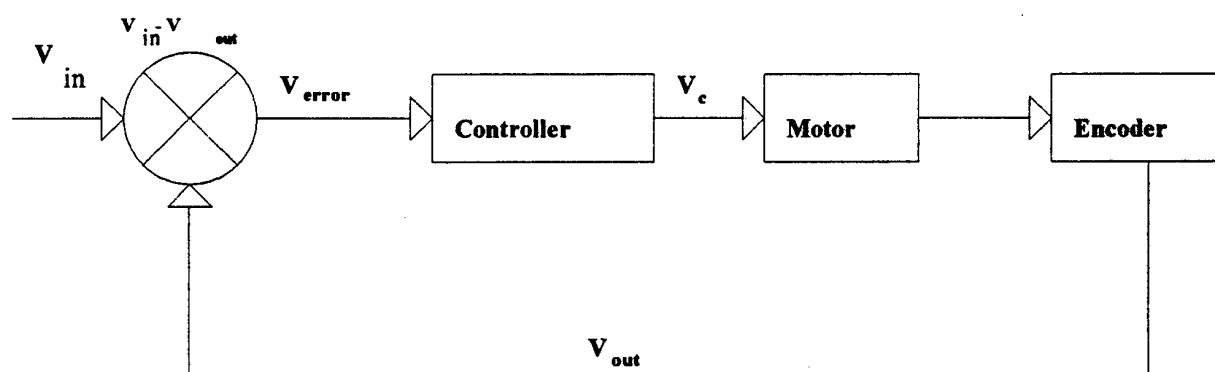


Fig. 1 Control sequence for a myokinematic controlled prehensor

Immediately prior to operating the prosthetic prehensor, a calibration routine had to be enacted. This calibration routine provided values of output from the myokinematic transducer for fingers closed and fingers fully open. These values were taken from several hundred outputs obtained whilst the operator closed and opened his or her hand. Once calibration had been completed, the control sequence was initiated. When errors occurred outside the range of calibration, the calibration range automatically moved so that the value of  $m$  causing the error became the maximum or minimum value of the new range. This roving calibration facility compensated for minor environmental variations in the vicinity of the transducer such as small movements of the forearm cuff relative to the forearm.

To demonstrate that the prosthetic prehensor could accurately track the grasping action of an automatic prehensor, a video recording of both the prosthesis and the anatomical hand was made. a series of angles for both the anatomical hand and the prosthetic prehensor was measured from the video recording and the degree of synchronization was quantified.

**Results.** The relationship between the grasp angles of the prosthetic and anatomical prehensors for ten successive grasping procedures is represented graphically in Fig. 2 and the error between the two is shown in Fig. 3

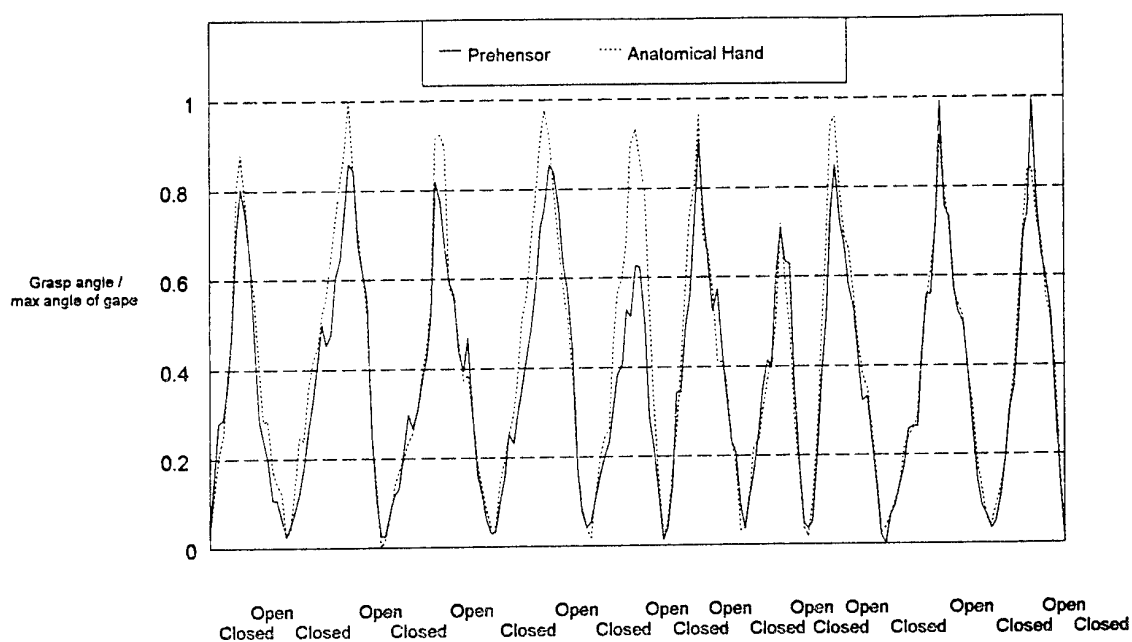


Fig. 2 Position of a myokinematic controlled prehensor vs position of the anatomical hand (Angles expressed as a proportion of the maximum angle of grasp by the prehensor and the fingers tot he thumb, during the trial)

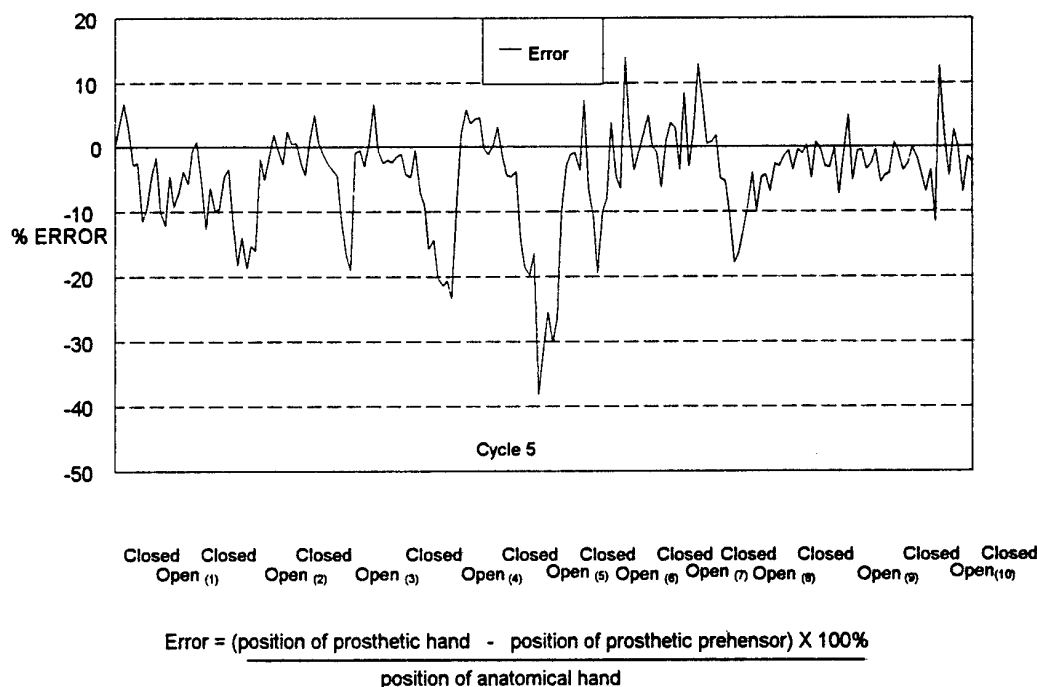


Fig. 3 Error between the position of the anatomical hand and the prosthetic hand.  
(Error expressed as a percentage)

Parameters effecting the accuracy of the control system that can be quantified are:

Error caused by incremental positioning of prehensor (21 positions between fully closed and fully open), +or - 2.35%

Error caused by digitization, + or - 5.15% (5% confidence limits (1.96 S.D.);  
n= 20)

Those parameters that can not be quantified are movement of the anatomical thumb, errors caused by compliance of the human tissue lying above the muscle and errors caused by inputs into the transducer.

**Conclusion.** Figure 2 shows a good degree of correlation between the angular positions of the prehensor and the anatomic hand. The grasping sequence is repeatable and the control strategy is capable of restoring the tracking of the anatomical hand by the prosthesis after an unknown event that has affected the system accuracy (Cycle 5, Fig. 3). Although the magnitude of this error in Fig. 3 is significant, further work is being undertaken to quantify the possible elements contributing to this error, such as the error incurred by digitization.

**Discussion.** This research has achieved synchronization of grasping between an anatomical hand and prehensor, controlled from the contraction of muscles associated with anatomical grasping. It may be possible for an amputee to control a prosthetic hand in a similar manner.

It is proposed that myokinematic control of prosthetic prehensors by amputees, may require less conscious effort than those traditionally used. This is because myokinematic control uses displacement inputs from muscles associated with anatomic prehension.

### **References.**

1. Wilms, E. (1951) Der Technik der Vaduzer Hand. (The technology of the Vaduz Hand). Orthopadie Technik, 2, 7-9, February.
2. Kobrinski, A. E., S. V. Bolkhovitin, L. M. Voskoboinikova, D. M. Ioffe, E. P. Polyan, B. P. Popov, Ya. L. Slavitski, A. Ya. Sysin, Ya. S. Yakobson (1960). Problems of bioelectric control. Automatic and remote control. Proceeding of the First International Congress of IFAC, Moscow. pp. 619-623.
3. Heath, G. T., B. Heravi, K. Taylor (1995). Patent Application Number GB 9513454.0, A Transducer, September.
4. Brand, P. W., A. Hollister (1992). In Clinical Mechanics of the Hand, Ch. 6, pp. 67-73

# COMMON SYSTEM OF INFORMATION MAINTENANCE OF INFECTIOUS-EPIDEMIOLOGICAL SERVICE AND MANAGEMENT OF EPIDEMIOLOGICAL PROCESS IN ST.-PETERSBURG

Boris M. Taits

St.Petersburg Administration Public Health Committee

1, Sadovaj Str., 191011, St.Petersburg, Russia

e-mail: igorkras@infopro.spb.su

**Abstract.** The new method of immunisation management show serious positive changes; increase immune layer among all age groups of children, decrease of morbidity and mortality from infectious diseases.

**Keywords.** Regional program, information system, management of epidemiological process, vaccination, morbidity, mortality.

The change of social-economical conditions has sharply deteriorated situation on infectious morbidity in entire Russia and particularly in large cities as Moscow and St.-Petersburg. Thus high growth of morbidity is marked in group of so-called controlled infections: diphtheria, whooping cough, epidemic parotitis and others. It causes serious concern and requires development of new methods of management of immunisation quality.

The existing system of collecting and processing of statistical data of infectious-epidemiological service could not satisfy the growing requirements and promote the effective solution of setting problems. Creation of information infrastructure for management of epidemiological process is especially actual today, with leading role of vaccinal prevention. The perfection of information-analytical system of epidemiological surveillance of infectious diseases controlled by vaccination and also for post-vaccinal complications was the basis of the regional program «Vaccinal Prevention». Automated system of information maintenance of management structures and establishments of St.-Petersburg Administration Public Health Committee and City Centre of State Sanitary-Epidemiological Surveillance (CSSES) was created by the Public Health Committee for the prophylactics of infectious morbidity for in-time collecting of information, analysis and acceptance of the administrative decisions.

Following blocks are provided as subsystems of information system «Vaccinal Prevention»:

- Registration of infectious morbidity;
- Registration of mortality from infectious diseases;
- Registration of vaccination;
- Preparation of reports.

Computers with the fax-modem were established in all children's polyclinics for realisation of the system. Local computer networks are created in epidemiological department of the City Desinfection Station and in infectious-epidemiological, organisation-methodical departments of Public Health Committee. The data exchange inside information network will be provided by global telecommunication network (GTN).

Complex of installed programs allows to carry out managing of personal cards of preventive vaccines, planning of children's immunisation, reception of accounting documents that are necessary for representation at higher organisations and for more effective analysis and management of polyclinic immunological work. This complex of programs completely replaces paper card file of inoculations. The databases contain the information about all children registered in polyclinic, all carried out inoculations, medical removals, individual inoculation plans for each infant. The complex has the information on inoculations calendar that can be flexibly changed according to the requirements of organisation of immunological work.

The formation of reports provides reception of children alphabetic lists, reports and distribution of children by region, age, children's establishment, risk group, packages of monthly and annual reports. Managing the address information allows effective updating of information base when polyclinic organisational structure changes. The archive will be established for the monthly and weekly reports, allowing to look through and print received earlier documents. The information on vaccinal prevention work is transferred by post server to an information network the infectious-epidemiological organization-methodical division (IEOMD).

The passing of effective administrative decisions in infectious-epidemiological service cannot be based only on the analysis of population vaccinal prevention. Therefore, for the first time we form common global city information system with reporting of infectious morbidity. Therefore the epidemiological department of the City Desinfection station enters to information system. The basic tasks of epidemiological department information system are collecting and processing of the primary information on cases of infectious diseases, dispatch of operative information to other subjects of city information system, managing database in Epidemiological Bureau and its copy in ИЭОМО, preparation of periodic reports on infectious-epidemiological situation in the city.

Until recently all the information on infectious morbidity in St.-Petersburg was on paper carriers and was processed manually. The primary registration document was a registration card BP 317/83/11 (PK), which is filled in primary registration department at reception of the information by phone. The information was transferred from PK to a infectious diseases log-book - form 060/Y. Thus, database existed in card-files and journal variants, first one was the most complete, reflecting all changes of records on each case of disease. The following hereinafter information (diagnosis specification, laboratory researches data and others) are entered as in PK, and in form 060/Y.

The algorithm of City Desinfection Station work now is the following. In case of detection of infectious patient information is transferred to the epidemiological department operator by phone or by modem. The operator activates subsystem of input of the information for formation of new record in a database of infectious cases. The record is formed according to PK. The following actions are carried out at a stage of filling in the form: input of the information about infectious patient; formation of all attributes of infectious case (residence, place of detection, work / study and etc.) according to the common directories; assignment of epidemiological number; check of the entered information; preparation of the tasks for electronic mail on dispatch of the operative information about present infectious case to the following addressees: regional Centres SSES, IEOMD, epidemiological department of CC SSES, polyclinic of residence, polyclinic of study / work place.

The information about infectious case, electronic address of the addressee and message (urgently, usual, postponed) will form the post message, which is located in queue for transfer. The address of «Vaccinal Prevention» system is chosen from the directory of addresses. After termination of input the information about infectious patient is kept in a database.

The order of registration of death case from infectious disease now forms the same information flows, as registered infectious diseases. The message on death case comes to Epidemiological Bureau from polyclinic or hospital by phone, death case is assigned post-mortem epidemiological number. The data are filled in special card (analogue of PK) and are transferred to CC SSES by messenger and to regional Centres SSES by phone.

The new order of registration assumes, that the Centre on registration of death from infectious diseases will be pathology department of infectious S.P.Botkin hospital № 30 - City Centre of infectious pathology, having the majority of pathology openings of this category of dead.

Thus, filling in PK for all dead infectious patients in St.-Petersburg is achieved in S.P.Botkin hospital with assignment of post-mortem epidemiological number. The following information flows are formed: hospital, where infectious patient has died and was opened; or polyclinic, on territory of which infectious patient has died at home, transfers by phone (in the long term - by GTN) all data of information for filling in PK in S.P.Botkin hospital; Centre on death registration transfers complete data volume on all death cases by GTN to IEOMD; reduced volume on all death cases - to epidemiological department of the City Desinfection Station, where the data about cases without epidemiological number during life are also directed. The operator carries out automated input of the information, the doctor on duty makes a decision on necessity of a complex of anti-epidemic measures; reduced volume of data on death cases requiring realisation of a complex of anti-epidemic measures is transferred to the territory centres of city SSES by GTN or by phone.

If death is caused by infection (with achieved vaccine preventing), HCOMO requests polyclinics (in addition to listed higher flows) data about inoculation history of dead (extract from the form 063/Y).

The organisation of necessary telecommunication network with blocks of registration of vaccination, morbidity and mortality from infectious diseases allows to arrange in time measures on control, analysis, regulation of vaccine preventing, forbidding unreasonable medical removals; will help not to admit not recent cases, when few members of one family died stage by stage from infectious disease, allows to accept fast and adequate administrative decisions and to carry out measures with connection of wide infectious-epidemiological service, when it is yet possible to correct situation, not admitting distribution of infectious diseases and lethal outcomes.

The first results of new methods on immunisation management for the last years show serious positive changes: increase of immune layer among all age groups of children, decrease of morbidity and mortality from infectious diseases, first of all from diphtheria.

Thus, the common system of information maintenance of structures and establishments of public health and sanitary-epidemiological service allows to manage the quality of population vaccine prevention: to have the necessary information on mentioned problems, to choose more expedient tactics and in a result to receive the best direct and remote results.

# A TELEMETERING SYSTEM FOR NONINVASIVE MEASUREMENT AND INTELLIGIBLE-PICTORIAL IMAGING OF THE CARDIAC ELECTROPHYSIOLOGICAL FUNCTION

Leonid I. Titomir, Igor V. Blatov, Vladimir G. Trunov,  
Eduard A. I. Aidu, and Natalia E. Barinova

Institute for Problems of Information Transmission,  
Russian Academy of Sciences,  
All Russian Research Institute of Automatics,  
Moscow, Russia  
E-mail: titomir@ippi.ac.msk.su

**Abstract.** The paper describes a computerized electrocardiographic system TELE-DE-CARTO based on a telemetering measuring equipment and a new technique of intelligible-pictorial representation of the heart electrophysiological characteristics in the form of maps related to the anatomical parts of the heart (dipole electrocardiotopography method, abbreviated as DECARTO). The system is intended for short-term and long-term observation of the heart function under complicated conditions of professional activity, for recognition and prediction of cardiac diseases, estimation of the efficiency of treatment, control of state of the cardiovascular system in closed-loop mode, and for solving other problems of cardiac diagnosis, preventive inspection, and control.

**Keywords.** Biomedical informatics, computerized electrocardiography, telemetering systems.

The most widespread methods of the heart state observation and electrocardiographic diagnosis are based on an estimation of curves recorded from leads which are few in number and not necessarily synchronized. The shortage of the initial information and rather an abstract shape of these curves hamper the straightforward biophysical and electrophysiological comprehension of the data with taking into account the spatially distributed nature of the cardioelectric generator and its spatio-temporal evolutions in the excitable myocardium. This disadvantage of the standard electrocardiography is to a large extent overcome by development of techniques for representation of the electrocardiographic data in an intelligible-pictorial topographic form with superimposing the functional characteristics of the excitable myocardium on the respective anatomical areas of the heart. The best prerequisite for implementation of such an approach is provided by the so-called body surface mapping methods using multiple lead systems with several tens of chest electrodes for acquisition of the initial synchronous records of the cardioelectric potentials. However, a widespread use of these methods for cardiological diagnostics is limited, to some extent, in view of the cumbersome and laborious procedure of measurement.

To overcome this difficulty, we developed a computerized telemetering system for mapping of the heart electrophysiological states and characteristics on the basis of the data recorded by means of a "small-dimension" lead system with the number of electrodes not exceeding that of the standardized electrocardiographic lead systems. In particular, 7 electrodes of the corrected orthogonal Frank lead system and 2 electrodes on the arms are used (Fig. 1). The potentials of all electrodes with respect to the right hand site are synchronously measured and then the eight unipolar signals acquired are used to synthesize, by means of well-known formulas [1], the three components of the cardiac dipole moment (heart vector), as well as the potentials of the individual Frank electrodes with respect to the Wilson central terminal.

The hardware of the system consists of a measuring-transmitting part weighing no more than 1 kg and designed in the form of a miniature unit placed on the patient (e.g., attached to his belt) and a receiving-recording part placed at any distance up to several tens of metres away from the patient and connected to a computer (Figs. 1, 2). The measuring-transmitting part involves, along with the pick-off electrodes, biopotential amplifiers, information transformer (including commutator, analog-digital converter, and coder), and transmitter of electromagnetic signal. The receiving-recording part includes a receiver of electromagnetic signal and decoder connected to a personal computer. A communication channel in infrared-frequency range or radio-frequency range can be used. The synchronous signals measured and entered into the computer are subjected to computational processing.

The method of data processing implemented in the system is a refined version of the dipole electrocardiotopography technique (DECARTO) proposed in our previous works for providing a proper trade-off between the demand for representation of the cardiac electric activity in an intelligible-pictorial topographic form with reference to anatomical landmarks of the heart surface, and the requirement for the minimal number of leads used for clinical diagnosis [2, 3]. The method uses a 3-orthogonal-lead system and simplified models of the cardioelectric generator adequately fitting the restricted measured data (three components of the heart dipole moment). These models (equivalent generators) have a surface-distributed configuration (for the ventricular depolarization period and conditions of injured myocardium) or a space-distributed configuration (for the ventricular repolarization period). In any case, the simple shape of the models, along with additional assumptions, correspond to the restricted available information on the true cardioelectric generator. The models are used to construct instantaneous and summary maps (decartograms) that display the main electrophysiological states and characteristics of the heart ventricles, along with the anatomical landmarks of the heart surface, in projection onto the spherical quasiepicardium (referred to as the image sphere).

Here, we consider in more detail the ventricular depolarization phase (QRS complex of the electrocardiographic cycle) during which the equivalent generator in the form of an uniform double layer with a circular edge is used. Then the regions of the heart wall possessing the following three basic electrophysiological states are projected onto a sphere enclosing the heart (image sphere): the state of resting (polarized) myocardium, or non-excitability region; the state of wall activation (corresponding to the depolarization front traversing the wall); and the state of completely excited (depolarized) myocardium after propagation of the depolarization front. The activation state corresponds to situation of the points inside of the circular depolarization front edge with the radius

$$a(t) = a_m \sqrt{\frac{D(t)}{D_m}},$$

where  $a_m$  is the radius of the image sphere,  $D(t)$  is the instantaneous magnitude of the heart dipole moment, and  $D_m$  is the maximal magnitude. The distribution of these regions on the image sphere at a given time instant is denoted instantaneous map (decartogram) of ventricular depolarization and is constructed as follows. The state of a particular point of the map at a time instant  $t$  is defined as the function

$$S_t = \begin{cases} \text{Rest, if the particular point acquires the resting state} \\ \text{Act, if the particular point acquires the activation state} \\ \text{Dep, if the particular point acquires the completely depolarized state} \end{cases}$$

depending on the previous state of the point and on the variable

$$F_t = \begin{cases} 0, & \text{if the particular point lies outside of the depolarization front} \\ 1, & \text{if the particular point lies inside of the depolarization front} \end{cases}$$

Each point of the map is supposed to be in the resting state before the onset of the QRS complex. Then the distribution of the electrophysiological states can be obtained from the following equations:

$$\text{if } F_t = 0 \text{ and } S_{t-\Delta t} = \text{Rest, then } S_t = \text{Rest}$$

$$\text{if } F_t = 0 \text{ and } S_{t-\Delta t} = \text{Act, or } S_{t-\Delta t} = \text{Dep, then } S_t = \text{Dep}$$

$$\text{if } F_t = 1 \text{ and } S_{t-\Delta t} = \text{Rest, or } S_{t-\Delta t} = \text{Act, or } S_{t-\Delta t} = \text{Dep, then } S_t = \text{Act,}$$

where  $t - \Delta t$  designates the previous time instant. The instantaneous decartograms of depolarization are drawn as maps of distributions of the aforementioned heart wall states on the image sphere cut along the right meridian and unrolled on a plane with retaining the elementary areas.

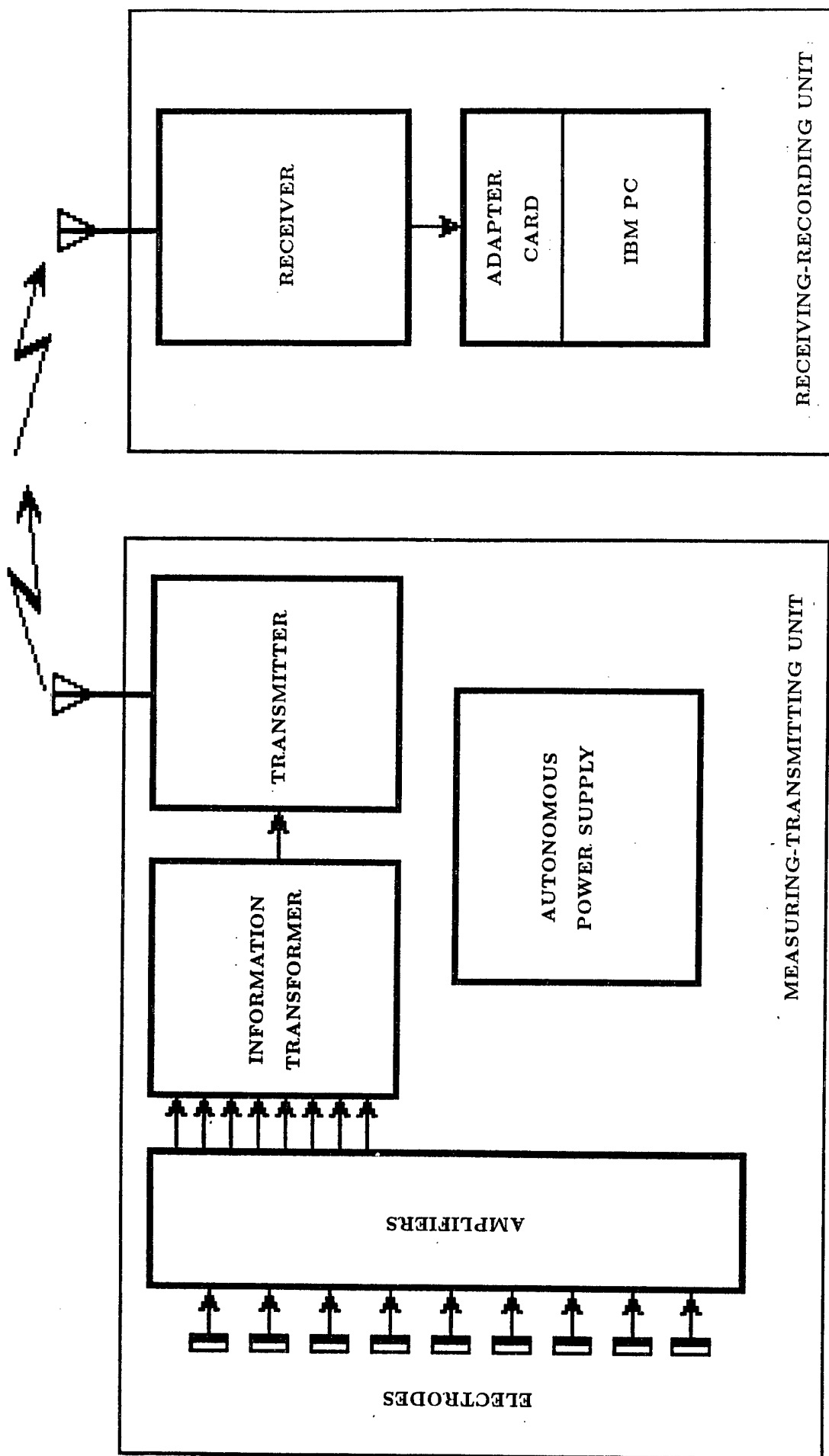
According to the properties of the total dipole moment of the cardioelectric generator, the heart vector measured by the corrected lead system does not depend, in essence, on the spatial position and translational movements of the generator elements. Therefore, for the depolarization period, the decartograms represent only the overall area and orientation, i.e. general size and direction of propagation of the depolarization front. However, the estimation of spatial position and movements of the cardioelectric generator is also of great importance for diagnostical interpretation of the data.

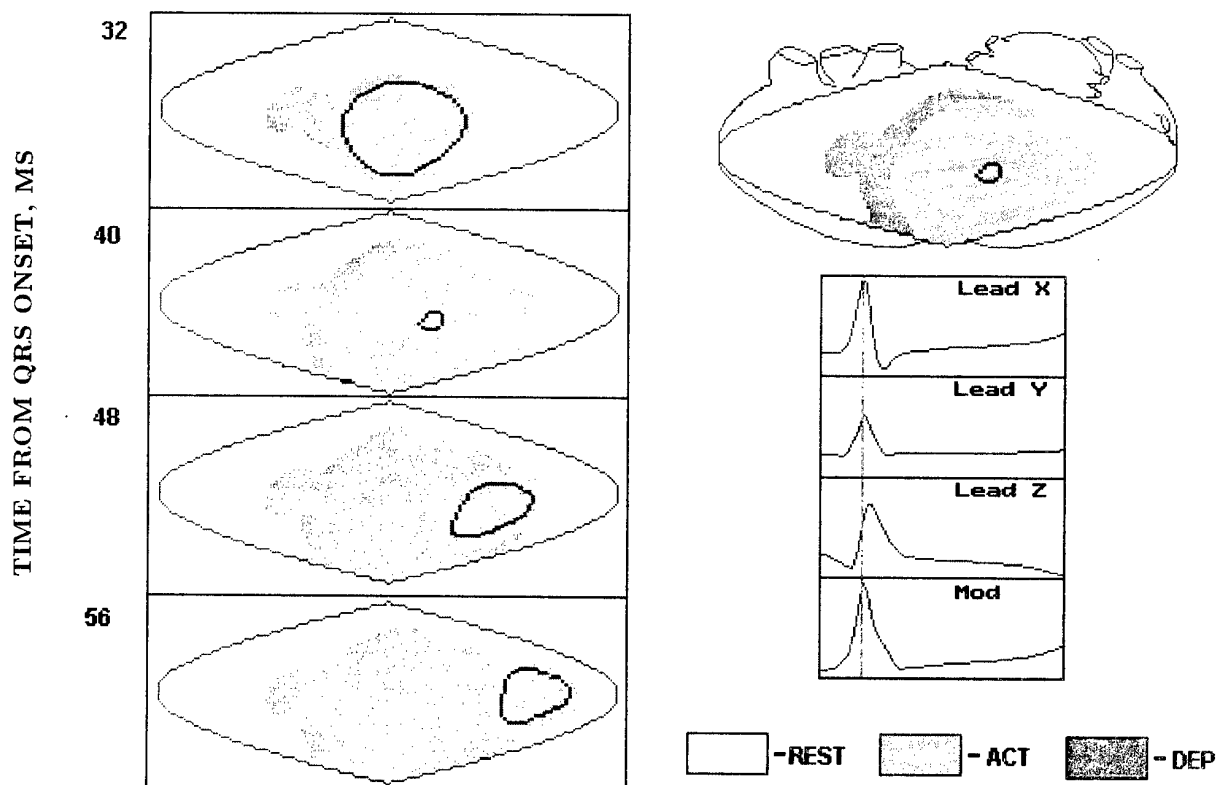
To solve this problem, we start from the following considerations. If an electric generator of arbitrary configuration, while with a significant predominance of the dipole constituent (which is typical for the cardioelectric generator), is situated within a confined region of a volume conductor, then it may be assumed, with practically acceptable accuracy, that the absolute value of the potential in the external region of measurement is inversely proportional to the distance between a midpoint of the generator (referred to as the electric center of the heart) and the measurement site. Applying this dependence to the body surface potentials, we can approximately estimate the position of the electric center at any time instant.

To implement the aforementioned approach on the basis of the Frank leads system, we use, as the initial data, not only the signals proportional to the heart vector components, but also the unipolar potentials of the individual Frank electrodes located on the diametrically opposite sides of the body (in particular, the left, right, anterior, posterior, inferior, and superior electrodes). From these potentials, the coordinates of the electric center in the standard Frank coordinate system  $xyz$  with the origin at the geometrical center of the mean transversal section of the chest are calculated for each time instant by the formulas

$$\begin{aligned} x_0 &= \frac{0.5a(\sqrt{|\varphi_A|} - \sqrt{|\varphi_I|})}{\sqrt{|\varphi_A|} + \sqrt{|\varphi_I|}}, \\ y_0 &= \frac{0.5c(\sqrt{|\varphi_F|} - \sqrt{|\varphi_H|})}{\sqrt{|\varphi_F|} + \sqrt{|\varphi_H|}}, \\ z_0 &= \frac{0.5b(\sqrt{|\varphi_M|} - \sqrt{|\varphi_E|})}{\sqrt{|\varphi_M|} + \sqrt{|\varphi_E|}}, \end{aligned}$$







**Fig. 3.** Instantaneous decartograms of a normal subject at four consecutive time instants of the middle part of the ventricular depolarization period (left) and anatomical relation of the map to the heart parts for time instant 40 mc indicated on the ECG curves by a vertical line (right). The heart is cut along the right-side meridional line and unrolled on the plane. The position of the plane containing the electric center is shown by a closed black curve

where  $\varphi_A$ ,  $\varphi_I$ ,  $\varphi_F$ ,  $\varphi_H$ ,  $\varphi_M$ , and  $\varphi_E$  are the unipolar potentials of the corresponding Frank electrodes;  $a$ ,  $c$ , and  $b$  are the principal sizes of the chest along the axes  $x$ ,  $y$ , and  $z$ , respectively. A plane which is perpendicular to the heart vector and passes through the electric center is indicated on the image sphere as the circle of intersection of this plane with the image sphere. Such an indication characterizing the instantaneous displacement of the electric center with respect to the geometric midpoint of the heart is of major interest here, inasmuch as the radial direction usually dominates for both the moments proper of the elementary dipole generators of the myocardium and their spatial movements.

An investigation on simple models of the generator (dipoles, uniform double layers of regular geometrical shape etc.) situated in an infinite or bounded volume conductor showed that the electric center determined by such a technique tends to fall within the neighbourhood of a "center of gravity" of the elementary electric generators of the myocardium. Moreover, in the event that the depolarization front is relatively small in size and monolithic in configuration, the electric center coincides with the geometric midpoint of the front and, hence, it characterizes the spatial position and movements of this front. The errors of determination of the electric center which are caused by the effect of actual inhomogeneity of the volume conductor and some other factors not taken into account are of a systematic nature, so that they can be neglected in estimating the relative movements of the electric center.

However, at some instants of the heart excitation cycle, the electric center determined by the method described above may come out of the heart region because of peculiar features of the actual generator configuration. Nevertheless, under such conditions the electric center displacement, as before, correlates with the dominating spatial position of the generator.

Our preliminary experimental studies on healthy subjects showed that the position of the electric center and its displacements during the ventricular depolarization period correspond to the major trends of the spatial propagation of the depolarization through the heart. This is illustrated by an example of the decartograms for an actual subject examined (Fig. 3).

Thus, the presented system offers the following favourable features: simplicity and convenience of practical execution of the measurement procedure; absence of the wires hindering movements of the patient; determination of a set of conceptually new characteristics that have a particular electrophysiological meaning; intelligible-pictorial representation of the data with attachment to the heart anatomy. The system provides new capabilities in the area of cardioelectric signal processing and diagnostic interpretation.\*

## References

1. Macfarlane P.W. (1989). In: Macfarlane P.W. and Lawrie T.D.V. (eds.) *Comprehensive Electrocardiology. Theory and Practice in Health and Disease*, Pergamon Press, New York etc. pp. 315–352.
2. Титомир Л.И., Руткай-Недецкий И. (1990). *Анализ ортогональной электрокардиограммы*. Москва. Наука.
3. Titomir L.I., Trunov V.G., Nagornov V.S., Danilushkin M.G., Shamarin V.M., Muromtseva G.A. (1995). In: Kneppo P., Tysler M. (eds.) *Model Based Biomeasurements*, CUPress, Bratislava, pp. 314–317.

---

\* This research is conducted within the research grant No. 96-01-01080 from the Russian Foundation of Fundamental Research.

**HIGH-RESOLUTION ELECTROCARDIOGRAPHY  
IN THE PROGNOSIS OF DEVELOPMENT AND EVALUATION  
OF PREVENTION EFFICIENCY OF LIFE THREATENING ARRHYTHMIA**

**Gennadii G.Ivanov, Leonid I.Titomir, Vladimir E.Dvornikov,  
Almas U.Eleuov, Serill Wambo**

Research Institute for Reanimatology, Russian Acad. Med. Sci.,  
Institute for Problems of Information Transmission, Russian Acad. Sci.,  
Faculty of Medicine, Russian University of People Friendship  
Moscow, Russia  
E-mail: ivanov@fbm.msu.ru

**Abstract.** Rapid development of the measuring instruments and computer technology during two last decades offers ample opportunity for improving automatized interpretation of the electrocardiographic signals, thereby overcoming the limited diagnostical possibilities of the standard electrocardiography. In particular, significant increase in accuracy of recognizing latent ischemic alterations and electric instability of the myocardium, disturbances of the depolarization and repolarization processes, is provided. The aim of this paper is to describe some results of investigation of the ventricular and atrial depolarization processes with the use of high-resolution electrocardiography (HR ECG) in patients with various pathological states, for estimating possibilities to predict electric instability of the myocardium and find new diagnostical indices. The population of subjects studied comprised 840 patients. The present study demonstrates that the HR ECG can be used for analysis of latent and small alterations of the myocardium electrophysiological properties, determination of zones with fractionated high-frequency activity, early diagnosis of electric instability of the myocardium and disturbance of the depolarization processes.

**Keywords.** Computerized diagnostics, high-resolution electrocardiography.

Numerous electrophysiological and pharmacological research works carried out under clinical conditions and on experimental animals during the past decades have allowed to expand our understanding of development mechanisms of arrhythmias hazardous for human life. The determining condition for the occurrence of fatal arrhythmias is considered to be the presence of a structural heart pathology which transforms into an unstable substratum under the influence of various functional factors. Such structural changes predetermining ventricular tachycardia (VT) may include myocardial infarction, hypertrophy and dilatation of ventricles, inflammation and oedema of myocardial tissue. In the opinion of many researchers, such changes constitute an anatomic substratum for the occurrence of VT with the interaction of different mechanisms. It is believed that various electrophysiological mechanisms, such as heightened automation and recurring input, take part in development and sustenance of VT. The presence of a long path of rotation of the impulse in this case is not obligatory, it would be sufficient to have a myocardium tissue of small diameter that had its electrophysiological properties altered as a result of acute ischemia of myocardium or heterogeneity of its structure caused by fibrous and necrotic changes.

Notwithstanding immense efforts put into the studies of the problem of electric instability of myocardium, reliable prognosis and preventive measures against sudden cardiac death (SCD), all of those are far from their final solution. This is explained by a set of theoretical, methodological, and practical factors. There is the necessity of further verification of structural causes of SCD having different origin in new born, young, aged,

and senile persons. These mechanisms should be analyzed in the cases of non-ischemic heart diseases, in sportsmen, and in the register of "inexplicable heart failure", where all subsequent methods of diagnostics reveal no pathology. Clarification of correlation between structural and functional disorders that made the basis of the modern model of development of arrhythmias hazardous for life is one of the main factors in solution of that problem.

Solution of the above problems is impossible without the most exhaustive information about the heart electric potential. This information allows expanding the diagnosis of pathological conditions of myocardium and its electrophysiological properties. Wide use of computer technologies and advanced methods of digital processing of the data have allowed to qualitatively improve acquisition and processing of electrocardiographic signals.

One of the achievements in this sphere is the method of high-resolution electrocardiography (HR ECG) which makes it possible to detect and analyze an important information invisible in the standard electrocardiogram. This method allows recording the low-amplitude high-frequency signals at the end of the QRS complex and at the beginning of the ST segment, the so-called late ventricular potentials (LVP). It has been established that the LVP reflect the existence of zones with retarded conduction in the ventricular myocardium. These zones can serve as anatomic-electrophysiological substratum of ventricular arrhythmias occurring on the basis of the re-entry mechanism.

The method under consideration was based on the recording 3 Frank orthogonal leads using low-noise amplifiers, accumulation and averaging of the signal, and its subsequent processing with an appropriate software. Various methodical approaches were suggested to analyze and qualitatively estimate LVP. Generally, they could be divided into two groups, the time domain analysis and spectrum domain analysis. The time domain analysis is widely used at present and serves as the "standard" method of discerning LVP. However, it has a number of serious drawbacks, in particular, a low prognostic value of positive result, lack of possibility to examine persons with His bundle branch blocks, and dependence of the examination result on the noise level.

Among the further developed processing methods which improve the analysis of the averaged ECG signal, one should note the methods of spectral analysis, as well as spectro-temporal mapping, which have certain advantages over the time domain analysis. However, these methods so far have not been sufficiently studied and there is no ample information to strictly confirm their clinical significance. The current research work including the spectro-temporal endocardial, epicardial, and intramural mapping are indicative of the necessity of frequency domain analysis of ECG signals over the whole QRS complex in a wide frequency spectrum.

In recent years, numerous works have been devoted to studying the possibility of using the HR ECG for prognosis of development of paroxysmal atrial fibrillation (PAF). To evaluate the prognosis of PAF, signs of late atrial potentials (LAP) are used. They are, similarly to LVP, low-amplitude high-frequency oscillations in the terminal part of the P wave. Therefore, there exists a number of different approaches to determining the electric instability of the atrial and ventricular myocardium.

Our research was aimed at studying the clinical significance of the time domain analysis and spectro-temporal mapping for estimation of the electric instability of atrial and ventricular myocardium in patients with various kinds of ischaemic heart disease and at assessment of how some pharmaceutical preparations affect the indices of LVP and LAP. A number of tasks were considered in the course of research: (1) to analyze prevalence and diagnostic value (sensitivity, specificity, and prognostic value, of the LVP determined

by the method of temporal analysis (Simpson method) in patients suffering angina pectoris and acute myocardial infarction; (2) to study properties of the spectro-temporal characteristics obtained from the spectro-temporal mapping of the whole QRS complex in patients with potentially dangerous and life threatening arrhythmias; (3) to analyze the distribution of frequency extrema and to compare spectro-temporal maps of the patients with post-infarction cardiosclerosis while without arrhythmias, and the maps of the patients with ventricular paroxysmal tachycardia; (4) to analyze parameters of LVP in patients having PAF, to assess the possibility of the method in prediction of their development, and to determine the frequency of LVP occurrence in the patients suffering PAF; (5) to study the impact of the Sotalol beta-blocker on the HR ECG parameters used for identification of the presence of LVP. In a series of observations, we examined 840 patients having ischemic heart disease, including those who underwent aortocoronary shunting, patients with ventricular arrhythmias and PAF, patients after various critical states, etc.

As it follows from the data obtained, among the patients having angina pectoris of II-IV functional classes, the LVP were recorded in 3 cases (15%), while among the patients having acute myocardial infarction the LVP were recorded in 30 to 35% of cases. The Pearson association coefficient for LVP and potentially dangerous arrhythmias was  $r = 0.003$  ( $p > 0.05$ ), and for LVP and unstable VT it was  $r = 0.055$  ( $p > 0.05$ ). Thus, the patients with unstable angina pectoris have a low incidence of potentially dangerous arrhythmias and signs of LVP.

The analysis of data in the myocardial infarction group showed that the greatest variations were observed in the patients with combined myocardial infarction.

The presence of LVP was correlated with various versions of the clinical course of acute myocardial infarction. It was established that among 87 patients with myocardial infarction, 13 patients (15%) had acute left ventricular insufficiency, 11 of the latter group (85%) had signs of LVP. The recurrent pain syndrome and/or recurrent course of the myocardial infarction was observed in 10 cases (11%), while in 80% of these cases signs of LVP were detected.

In the group of patients having the unstable VT, the signs of LVP were observed in 59% of the patients with postinfarction cardiosclerosis, in 32% of the patients with idiopathic forms of arrhythmia, and in 37% of the patients with "short term" VT. An evaluation of the parameters of the total spectral density (TSD) showed a decrease of their values. This was manifested in increasing the ratio of the low-frequency and high-frequency components. Particularly, in the group of patients with LVP having postinfarction cardiosclerosis this ratio amounted  $12 \pm 3$ , in patients having the idiopathic form of the unstable VT it was  $10 \pm 3$ , and in patients with "short term" it was  $8 \pm 2$ .

The analysis of the spectro-temporal characteristics of the QRS complex showed that there can usually be distinguished three frequency peaks in the control group: the 1st peak in the range 8-16 Hz, the 2nd peak in the range 32-65 Hz, and the 3rd peak in the range 70-110 Hz. In the patients with rheumatic heart disease and ischemic heart disease during the postoperational period, it was found that the decrease in TSD was accompanied by a decrease in the values of the 2nd peak. To a lesser degree, similar changes held true for the 3rd peak. In the patients with unstable VT, a certain increase of the frequency values of the 2nd peak and its shift towards the end of the QRS complex were observed. It is important to note a tendency for reduction of the mean values of the extremum in the range 20-60 Hz in the group with VT and an increase of these values up to  $58 \pm 5$   $\mu$ V in the group with postinfarction cardiosclerosis without arrhythmia.

In the group with VT and LVP, we observed 3 to 5 basic modulations of the frequency maximum patterns which were distributed in a much wider time range compared with the control group and the group without VT, namely in the interval between 30 and 70 ms. Moreover, in the interval 40–65 Hz, there were 3 distinctive patterns of maxima. In addition, a distinctive pattern of frequency maxima was observed at the beginning of the QRS complex.

One part of this research was a study of the influence of Sotalol beta-blocker in an individually prescribed dosage on the indices of time analysis of HR ECG. A preliminary analysis of the results showed that in the complex therapy of unstable angina pectoris, there were significant changes of the HR ECG parameters on the background of the therapy including Sotalol. Also, a distinctive tendency to an increase in duration of the filtered QRS complex and decrease in the amplitude values during the last 40 ms of the QRS complex was observed. It is important to note that such changes were accompanied by a decrease in values of the total spectral density of the filtered signal.

Thus, the research has shown that the HR ECG can be used for analysis of latent and insignificant changes of electrophysiological properties of the myocardium, for detecting the zones of fractionated high-frequency activity and disruption of the front of the excitation wave. Of importance is the detection of distinctions between the distribution patterns discerned by the spectro-temporal analysis of the frequency maxima during the QRS complex in the patients after myocardial infarction without arrhythmias and with the paroxysms of VT. It has been shown that to detect the patients with a high risk of development of life threatening arrhythmias, it is important to combine the time domain analysis, index of the normality, and parameters of spectro-temporal mapping. Values of LAP separating groups of patients with PAF and a group of healthy persons have been determined. Possibilities to use the method for estimation of the effect of antiarrhythmic preparations have been confirmed.

As can be seen from the literature and the results of our research, there is a great number of different approaches to determination of the electric instability of the atrial and ventricular myocardium, among which the method of HR ECG are rather important. Moreover, it is obvious that to have reliable prognosis, one should use a complex approach to estimation of the electric instability of the myocardium, including, in particular, the detection of the morphological substratum (late potentials) and modulating factors (state of vegetative regulation of the heart function and disturbances of the neurohumoral regulation), as well as evaluation of possible trigger mechanisms (extrasystoles) provoking the life threatening arrhythmias and SCD. Of some interest in future will be the active use of exercise and functional tests to clarify the role of the methods considered in prognosis of SCD and in estimation of the efficiency of therapy applied.

# ABOUT THE SOLUTION OF THE REVERSE ELECTROCARDIOGRAPHIC PROBLEM

Galina A. Kuteeva  
St. Petersburg State University  
Mathematics and Mechanics Department  
2 Bibliotechnaya sq.  
St. Petersburg 198904, Russia  
e-mail: star@kga.stud.pu.ru

**Abstract.** The reverse electrocardiographic problem is the problem where E.M.F. (electromotive force) of heart is defined by given values of electric potential at points on border of a human body. In this paper the approximate plane problem is discussed. Analog of a human body is external circle. A heart is presented as a circle with displaced center from a center of external circle. At the beginning assume that potential on a boundary of external circle is given as continuous function (it is "the continuous problem"). Then this potential is given as discrete function (it is "the discrete problem"). Number of points in which the electric potentials must be given for discrete case is defined the difference between the solutions of the first ("continuous") and the second ("discrete") problems.

**Keywords.** Heart E.M.F., electric potential, number of points, conformal mapping, eccentric and concentric circles.

Number of works is devoted to the consideration of the problems connected with mathematical investigation of electrocardiological questions. The International congresses on electrocardiology are held (for example, in 1994 the XXI International Congress on Electrocardiology was held in Japan). In this paper the plane electrocardiographic problem is solved. A heart is modeled as a circle with radius  $r_1$ . A human body is modeled as an external circle with radius  $r_2$  ( $r_1 < r_2$ ). These circles are eccentric circles. The distance  $a$  is the value of which the center of internal circle is displaced from the center of external circle. The function  $F$  presents the given potential on the boundary of external circle. It may be continuous or discrete. With the help of conformal mapping this problem with eccentric circles transforms into the problem with concentric circles of radiuses  $R_1, R_2$  ( $R_1 < R_2$ ).

Eccentric circles are situated in "physical" plane  $z$ . The origin of coordinates is the center of external circle. These circles transform into concentric circles in plane  $w$ . The electric potentials  $u_i$  in plane  $z$  (where  $i = 1$  for internal circle,  $i = 2$  for the ring between the circumferences) are harmonic functions [1]. In this way functions  $u_i$  satisfy Laplace equation

$$\Delta u_i(r, \psi) = 0, \quad i = 1, 2 \quad (1)$$

and boundary conditions

$$\begin{aligned} \frac{1}{\rho_1} \frac{\partial u_1}{\partial n} &= \frac{1}{\rho_2} \frac{\partial u_2}{\partial n}, & (r = r_1), \\ u_2(r, \psi) &= F(\psi), & (r = r_2), \end{aligned} \quad (2)$$

where  $\Delta$  is Laplace's operator;  $(r, \psi)$  are polar coordinates in plane  $z$ ;  $\rho_i$  is specific resistance inside of  $i$ -th domain ( $i = 1, 2$ );  $n$  is exterior normal with respect to the internal circle;  $F(\psi)$  is given electric potential on the boundary of external circle. The potential in plane  $w$  for the internal circle is denoted by  $w_1$ , for the concentric ring is denoted by  $w_2$ . Coordinates of a point in plane  $w$  are  $(R, \varphi)$ , where  $\varphi = \text{Arg}(w)$ ,  $R = |w|$ .

The conformal mapping [2]

$$w = \frac{sz - 1}{z - s} \quad (3)$$

transforms the domain with eccentric circles ( $r_2 = 1$ ) into the domain with concentric circles ( $R_2 = 1$ ), here

$$s = \left(1 + x_1 x_2 + \sqrt{(1 - x_1^2)(1 - x_2^2)}\right) / (x_1 + x_2), \quad (4)$$

$$\begin{aligned} x_1 - x_2 &= 2r_1, \\ x_2 + r_1 &= a. \end{aligned} \quad (5)$$

After this transformation the radius of internal circle in plane  $w$  is

$$R_1 = (x_1 - x_2) / \left(1 - x_1 x_2 + \sqrt{(1 - x_1^2)(1 - x_2^2)}\right). \quad (6)$$

Without losing generality suppose that the function  $F(\varphi)$  defining the given potential on the boundary of external circle for plane  $w$  is even function. Then the solutions of Laplace equation can be searched as

$$\begin{aligned} w_1(R, \varphi) &= a_0 R + \sum_{j=1}^n a_j R^{j+1} \cos j\varphi, & 0 \leq R \leq R_1, \\ w_2(R, \varphi) &= b_0 R + \sum_{j=1}^n b_j R^{j+1} \cos j\varphi, & R_1 \leq R \leq R_2. \end{aligned} \quad (7)$$

Boundary conditions allow to find coefficients  $a_j, b_j$ :

$$\begin{aligned} b_0 &= \frac{1}{2\pi} \int_{-\pi}^{\pi} F(t) dt, \\ b_j &= \frac{1}{\pi} \int_{-\pi}^{\pi} F(t) \cos j\varphi dt, & j = 1, \dots, n, \\ a_0 &= k b_0, \\ a_j &= k b_j, & j = 1, \dots, n, \end{aligned} \quad (8)$$

where

$$k = \rho_1 / \rho_2.$$

The function of E.M.F. on the boundary of the internal circle is found from the relation

$$E(\varphi) = w_2(R_1, \varphi) - w_1(R_1, \varphi). \quad (9)$$

Let us consider the discrete function  $F(\varphi)$ . This function is given at points  $\varphi_i$  ( $i = 0, \dots, m$ ). Potentials of electric field  $w_1^*$ ,  $w_2^*$  will be searched as

$$\begin{aligned} w_1^*(R, \varphi) &= a_0^* R + \sum_{j=1}^n a_j^* R^{j+1} \cos j\varphi, & 0 \leq R \leq R_1, \\ w_2^*(R, \varphi) &= b_0^* R + \sum_{j=1}^n b_j^* R^{j+1} \cos j\varphi, & R_1 \leq R \leq R_2, \end{aligned} \quad (10)$$

where  $w_1^*$ ,  $w_2^*$  for discrete function  $F(\varphi)$  are analogs of  $w_1$ ,  $w_2$  for continuous function  $F(\varphi)$ ;  $a_j^*$ ,  $b_j^*$  are analogs of  $a_j$ ,  $b_j$  in formulae (7).

The next boundary conditions are satisfied

$$\frac{1}{\rho_1} \frac{\partial w_1^*}{\partial R} = \frac{1}{\rho_2} \frac{\partial w_2^*}{\partial R}, \quad (R = R_1), \quad (11)$$

$$w_2^*(R, \varphi_i) = F(\varphi_i), \quad i = 0, \dots, m, \quad (R = R_2). \quad (12)$$

The relation (12) is the system of equations with respect to  $b_j$ :

$$AD = W, \quad (13)$$

where

$$A = \begin{pmatrix} 1 & \cos \varphi_0 & \cos 2\varphi_0 & \cos 3\varphi_0 & \dots & \cos n\varphi_0 \\ 1 & \cos \varphi_1 & \cos 2\varphi_1 & \cos 3\varphi_1 & \dots & \cos n\varphi_1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \cos \varphi_m & \cos 2\varphi_m & \cos 3\varphi_m & \dots & \cos n\varphi_m \end{pmatrix}, \quad (14)$$

$$D = (b_0^*, b_1^*, \dots, b_n^*)^T, \quad W = (F(\varphi_0), F(\varphi_1), \dots, F(\varphi_m))^T. \quad (15)$$

Let us consider  $m = n$ . Points  $\varphi_i$  have to be chosen so that the determinant of system (13) is not equal to zero  $\det A \neq 0$ . Then after substituting  $b_j^*$  into the second relation of formulae (10) the value of potential  $w_2^*$  has been obtained. The potential  $w_1^*$  has been found using formulae (10) and boundary condition (11).

The function of E.M.F. on the boundary of internal circle is defined as

$$E^*(\varphi) = w_2^*(R_1, \varphi) - w_1^*(R_1, \varphi). \quad (16)$$

The next example with the parabolic function  $F(\varphi)$  is analysed:

$$F(\varphi) = \varphi^2 - \pi^2. \quad (17)$$

Points  $\varphi_i$  ( $i = 0, \dots, m$ ) are assumed as equidistant between 0 and  $\pi$ , i.e.

$$\varphi_i = i \frac{\pi}{m}, \quad i = 0, \dots, m. \quad (18)$$

It is important that points  $\varphi_i$  are situated so that  $\varphi_i \in [0, \pi]$ .

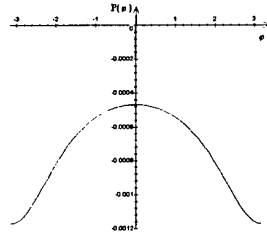


Fig. 1  $k = 0.2$ ;  $m = 20$ ;  $R_1 = 0.225148$ ;  $R_2 = 1$ .

The following difference is considered

$$P(\varphi) = E(\varphi) - E^*(\varphi) = \sum_{j=0}^n (b_j - b_j^*) (1 - k) R_1^{j+1} \cos j\varphi. \quad (19)$$

Let us introduce the notation  $S = \sum_{j=0}^n (b_j - b_j^*)^2$ . The coefficients  $b_j^*$  are chosen (by changing the number  $m$ ) so that  $S \leq 2 \cdot 10^{-3}$ . The necessary number of points  $\varphi_i$  ( $i = 0, \dots, m$ ) is 21. The function  $P(\varphi)$  for this case is shown in Fig 1. It is noted that  $|P(\varphi)| \leq 1, 2 \cdot 10^{-3}$ .

The reverse conformal mapping is

$$z = g(w) = \frac{1 - s w}{s - w}. \quad (20)$$

After this transformation points  $\varphi_j$  ( $R_2 = 1$ ) in plane  $w$  transform into points  $\psi_j$  ( $r_2 = 1$ ) in plane  $z$ :

$$\psi_j = -I \operatorname{Ln} \left( \frac{1 - s e^{I(j\pi)/m}}{s - e^{I(j\pi)/m}} \right), \quad I^2 = -1, \quad j = 0, \dots, m. \quad (21)$$

If points  $\varphi_i$  in plane  $w$  are equidistant points then they aren't equidistant points  $\psi_i$  in plane  $z$ . And opposite, if points  $\psi_i$  are equidistant points in plane  $z$  they aren't equidistant points  $\varphi_i$  in plane  $w$ . The function  $F$  is transformed by the similar way:

$$F(\psi) = F \left( \operatorname{Arg} \left( \frac{1 - s w}{s - w} \right) \right). \quad (22)$$

The difference  $P(\psi)$  obtaining after the transformation (20) is shown in Fig. 2.

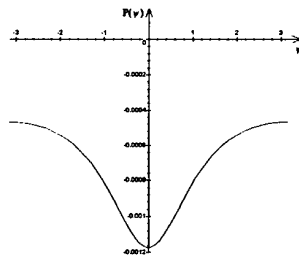


Fig. 2  $a = 1/2$ ;  $r = 1/6$ ;  $r_2 = 1$ ;  $x_1 = 2/3$ ;  $x_2 = 1/3$ .

**Remark.** There are conformal mappings with the help of which the double-connected domains are transformed into the interior of concentric ring [3]. This statement permits to conform more complicated domains modelling a heart and human body (for example domains may be the combination of a circle and an ellipse or two ellipses).

### References

1. Tamm I.E. (1966) The basis of the electricity. Nauka, Moscow. (in Russian)
2. Lavrentyev, M.A., B.V. Shabat (1973) Methods of the theory of complex variable. Nauka, Moscow. (in Russian)
3. Volkovysky, L.I., G.L. Lunts, I.G. Aramovich (1970) The problem book of the theory of complex variable. Nauka, Moscow. (in Russian)

# CONTROL SIGNALS FOR BIOLOGY PROCESSES.

R.I.Polonnikov

St.Petersburg Institute for Informatics and Automation of the Russian Academy of Sciences,  
e-mail: polon@mail.iias.spb.su

**Abstract.** This paper presents investigation of the methods of synthesis of the flicker-noise like signals on the base of Fibonacci sequence and Weiershtrass function. The power spectra of such signals which was calculated with the help of Fourier transformation is analysed. A simple method of control of the fine structure of the signal is suggested, which is based on the manipulation with the parameters of the initial signal.

**Keywords.** Modulation, spectrum, Fourier transformation, Weiershtrass function, flicker-noise like signals, cosine fractal function.

**Introduction.** It is known [1], that electromagnetic fields (EMF) of the technogen origin have the effect on the biology objects including human beings. The result of such effect strangle depends on the mode of EMF modulation. The membrane of the cell and the fourth phase of the water as it exists in the cell - the quasi crystal, are non-linear devices referring the external EMF. They posses for example the detection ability. If technogen EMF is modulated so that after passing the quadrate detector it will have power spectrum in the range of infra low frequencies ( 0-100 Hz) with the power spectrum bending curve strongly differing from the low  $f^{-\beta}$ , where  $f$ -frequency,  $\beta$ - power index, which takes the values in the range  $0.8 < \beta < 1.6$ , that may cause strong dysfunction in living beings performance, free the active untied radicals. The last effects the DNA and RNA in the same manner as radiation does and may cause very negative effect in the following up to genotype decrease. The danger of such effect is increased by the fact that the named consequences of such effect are realised even for the intensities of EMF under those which are set as save by safety regulations.

On the other hand it is known [2] that living beings and their cells the process of evolution have perfectly adopted to natural EMF of the Earth. These fields have a fracture structure and their power spectra in the range of infra low EMF follows the low of  $f^{-\beta}$  and they posses scale invariance. More then that, the artificial weak EMF of the infra low range, which reproduce the structure of spectrum of the natural EMF have the therapy effect and are used nowadays for treatment of some heavy diseases [2]. That is why synthesis of the signals with a structure close to flicker-noise is important.

**Fibonacci signals.** Let us call the signal Fibonacci if it has one impulse on the segment  $[0,T]$ , which has the front and rear edges formed by the segment of Fibonacci sequence. To form the front edge a natural direct sequence is used and an inverse sequence is used for the rear edge. Let us set that the segment  $[0,T]$  includes  $n=2^m$  counts, the  $q$  of which are making impulse itself and the rest  $(n-q)$  are zero. Let us analyse the impulse signal with the front edge formed by the first Fibonacci numbers  $u_k$ :

$$u_k = \frac{2^{-k}((1+\sqrt{5})^k - (1-\sqrt{5})^k)}{\sqrt{5}} ; \quad k=1,8 \quad (1)$$

That means that vector  $u^T = [1, 1, 2, 3, 5, 8, 13, 21]$  determines impulse front edge. The rear edge may be set by vector  $y^T = [13, 8, 5, 3, 2, 1, 1]$ . Let us set operator  $\text{stack}(u, y)$  so that it forms vector by combining vectors  $u$  and  $y$  so that vector  $y$  is located under vector  $u$ . In this case vector  $t_1^T = [1, 1, 2, 3, 5, 8, 13, 21, 13, 8, 5, 3, 2, 1, 1]$ , where  $t_1 = \text{stack}(u, y)$  is Fibonacci impulse with amplitude equal to 21. Let us combine the zero vector  $r_{1i} = 0$  ( $i = 0..111$ ) using stack operation with vector  $t_1$ . This gives the vector of the searched Fibonacci signal formed by Fibonacci impulse  $t_1$  and pause  $r_1$ . Let us analyse spectrum of such signals. Each subsequent signal let form from the preceding one by adding one more Fibonacci number. So the next signal will contain new Fibonacci impulse with the increased amplitude and duration:  $t_2^T = [1, 1, 2, 3, 5, 8, 13, 21, 34, 21, 13, 8, 5, 3, 2, 1, 1]$ . The length of the segment  $[0, T]$  is left unchanged  $n = 128$ . That means that dimension of vector  $r_2$  will decrease ( $i = 0..109$ ) and analysed second Fibonacci signal will be  $e_2 = \text{stack}(t_2, r_2)$ . The spectrum  $A$  of signals  $e$  were determined with the help of fast Fourier transformation (operator FFT):

$$A = \text{FFT}(e) \quad (2)$$

After that the spectrum density of power was determined:

$$B_j = |A_j|^2 \quad (3)$$

The low frequency part of the relation (3) was approximated by the functions of the type of:

$$B_j = K \cdot (j)^{-\beta}, \quad j = 2..10 \quad (4)$$

And by means of non-linear regression the estimations for  $K^*$ ,  $\beta^*$  were found. Results are summarised in Table 1.

Table 1

Amplitude of pulse	$K^*$	$\beta^*$	Maximum of approximation error	s	$K^* - K^*_{s-1}$
21	0.738	0.613	0.06	1	-
34	1.981	0.634	0.15	2	2.68
55	5.255	0.646	0.37	3	2.65
89	13.836	0.651	0.95	4	2.63
144	36.293	0.653	2.43	5	2.62
233	95.032	0.654	6.26	6	2.618

It can be seen from Table 1, that with the accuracy of calculations the analysed Fibonacci signals possess the property of scale invariance, so they may be qualified as fractal. By changing the duration of the segment  $[0, T]$ , in our case value of  $m$  ( $n = 2^m$ ) and saving the same amount of counts, forming the front and rear edges of Fibonacci impulse, it is possible to control the values of  $K^*$ ,  $\beta^*$ . This is demonstrated in Table 2.

Table 2.

Amplitude	m	$K^*$	$\beta^*$	Maximum of approximation error
21	6	3.94	1.34	0.185
21	7	0.738	0.613	0.059
21	8	0.137	0.194	0.007

It was said above that the signals of the presented type may have a positive effect on the living beings. Taking into consideration a simplicity of realisation of the Fibonacci signals it is possible to hope for there wide application. It is necessary to mention that the fractal structure of biology processes may be formed due to dynamic or noise mechanism. This makes additional reason for simulation of flicker-noise like signals.

**Flicker-noiselike signals.** The character of the power spectrum on the infra low frequencies  $f$  in accord with the law  $f^{-\beta}$  ( $0.8 < \beta < 1.4$ ) proves that described above signals may be classified as flicker-noise. There is a lot of systems around us in which the processes controlled by flicker-noise signals play an important role. Some of the examples are: electric current in carbon resistor, semiconductor, in contact of two metals, vacuum discharge, Ziner diode (stabilitron), bipolar or polar transistor, voltage on the thermocouple, accumulator, frequency in quartz oscillatory counter, Earth rotation frequency if it is averaged on the five day interval, the level of sound of human speech, potential on the nerve cell membrane and plenty other cases. At the same time there is a significant interest in mathematical simulation of such signals. There are not many models of flicker-noise known at this moment [3]. A well known is the model of change laminar flow of the process ( with smooth increase) and arbitrary break to chaos. But here we have used an other approach which is based on the combination of the Fibonacci signal with the signal in the form of modified Weiershtrass function.

The Weiershtrass function [4] is a rather interesting noise-like object from our point of view:

$$f(x) = \sum_{i=1}^{\infty} a^i \cos(b^i \pi x) \quad (5)$$

This function is invariant referring co-ordinate system transformation ( the zoom in by “ $b$ ” times along the  $x$ -axis and by “ $a^{-1}$ ” times along  $Y$ -axis) which means that it is a fractal . In addition to this the raw (5) generates a strange attractor. The noiselike form of the  $f_n$  function enables to synthesised a signal in a form of Fibonacci impulse on the basis of artificial noise  $f_n$ . Analytically it may be expressed as the sum of two vectors:  $D = e + c$ , where  $c_n = 1 + f_n$ ,  $n = 1..128$  and  $f_n$  is a modified Weiershtrass function, such, that

$$f_n = \sum_{i=1}^r a^i \cos(b^i \pi n) , \quad (6)$$

where  $r \neq \infty$ . Such signal  $D$  may be called flicker-noise-like, as in the low frequency range it has power spectrum bending curve close to that determined by (4) and in the time scale it contains a strange attractor. A significant advantage for the practical usage of such signals is the simplicity of control of the spectrum. It is enough to realise a very low control effect changing parameters  $a$  and  $b$  from (6) in the second digit after the decimal point, to cause some changes in the form of power spectrum bending curve  $W_j$ , where

$$W_j = |Q_j|^2, \quad (7)$$

$$Q = \text{FFT}(D) \quad (8)$$

These changes in the spectrum cause a corresponding response in the approximation parameter  $\beta$  and are clearly traced in the matrix  $\beta$  and on the Fig.1. The elements  $\beta_{ij}$  of matrix  $\beta$  are the functions of parameters of the modified Weiershtrass function, used in formation of

signal D:  $\beta_{ij} = \varphi(a_i, b_j)$ , and  $a_i = 0.89 + 0.01 \cdot i$ ,  $i = 0 \dots 9$ ;  $b_j \in [1.59, 1.69, 1.79, 1.8]$ , at  $j=0 \dots 3$ . Matrix  $\beta$  is presented below:

0.643	0.58	0.649	0.399
0.647	0.58	0.661	0.396
0.653	0.581	0.676	0.393
0.66	0.583	0.693	0.391
0.67	0.585	0.714	0.389
0.683	0.587	0.739	0.388
0.699	0.59	0.768	0.387
0.717	0.593	0.801	0.387
0.74	0.596	0.84	0.386
0.767	0.597	0.884	0.386

The 3D diagram of in Fig.1 presents a surface of matrix  $\beta$ . Via X-axis  $a_i$  variable is plotted, Y-axis - variable  $b_j$  and via Z-axis the values of function  $\beta_{ij}(a_i, b_j)$  are plotted. The diagram demonstrates a rather complicated surface which additionally emphasises the advantages of the presented method, which may be summarised as follows:

- the digital approach provides a full and exact reproducibility of the synthesis of a signal processing the necessary properties;
- a possibility of detailed search and efficient visualisation of the results of this search by means of calculation of matrix  $b$  in the vicinity of point  $(a_i, b_j)$ ;
- an efficiency of control of the form of the power spectrum bounding curve of the synthesised signal.

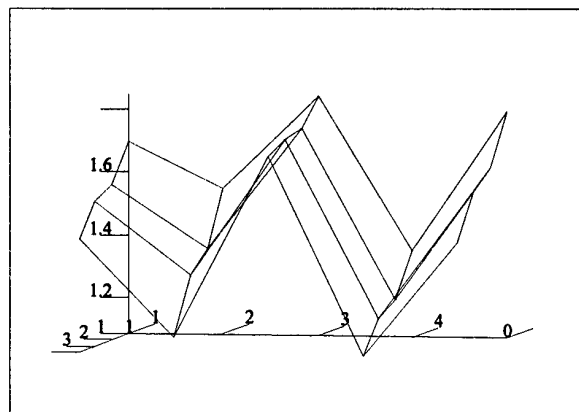


Fig. 1

**The signals on the base of cosine fractal function on Weiershtrass-Mandelbrot.** This function is analysed in some of the publications [5]. It is commonly set in a form

$$C(t) = \sum_{n=-\infty}^{\infty} (1 - \cos b^n t_i) / b^{(2-D)n} \quad (9)$$

At  $D \rightarrow 2$ ,  $C(t)$  has strong fluctuations, which resemble a noise in electric chains.  $C(t)$  matches the relation of uniformity:

$$C(bt) = b^{2-D} C(t) \quad (10)$$

As it is mentioned in [5]  $C(t)$  is not selfsimilar, but it is selfaffine. In the practice of computer simulation we have used a modified cosine fractal function of Weiershtrass-Mandelbrot  $F(t)$ , which was formed in the following way:

$$q(t_i) = \sum_{n=-50}^{50} (1 - \cos b^n t_i) / b^{(2-D)n}, \quad (11)$$

where  $t_i = i / 256$ ,  $i=0...255$ ,

$$F(t_i) = F_i = \alpha - q(t_i), \quad (12)$$

where  $\alpha = \text{const} > 0$ . After that was determined a spectrum of  $F$  function:

$$g = \text{FFT}(F) \quad (13)$$

and the corresponding power spectrum:

$$H_j = (|g_j|)^2, \quad j = 0...127. \quad (14)$$

After that the approximation of power spectrum was repeated in the range of harmonicas adjacent to zero harmonic ( from second to 15-th) with the help of function of the type of (4) by minimising a functional:

$$\text{SSE}(K, \beta) = \sum_{p=2}^{15} (H_p - U(p, K, \beta))^2, \quad (15)$$

where  $U(p, K, \beta) = K p^{-\beta}$ . The problem (15) was solved several times for different values of parameters  $a, b, D$  and for  $\alpha=18$ ,  $b \in [1.3 \ 1.4 \ 1.43 \ 1.48 \ 1.5]$ ,  $D \in [1.75 \ 1.83 \ 1.85 \ 1.9]$   $\beta$  matrix was determined and a surface corresponding to this matrix was plotted ( Fig.2)

$$\beta = \begin{bmatrix} 1.5; 1.35; 1.65; 1.16; 1.61 \\ 1.38; 1.18; 1.57; 1.03; 1.45 \\ 1.35; 1.12; 1.55; 0.99; 1.4 \\ 1.25; 0.94; 1.52; 0.89; 1.25 \end{bmatrix}$$

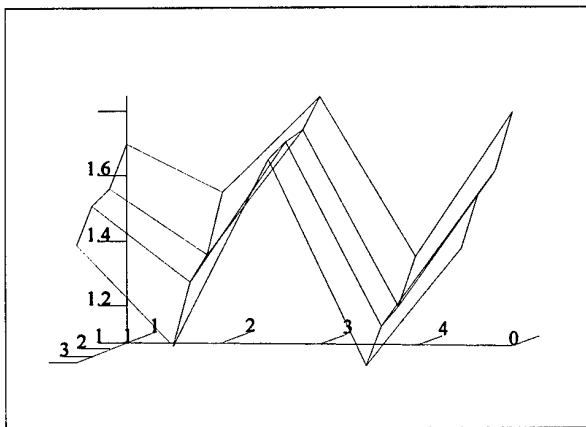


Fig.2.

**Conclusions.** In all these cases it is possible to held an efficient control of the form of power spectrum bending curve of the synthesised signal, used for therapy purposes.

### ***References.***

1. Kontorov D.S., Kontorov M.D., Sloka V.K. Radiocomputer science. The Radio and Connection .1993, 294 p.
2. Biomedical computer science and eniology (under edition R.I.Polonnikov and K.G.Korotkov) St.Petersburg. "Olga", 1995, 236p.
3. Shuster G. Determined chaos.. The World, p.95-102.
4. Kurdyumov S.P., Malinetsky G.G., Potapov A.B. Non-stationary structures, dynamic chaos, cage automatic devices. // New in sinergetice. Riddles of the world of nonequilibrium structures. |. Science, p.145.
5. Feder J. Fractals . Peace. 1991, p.35-38.

# REGISTRATION AND PROCESSING OF THE DIGITAL INFORMATION IN BIOTECHNICAL SYSTEM BY COMPUTER ENCEPHALOGRAPHS

R.I.Polonnikov, V.P.Kobrin

St.Petersburg Institute for Informatics and Automation of the Russian Academy of Sciences,  
I.I.Mechnikov St.Petersburg State Medical Academy, Russia  
e-mail: polon@mail.iias.spb.su

**Abstract.** The description new methodology of registration and processing encephalograms is given by transfer of the digital information with the help of biotechnical systems. The description of organization of experiment, the used equipment and received results is resulted also.

**Keywords.** Computer encephalographic, Baker's code, demodulation, decoding, non-linear filtration, correctness code, estimation, receiving of code.

**Introduction.** In 1992 new methodology for study and further utilization of the information transfer processes along the extrasensory channel of communication was developed by Prof. R.I.Polonnikov. Its novelty consists in the following: it does not use the total knowledge of a physical nature of the communication channel, mental-mode of both inductor and percipient do not have any information contained in the transmitted message; it is important for process of message transfer to have only the fact of their presence or absence, inductor and percipient have no information about the contents of message before and after of communication session, the message is coded by the third person before the start of communication and the received code operates the alarm device influencing the inductor. The experiments were made in the Laboratory of the Chair of Pathological Physiology of I.I.Mechnikov St.Petersburg State Medical Academy. Registration of encephalograms both of inductor and percipient were made continuously and synchronously with the help of two computer encephalographs «Encefalan 131-01». The received values of spectral density of capacity of signal EEG were then exposed to special processing, which included: nonlinear filtration, demodulation, decoding, calculation of probabilities of correct reception of symbols of systematic (n,k)-code and code word as a whole. Distance between inductor and percipient was about 15 km. The probability of correct reception of a code word was in the interval from 0.74 to 0.99 in 75% of cases.

The first person forms a message which is necessary to transfer. For example, that can be any sentence, which serves as input for computer. In computer this message is coded into the binary symbols according to the code which is described below. Moreover, computer has information about Moscow time of the start of work which is agreed with receiving side. Code of message begins to control modulating coding device in the prescribed moment of time. The simplest realization of that may be a lamp. Zero symbol corresponds to lamp switched on and unit - to lamp off. Zero symbol is interpreted by the inductor as a signal to start his normal work. If the extrasensor is a healer he is trying to help his patient mentally in this regime. If the lamp is off extrasensor does not work and relaxes. These operations of extrasensor (regime on-off) reproduce the code of message, that influence the change of power spectrum of his encephalogram and, if channel of communication is functioning, of power spectrum of

percipient encephalogram as well. The description of algorithms of the primary and secondary processing of encephalogram signals of inductor and percipient is in the following part.

**Experiment description and used equipment.** The experiments were held in Laboratory of the Chair of Pathological Physiology of I.I. Mechnikov St. Petersburg State Medical Academy with participation of Dr. V.P. Kobrin. Before and after experiment all tested persons had to pass a psychological test according to common method used in this laboratory. The test included the color test of Lusher in its classic version of computer psychological testing (SMIL, Kattel and etc.), the count of the Internal time. Computer psychological testing was preceded by the registration of the clinical anamnez data. In all experiments as the inductors were used extrasensitives which has been participating in the test in previous series and their abilities for extrasensory action and mental health were out of questions.

Inductor was sitting in the comfortable armchair in the sound and light screened camera. The encephalogram was registered by twenty silverchloride electrodes, located according to the international scheme '10-20' with the usage of the standard helmet. EEG was registered in monopolar output, indifferent electrodes were placed on the lobes of both ears. The neutral electrode for reading ECG was placed over the left precordial area, the active electrocardiographic electrodes were placed on the back side of the lower third part of the left and right forearms. Except two cases when inductor and percipient were the married pair, in all other cases a 'chance' persons of both sexes were taken for the percipients. Their age varied in the range from 21 to 45 years. The acquaintance of the inductor with the percipient was held in 30 -40 minutes before the session of communication was started. Percipients were subjected to the extended psychological, clinical, and physiological testing, which included the analysis of brain blood circulation with the help of reoencephalography, rhythm-cardiograph, meridional diagnostics (by Nakatani) and some other. The experiment was held as the double blind. After the electrodes were fixed and the encephalograph control run, inductor was acquainted with the terms of the experiment. Inductor was free to create the image of his own, which he had to transfer to the percipient in continuation of the control lamp being switched. Nobody was permitted to come into the camera during the experiment. The operation of the control lamp was held from the outside of the camera by the head of the experiment.

The percipient was also placed in the armchair in the neighbouring to the camera room from which he could not have any audio-visual contact with the inductor or the control lamp. Condition of the percipient was controlled by the staff of laboratory, who did not know anything about the experiment details and the contents of the transferred message. The control apparatuses and the scheme of data acquisition was the same as for the inductor. After the electrodes were fixed and the apparatuses test the percipient was asked to place himself most comfortable in the armchair, close the eyes and to relax as deep as he was able. Percipient was told that if he would feel sleepy, not to resist this desire. To exclude the interrupt from the side of chance visitors the room was locked for the time of the experiment.

The encephalograms of inductor and percipient were continuously and synchronously registered by two computerised encephalographs 'Encefaln 131-01' type from the Medicom, Ltd. (Taganrog, Russia). The technical parameters of the encephalogram channel were as follows: the range of the registered signal amplitudes 5 - 500 mV, the

bandpass has from 0.5 to 30 Hz (3 dB), the coefficient of sinphase nose suppression on the frequency of 50 Hz - no less than 110 dB, the level of the internal noises reduced to the input - no more than 2 mV. The period of encephalographical measurements was divided into three intervals. The first interval was used for measure the background bioelectrical brain activity taking into account on inductor of some functional tests (closing - opening eyes, fotostimulation with the increasing frequency 3, 6, 9, 12, 15, 18, 21 and 24 Hz). The second interval was active ( the sequential code transmission according to the developed method) and the third interval was the registration of the aftereffect. During the experiment by the computer mark the different experiment phases were fixed. The following encephalogram processing was held with the help of the encephalograf standard software in the regimes of spectrum analysis, topography brain mapping in according to the power spectrum, amplitudes spectrum, kannogram etc..

After that acquired in this way data on spectral power density were used for the special secondary processing according to the method and programm developed by Prof. Polonnikov. This processing included the nonlinear filtration, decoding, calculation of probabilities of the correctness of the receiving of the simbole systematic (n,k) - code and the code signal in whole.

**Coding.** The systematic (n,k) - code for code of the transmitted messages is used, which means that it was an n - digit code composed of  $N = 2^k$  code vectors. Among n symbols forming code vector, k are information ones, and (n-k) - surplus, determining the correction power of the code. Let us take n=5, k=3. The full code table for the selected systematic code (5,3) - is presented in Table 1.

Table 1

<b>00000</b>	<b>00011</b>	<b>01101</b>	<b>11010</b>	<b>01110</b>	<b>10111</b>	<b>11001</b>	<b>10100</b>
00001	00010	01100	11011	01111	10110	11000	10101
00100	00111	01001	11110	01010	10011	11101	10000
01000	01011	00101	10010	00110	11111	10001	11100

The first line is the (5,3) code itself, which contains  $2^3=8$  code vectors. The general method of decoding have the following procedure. After some code combination has been received it is found in the table and identified with the code vector written in the first row of the table in the same column. In our experiments we have used the third in the first row code vector 01101 and also the fifth 0111, which are marked in the table by bold font. The full code word accepted by the inductor from the control lamp cab be for example the following:

-1-1-11-1 111-11 111-11 -1-1-11-1 111-11

0 1 1 0 1

Symbol "-1" corresponds to the lamp 'On', symbol '1' - corresponds to the lamp being 'Off'. The whole code sequence ( the upper string of 25 symbols) is divided in to five symbol sequence, each of them presenting direct ( 111-11) or reverses (-1-1-11-1) R.Barker's code. A direct corresponds to '1' of (n,k) - code, and the reverse to symbol 0.

**Formatting data for the secondary processing.** The secondary processing included the following stages:

- \* non-linear filtration and centering,
- \* demodulation,

- \* decoding,
- \* building the histograms of the frequencies of meeting the codes,
- \* calculation of probabilities of the correctness of (n,k) - code symbol and word receiving.

The detailed description of these stages is below.

Before starting secondary processing the data were formatted in a special manner - they were cut to segments of certain longitude. An important moment is the selection of the initial point for which the time marker indicating the code transmission start was taken. The whole period of the session, which was equal to 25 minutes, was split into five intervals according to the longitude of the Barker's code transmission. In its turn each five minute interval was split into three intervals. In the result 15 time intervals were formed, five of them were during three minutes and ten one minute. Let us call in the following these intervals as samples and count them by numbers in the sequence: sample 1, sample 2, ..., sample 15. For each of them for all 19 take - off's a power spectrum density for mean frequencies  $\delta$ ,  $\theta$ ,  $\alpha$  and  $\beta$  - rhythms was calculated with the help of EEG apparatus software.

**Demodulation.** For decoding decipher a message on the receptor side it is absolutely necessary to know two things. A moment when the transition starts and use type of the Barker's code (a five element code in our case). The received information is formatted in time intervals as it was described above and is filtered by median filter. Only after that is possible to start the stage of demodulation. Demodulation processes is reduced to the codes convolution or ( that is just the same for the discreet signals) to the operation of scalar multiplication of two vectors.

**Decoding.** Each (n,k) - code symbol decoding was realized with a help of the original algorithm. In the result on the decoder output one of three possible symbols was appearing: '1', '-1' and '?'. The '?' means that the situation is undetermined and to take final decision an additional information is wanted. If in the result of decoding of all five symbols of (n,k) - code '?' is met more then for one time such result is thought to be wholly undetermined, and the received code word is excluded from the consideration. If symbol '?' met only on the place of one from five code symbols, then for solution of infinity a special algorithm is used.

**A posterior probability of code receiving.** According to point 3. ( Table 1) it is possible to expect receiving of eight code groups ( amount of table columns). Let us designate these groups as a,b,c,d,e,f,g,h. Let us introduce into consideration one more ninth group, which will be designated as Z. To this group the codes will be placed which have been received ( on the results of coding and decoding) with two or more '?' symbols or one symbol if the special processing is applied.

It is necessary to put attention to the fact that Z group contains a determined uncertainties, which can be excluded from the following analysis.

If all 19 EEG take - offs were taken into consideration , that means that in the result of decoding  $M=19 \cdot 4=76$  codes will be received. Where 4 is the amount of analyzed rhythms ( $\delta, \theta, \alpha, \beta$ ) which cause appearance of the analyzed processes. It is obvious that

$$M=M_a+M_b+M_c+M_d+M_e+M_f+M_g+M_h+M_Z=\sum M_i; \quad i=a,b,c,d,e,f,g,h,Z, \quad (1)$$

where  $M_i$  - is the amount of codes of  $i$  - class registrated on the results of decoding in one session. These data make is possible to build the histogram of relative frequencies distribution

$v = F(i)$  of the events a,b,c,d,e,f,g,h, where

$$F(i) = \frac{M_i}{M - M_z} \quad (2)$$

On the histogram it is possible to determine the event which corresponds to the maximum of the relative frequency  $v$ :

$$\arg \max_i F(i) \quad (3)$$

Relation (3) helps to determine the code which can be considered to be received with the maximum probability.

**The estimation of the formation of the transmitted code by chance.** In the result of the decoding in the position of each of the five symbols should appear one of the possible symbols: "0", "1", "?". So the amount of all possible versions of decoding is  $3^5=243$ , from which only 9 are correct ( expected), four of them are in each column of Table 2 and five may appear in the result of the uncertainty elimination. In the result a probability of the correct result is  $p_v = 9/243 = 0.037$ . Now let us determine what is a probability to receive a wanted result  $k$  times in  $M=76$  test. For determination of  $k$  we can use experimental data and set  $k=(\max F(i)) (M-M_z)$ . For determination of probability of the chance appearance of  $k$  realizations of the expected version in  $M$  tests we use Poisson distribution:

$$P_\lambda(k) = \frac{\lambda^k}{k!} e^{-\lambda}, \quad (4)$$

where parameter  $\lambda$  can be estimated in the following way:  $\lambda = M p_v = 76 \cdot 0.037 = 2.8$

Using the tables for Poisson distribution ( we used [3]) for  $k=9$  and  $k=29$  the probabilities will be  $P_{2.8}(9) = 1.7 \cdot 10^{-3}$ ;  $P_{2.8}(25) = 10^{-6}$ . As it will be demonstrated in real experiments the real probabilities are much higher that wholly excludes the possibility of the chance formation of the wanted result on the message on the receiving side.

**Estimation of the correct receiving of the code.** It is well known [4] that in case of independent mistakes and if code is correcting all mistakes of ratio  $d$  ( in our case  $d=1$ ), then the probability of incorrect decoding is

$$P_{er} = \sum_{i=d+1}^n C_n^i p_0^i (1 - p_0)^{n-i}, \quad (5)$$

where  $p_0$  is a probability of a mistake in code symbol receiving. In our experiments the maximum value of  $p_0$  for the information take - off's (branches) on the corresponding rhythms did not exceed 0.2, which means that according to (5) a probability of mistake during code receiving doesn't exceed 0.26. Thus a probability of the correct code receiving turns to be no less then  $P_{tr} = 1 - P_{er} = 1 - 0.26 = 0.74$

**Experiments results.** Six sessions of transfer of digital information via a extrasensory channel with four pairs of participants were held during the period from 16.04.96 to 28.05.96. All

experiments has demonstrated a correct reproduction of the set code by the inductors and correct receiving of this code by percipients. In the following for illustration of the detailed results for only two sessions held by the same pair of persons are presented. This pair has been selected for the illustration also for the reason that with it a session was held as on small distances so on rather big distance. In first case the distance between inductor and percipient was about some meters between two rooms ( one of them with the inductor was isolated) where they were placed. In the second case a distance was no less then 15 km and in this case a percipient was placed in the isolated room. In the named test pair the role of inductor was held by a lady and percipient was a male who did not posses any extrasensory abilities. The first experiment ( in one building) was held on 19.04.96 and on the big distances on 28.05.96. the results of experiment in the tables 2 - 8 are presented. From table 2 can be seen that inductor is most intensive and efficiently operates on  $\beta$ -rhythm ( 13 correct code reproductions) and on  $\alpha$ -rhythm ( 10 correct code reproductions) . Percipient receives the code on all the rhythms nearly with the same efficiency ( 6-8 correct receiving on each rhythm). The histograms of the table 4 demonstrate that for both of them the estimation of the maximum of the a posterior probability is located in the area of receiving of c-code, which was transmitted by the control device.

Table 2

Take - off	$\delta$ - rhythm		$\theta$ - rhythm		$\alpha$ - rhythm		$\beta$ - rhythm		Comments
F1	-		-	*	+	*	-		S. (ind.)
F2	-		-		-		-	*	U. (perc.)
F7	-	*	+	*	-	*	+	*	19.04.96. St.PSMA
F3	-	*	-	*	+	*	+	*	
Fz	-		-	*	-	*	+	*	
F4	-		-		+		-	*	
F8	-		-		+		+		
T3	+	*	+	*	-	*	+		
C3	-		-		+		+		
Cz	-		-		+		+		
C4	-		-		+		+		
T4	-		-		-		+		
T5	-	*	-	*	+	*	+	*	
P3	-	*	-	*	-	*	+	*	+ correct reproduction of the code by inductor
Pz	-	*	-	*	-		-		- a fault reproduction of the code by inductor
P4	-		-		+		+		* a correct receiving code by percipient
T6	-		-		-		-		
O1	-		-		+		+		
Total :	+	1	+	2	+	10	+	13	+ 26 / 72
	*	6	*	8	*	7	*	7	* 28 / 68
	-	18	-	16	-	8	-	5	- 47 / 72

Table 3

The estimations of the a posterior probabilities of appearance of codes at inductor S. 19.04.96 (St.PSMA)

Code Cipher	a	b	c	d	e	f	g	h	Z	$\Sigma$
Appearance	0	0	26	5	4	8	8	2	19	72
Frequency (inc. Z)	0	0	0.36	0.07	0.05	0.11	0.11	0.03	0.26	72
Frequency	0	0	0.49	0.09	0.07	0.15	0.15	0.04	-	53

Table 4

The estimations of the a posterior probabilities of appearance of codes at percipient U. 19.04.96 (St.PSMA)

Code Cipher	a	b	c	d	e	f	g	h	Z	$\Sigma$
Appearance	0	1	28	0	16	0	0	0	23	68
Frequency (inc. Z)	0	0.015	0.41	0	0.23	0	0	0	0.34	68
Frequency	0	0.02	0.62	0	0.35	0	0	0	-	45

Comments to tables 3,4: 1) a c-code was transmitted (01101),  
2) Z- code was a set of determined undetermined codes.  
3) 17 take-offs were processed for percipient and 18 for inductor.

Table 5

Take - off	$\delta$ - rhythm	$\theta$ - rhythm	$\alpha$ - rhythm	$\beta$ - rhythm	Comments					
F1	+	+	- *	+	S. (ind.)					
F2	+	+	* +	+	U. (perc.)					
F7	-	+	- *	+	28.05.96. St.PSMA - Str. Rubinshtein					
F3	-	- *	- *	+						
Fz	- *	+	-	+						
F4	+	+	* -	+						
F8	+	+	* -	+						
T3	-	- *	- *	-						
C3	- *	-	- *	-						
Cz	+	+	- *	+						
C4	+	+	- *	+	*					
T4	+	*	- *	+						
T5	-	-	-	-						
P3	-	- *	-	+	+ correct reproduction of the code by inductor					
Pz	- *	- *	-	+	- a fault reproduction of the code by inductor					
P4	+	*	-	+	* a correct receiving code by percipient					
T6	- *	-	-	+						
O1	-	- *	-	+						
Total :	+	8	+	8	+	1	+	15	+	32 / 72
	*	7	*	8	*	9	*	1	*	25/ 72
	-	10	-	10	-	17	-	4	-	41 / 72

From the table 8 ( experiment held on big distance) is follows that inductor was most intensively reproducing e-code ( as it was set by the control lamp) on the  $\beta$ -rhythm. The percipient is receiving information more or less equally ( 7-8 correct receiving for each rhythm) on  $\delta$ ,  $\theta$ , and  $\alpha$ -rhythms. On the b-rhythm percipient has only one correct receiving.

Table 6

The estimations of the a posterior probabilities of appearance of codes at inductor S. 28.05.96 (Str. Rubinshtein.)

Code Cipher	a	b	c	d	e	f	g	h	Z	$\Sigma$
Appearance	6	2	0	6	32	8	5	1	12	72
Frequency (inc. Z)	0.08	0.03	0	0.08	0.44	0.11	0.07	0.014	0.16	72
Frequency	0.1	0.033	0	0.1	0.53	0.13	0.08	0.016	-	60

Table 7

The estimations of the a posterior probabilities of appearance of codes at percipient U. 28.05.96 (StPSMA)

Code Cipher	a	b	c	d	e	f	g	h	Z	$\Sigma$
Appearance	0	18	1	0	25	1	0	0	27	72
Frequency (inc. Z)	0	0.25	0.014	0	0.35	0.014	0	0	0.37	72
Frequency	0	0.4	0.02	0	0.55	0.02	0	0	-	45

Notes to Tables 6,7: 1) e-code was transmitted (01110),

2) Z- code was a set of determined undetermined codes.

3) 18 take-offs were processed for percipient and 18 for inductor.

4 rhythms were processed for each take-off.

Histogram of tables 6, 7 demonstrates that again the estimation of the maximum of the a posterior probability belongs to the code which was set by the control device and transmitted by inductor for percipient.

Table 8

Inductor/ Percipient	Date	Information take-offs and rhythms	Number of information points	$p_0$	$P_{tr}$	$P_v$	$\frac{P_{tr}}{P_v}$
Ind. S	19.04.96	T3θ, T3β, F8α, F8β, C4α, T4β, C3β, Czβ, T5α, T5β, P3β, F3α, F3β, F1α, O1α, O1β	16	0.0125	0.9999	$10^{-6}$ ( $k=16$ , $\lambda=2.6$ )	$9.9 \cdot 10^5$
Perc. U.	19.04.96	F3θ, F3α, F3β, F7θ, F7α, F7β, F1θ, F1α, F1β, P3θ, P3α, P3β, Pzθ, T5θ, T5α, T5β, F4β, Fzθ, Fzα, Fzβ, T3δ, T3θ, T3α	23	0.2	0.74	$10^{-6}$ ( $k=23$ , $\lambda=2.5$ )	$7.4 \cdot 10^5$
Ind. S.	28.05.96	Pzβ, P3β, T6β, F1θ, F1β, F2δ, F2θ, F2β	8	0.025	0.9999	$3.84 \cdot 10^{-3}$ ( $k=8$ , $\lambda=2.6$ )	260
Perc. U.	28.05.96	F2θ, F7α, Pzθ, T3α, O1θ, Czδ, C4β, P4δ, F3α	9	0.2	0.74	$1.11 \cdot 10^{-3}$ ( $k=9$ , $\lambda=2.6$ )	666

**Comment:** information are meant such take-offs and rhythms, for which a received code held no more then one correctable mistake.

Table 8 presents the results of estimation of the probabilities of correct reproduction and receiving of the signal codes by most information take-offs and rhythms.

After determination of the received code by means of algorithm (3) applied to the histograms for it an informative take-offs and rhythms are found. Then for them  $p_0$ ,  $P_{er}$  and  $P_v$  are determined. For determination of  $P_{er}$  it is necessary to know the estimation of  $k$  ( to use Poisson distribution). It is natural to put  $k = m$ , where  $m$  is a number of the information points (take - off - rhythm).

In October - November 1996. and in January 1997 the experiments were continued and have confirmed high efficiency of a technique and good reproducibility of results. Below in tables 9 and 10 selective results of November experiment are resulted (brought), where in the old days inductor and percipient were on distance about 15 kilometers each from other, and one of them was placed at the shielded chamber.

Table 9

The estimations of the a posterior probabilities of appearance of codes at inductor S. 05.11.96 (Str. Rubinshtein)

Code Cipher	a	b	c	d	e	f	g	h	Z	Σ
Appearance	1	3	31	17	7	1	14	1	17	92
Frequency	0.013	0.04	0.413	0.23	0.09	0.013	0.187	0.01	-	75

Table 10

The estimations of the a posterior probabilities of appearance of codes at percipient U 05.11.96 (SPSMA)

Code Cipher	a	b	c	d	e	f	g	h	Z	Σ
-------------	---	---	---	---	---	---	---	---	---	---

Appearance	1	0	23	0	18	1	1	0	48	92
Frequency	0.02	0	0.523	0	0.41	0.023	0.023	0	-	44
	3									

The notes to Table 9,10: 1) c-code was transmitted (01101),

2) Z- code was a set of determined undetermined codes.

Below on Fig.1 the probabilistic-frequent characteristic, showing is resulted(brought), that at the given pair searching the transfer of the information occurs mainly on  $\alpha$ -rhythm.

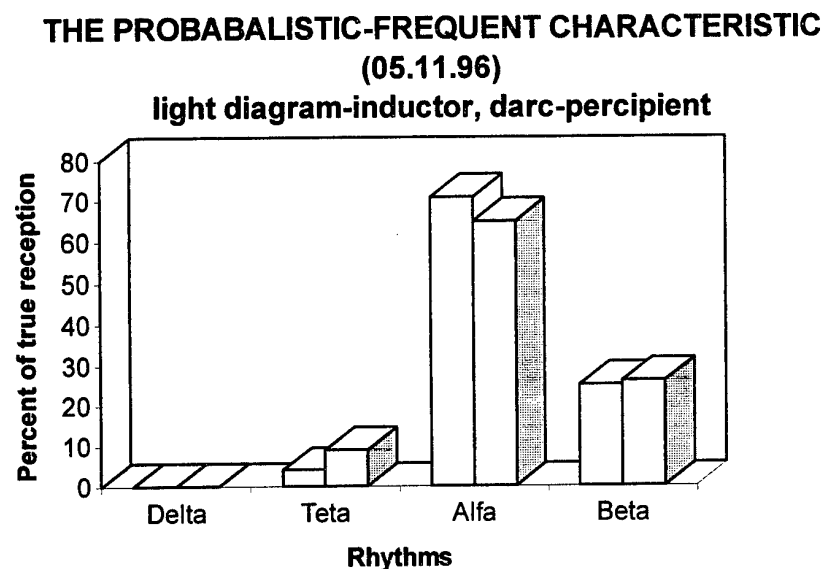


Fig. 1.

**Conclusions.** The most general and important experiment result can be formulated by following: way as far as we know it was for the first time the confidence transmitting of digital data via extrasensory channel of communication with registration of the results with the help of computer encephalographic and all estimations of probabilities of receiving correctness necessary for the digital transmission were estimated. Developed method can be reproduced in a wide experimental researches.

## References

1. Полонников Р.И. Методика применения помехоустойчивых кодов в экспериментах по экстрасенсорному восприятию. Известия вузов. Приборостроение. N 6, Санкт-Петербург. 1993, с. 24-28
2. Полонников Р.И. Управление передачей информации по экстрасенсорному каналу связи. Вторая межведомственная научно-техническая конференция. Проблемные вопросы сбора, обработки и передачи информации в сложных радиотехнических системах. Часть 1. Тезисы докладов. Пушкин, 1995, с. 10-11.
3. Большев Л.Н., Смирнов Н.В. Таблицы математической статистики. М. Наука. 1983, с. 298.
4. Самойленко С.И. Помехоустойчивое кодирование. М. Наука. 1966, с.14.

# ADAPTIVE BIOFEEDBACK CONTROL IN BIOLOGICAL SYSTEMS

Suvorov Nikolay B.  
Institute for Experimental Medicine of the RAMS  
Ecological Physiology Department  
Pavlov Str., 12 St. Petersburg 197376, Russia  
e-mail: iem@iem.spb.ru

**Abstract.** Oscillatory biofeedback methods have high physiological adequacy to biorhythmological processes of an organism. It is offered the adaptogenous range 5-30 sec as the basic temporary constants of biofeedback. In the limits of such periods a subject can control up and down any parameter of chosen function. Oscillatory training coordinates with the structure of physiological functions in time, loads different regulatory mechanisms. This idea is realized in the intelligent computer complex for biofeedback-assisted cardio-training. It is intended for systematic examination of persons, being in condition of stress, applications as an additional means of preliminary diagnostics. During cardiotraining the problem of a patient is periodic, at the expense of the special breath rhythm, set by a sine-wave curve, increase/downturn own heart rate at the continuous visual control. The basic difference of this complex is the presence of adaptive elements - the task for each following test is formed by the analysis of previous one. On a particular example is shown positive steady dynamics in arterial pressure, heart rate and other parameters of cardiorespiratory system. Analytical and graphic opportunities of the software are also demonstrated.

**Keywords.** Heart rate, arterial blood pressure, respiratory arrhythmia, oscillatory biofeedback control, efficiency factors.

Strain of separate systems and whole organism, caused by adverse ecological environment, results in nervous and visceral frustrations, and promotes development of steady pathological states. The weak factors of various origin also present a high danger because of the absence of primary negative reactions. If at the same time functional reserves of an organism are reduced, external influences will be considered as risk factors, resulting in disadaptation, prenosologic states, pathology. It is known, that early deviations from the norm have a functional character rather long time and are indicated in kind of infringement of biorhythmological processes in regulatory systems. At timely revealing correction of these disorders, as a rule, will be carried out by pharmacologic methods. Spreading of a medicamentous illness represents essential threat to health owing to superfluous and long-term pharmacotherapy. Nonpharmacologic receptions, such as psychotherapy, self-control, biofeedback control, autogenic training, raising reserves of an organism and lability of the regulatory mechanisms, have received a recognition in the last decades.

It is obvious that development of this rehabilitation field at present is defined by introduction of computer (microprocessor) techniques, ensuring realization of a principle of systemic regulation in involuntary (noncognitive) functions and directing to normalization of them biorhythmological structure and management of biorhythmological information.

The efficiency of biofeedback-assisted training depends on a great number of the factors, among which methodological, physiological and psychological aspects are considered.

The methodological factors:

1. Sensory informativeness of biofeedback signals;
2. Degree of the affinity of a chosen parameter to its natural maximum or minimum;

3. Necessity of gradual forming of the skills of voluntary (cognitive) function control, smooth transition by an established level in the combination with the small intermediate steps;
4. Necessity of the purpose, according to which it is necessary to ensure the certain positive result in first sessions of training cycle, as sense of the success stimulates efforts in the needed trend and mobilizes subjects;
5. Sufficient duration of each session and sufficient number of repeated training sessions;
6. Reproduction of the skill outside of a laboratory without biofeedback signals for fastening of the abilities to realization of internal efforts;
7. Restriction of parameters number, forming simultaneously or consistently the biofeedback signal;
8. Use of the oscillatory mode of biofeedback control or in consecutive sessions, or in each of them.

Among the physiological factors are taken into consideration:

1. Natural involuntary plasticity of the chosen function;
2. Sufficient variability of its biorhythm;
3. Initial destabilization of the subject condition;
4. Individual and typological characteristics of the biorhythmological parameters plasticity and so on.

The psychological factors:

1. Preliminary psychological encouragement (acquainting with by devices, problems and expected results, with possible manners of required effect receipt; the emphasizing of the necessities of concentration of attention or relaxation in certain periods of fulfilment of the tasks and so on);
2. Working out necessity of the own receptions of the chosen parameter control, the use for these purposes not only emotionally painted and sensual images, but also voluntary manipulations by muscular tonicity and breath;
3. Forming of the want to voluntary correction and self-regulation of functions, automatization of the skill of biofeedback control;
4. Verbal self-suggestion or self-inspiration;
5. Maximal rapprochement of feedback control with by autogenic training technique.

The mentioned above is essentially for realization of biofeedback technologies in clinical practice.

Biofeedback methods with alternative oscillatory modes have high physiological adequacy to biorhythmological processes in an organism. As the basic temporary constants of oscillatory biofeedback it is offered the adaptogenous range 5-30 sec [1,2]. Just these rhythms are connected to the intersystemic mechanisms of functions integration and as much as possible are adequate to the mechanisms of biofeedback control as one of the form of instrumental conditional reflex. Development of adaptation and homeostatic processes in an organism is determined by biorhythmological dynamics inside this range. In the limits of such periods a subject can smoothly up and down manipulate of any parameter of the chosen function. The simplicity and presentation, ease of mastering of the training program, the biorhythmological adequacy, game elements and other positive properties of such mode have strengthened the opinion about its expediency. Thus, alternative oscillatory regimen of trainings as much as possible coordinates with the functions structure in time, loads different regulatory mechanisms; has the patent protection [3].

The idea of an oscillatory mode of biofeedback-assisted training is realized in the intelligent computer complex for alternative cardiotraining and its universal use in scientific researches and introduction in clinical practice [4]. The complex, consisting of the cardiosignal converter, a personal computer and the software, is intended for systematic examination of persons,

being in a condition of professional, ecological, psychological or social tension (stress); sportsmen before and especially after competitions; self-checking and self-regulation under nervous and physical loads; applications as an additional means of preliminary diagnostics. The essence of oscillatory biofeedback control consists of simultaneous presentation of two functions on the subjects monitor - own cardiointervalogram and a target sine-curve. A subject has to draw together the periods and the amplitudes of both functions. The basic idea of oscillatory cardiotraining is submitted on Fig. 1.

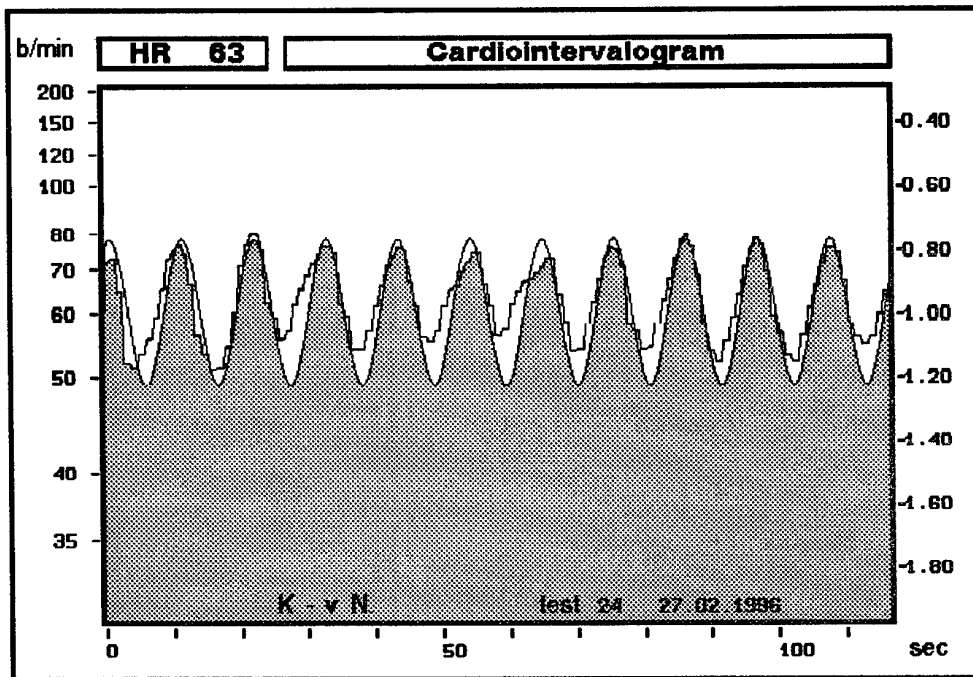


Fig. 1 Training test 24 - satisfactory combination of own cardiointervalogram and target sinusoid

A patient's problem is periodic (at the expense of the special rhythm of breath) set by the sine wave curve, increase/downturn (inspiration/expiration accordingly) of own heart rate at the continuous visual control. The adaptive elements are the principal peculiarity of this complex - the task for each following test is formed by the previous one analysis. The values of the amplitude, period and constant making of the target sine-curve are offered automatically. In the adaptive mode the software can, not allowing to exceed the limits of the individual physiological capacities, raise in steps the complexity of the current task, if the patient successfully coped with the previous one, otherwise the task is automatically simplified. The adaptive character of session control enables to carry out it without the intervention of a pdoctor, however in case of disagreement with by the recommendation of the program it is possible to change any parameters on any stage of any session. Automatic presentation of the task allows after appropriate advice from the physician to carry out the training independently (at home or at work) without direct medical observation. It is recommended to show the results to the expert after each three sessions.

The hystograms of the cardiointervals distribution, the frequent spectra of the cardiointervalograms, the intensity of the spectrum waves, the scatterograms of the cardiointervals and some of settlement parameters are calculated during each test. Complete training-cycle is recommended for restoration of the respiratory rhythmic component, normalization of heart

rate and arterial blood pressure at functional disregulation in the cardiorespiratory system, vegetovascular dystonia. The basic principles of cyclic organization consists of following: after the 1st background test it is carried out test-training with the parameters, nominated or by the software, or by the expert conducting this procedure. Further the subject should have a rest for 2 - 3 minutes, then test-training is offered, again the pause and so on. Next session can repeat previous one (the automatic character of the choice of the target sine-curve parameters is kept), but variants caused by the individual medical status of the subjects are possible. In this sense the system is opened for development of new techniques of human state correction. The patient instruction should by all means contain the request as much as possible to be relaxed during registration of his cardiointervalogram. Not each of the subjects is a success in several first tests, as the sine-curve tracing is a sort of the loading and it can be reflected in heart rate, in the index of the systems strain and other parameters. Besides the subject does not owe deliberately to increase of his breath depth (the software in no event does not provoke it) because it can result in the undesirable hyperventilation.

For controllable state recovery after sport trainings, competitions, heavy physical loadings can appear sufficient one or two training sessions, spent or in conformity with the mentioned above, or on the basis of the same principles, but in view of the loading specificity and of subject individual peculiarities.

The developed complex is used for the research of systematic training influence on processes of self-control of organism functions, being in the condition of unsatisfactory adaptation to loadings. The cycle of the cardiotraining consists of 8-10 sessions on 6-8 tests in each.

The period of the target function was limited by the temporary frameworks of the respiratory waves (3 - 13 seconds), as their presence into a cardiointervalogram is a favorable diagnostic attribute. We did not study the conditions and accuracy of target function tracing in view of its phase and period, as the problems of quality of the task fulfilment, the limits of regulations were deeply researched in earlier works [5,6].

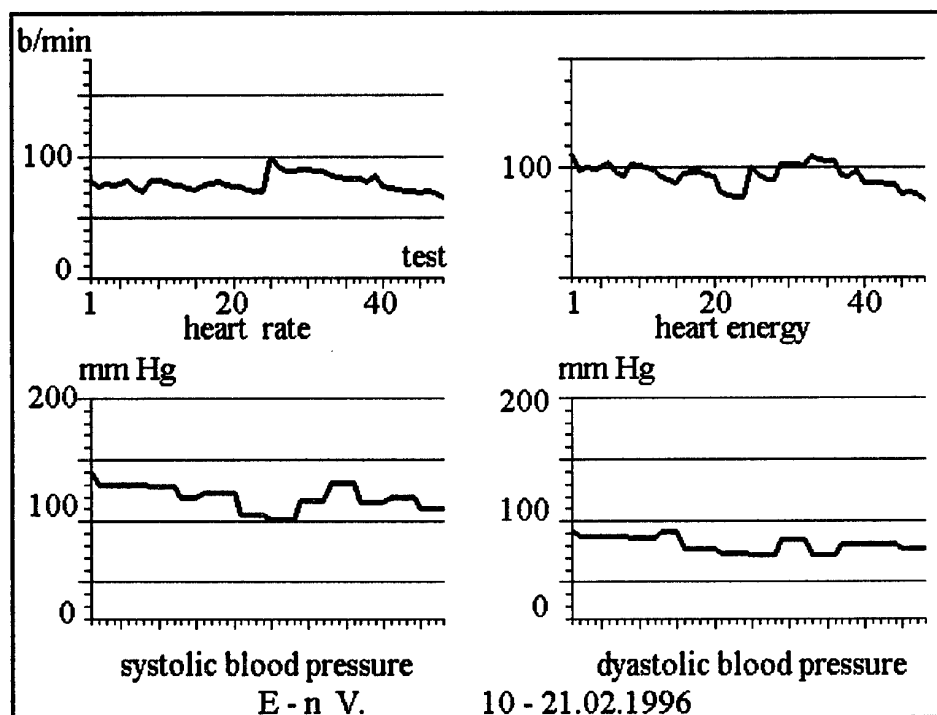


Fig. 2 Training-cycles dynamics of four parameters in the cardiorespiratory system (48 tests)

27 men aged by 28 - 50 years were examined. Functional deviations in the state of the cardiorespiratory system (functional hypertension, tachycardia, rigidity of cardiorhythm or hard pulse) were revealed by the analysis of their psychophysiological parameters.

The results of training-cycles at the majority are positive (Fig. 2), but are not equivalent, that it is possible to connect with by the irregular visiting to the nominated procedures by some patients. The systematic sessions and completed cycle of biofeedback-assisted cardiorhythm training give more effective results.

On a particular example is shown positive stable dynamics in the arterial pressure, heart rate and other parameters of the cardiorespiratory system (Fig. 2). 90% of the subjects had such positive changes for a year. The analytic and graphic opportunities of the software are also demonstrated.

**Conclusions.** Oscillatory adaptive biofeedback control, distinguished by the absence of monotony at the expense of the periodic change of activation and inhibition in the controlled function, realizes safe alternate training of the sympathetic and parasympathetic mechanisms in the cardiorespiratory system and helps to expand the dynamic range of regulatory processes. It allows "to shake" rigid functional states for transition them on the adequate level. The effect of alternative oscillatory biofeedback-assisted training can be kept durably, whereas unidirectional changes are compensated by natural homeostatic processes of an organism.

### **References**

1. Suvorov N.B., V.V.Uriash, V.V.Yakovlev (1980) Correlation of sensory-motor rhythm of the brain and heart rate in cat. *Fiziol. zhurnal SSSR*, 66, 5, 746-749.
2. Vassilevsky N.N., Z.A.Aleksanian (1982) Adaptive control of autonomic processes. *Fiziol. zhurnal SSSR*, 68, 7, 948-952.
3. Vassilevsky N.N., L.N.Gondareva, T.Z.Seissebekov, E.V.Antonenko, L.U.Assylbekova, I.M.Riklefs, B.K.Koichubekov (1992) Method of functional correction of blood pressure. Patent SSSR N 1745200, Bull. N 25.
4. Babich S.S., A.V.Belov, D.N.Bochkarev, S.V.Vassiliev, M.S.Mishin, N.B.Suvorov, V.F.Usun, N.L.Frolova, A.S.Shmygol (1996) Individual-typologic features of the central nervous system, functional reserve of the brain, human state correction. In *Vestnik Akademii technicheskogo tvorchestva*, SPb, pp.21-39.
5. Vashchillo E.G., M.A.Konstantinov (1993) Voluntary cardiorhythm regulation. In *Problemy neurokibernetiki*, Rostov na Donu, izd. RGU, pp.80-87.
6. Vassilevsky N.N., N.A.Migalovskaya, S.B.Nikitina, A.M.Zingerman (1993) Method of alternating biofeedback control and efficacy training criteria. In *Biofeedback - 2. Theory and practice*, Novosibirsk, pp.65-76.

# MATHEMATICAL MODELS OF TWO BIOSYSTEMS

Natalia V. Blinkova, Tatyana V. Voloshinova,  
Boris A. Ershov and Vladimir E. Pasyunkov  
St. Petersburg State University  
Mathematics and Mechanics Department  
2 Bibliotchnaya sq.  
St. Petersburg 198904, Russia  
e-mail: bae@ersh.usr.pu.ru.

**Abstract.** A mathematical model of a flexible flapping wing with two controls is created. These controls are entered into equations of a model in nonlinear way. Mechanical model, usually used in hydrodynamics and hydroelasticity for study of a flapping wing, is represented by a carrying surface, which transfers in fluid flow at designated manner. The dynamic model of the hydroelastic problem consists of three parts, namely, a flexible profile, a fluid flow and a flexible profile interacting with a fluid flow. The mathematical model is represented by a set of integro-partial differential equations, which are generalizations of Fredholm and Volterra equations.

**Keywords.** Mathematical models, hydroelasticity, flapping wing.

Flapping wing of a bird, caudal fin of a fish are unique creations of nature, forming propulsive and lifting forces in its motion. Mechanical model, usually used in hydrodynamics and hydroelasticity for study of a flapping wing, is represented by a carrying surface which transfer in fluid flow at designated manner. Refinements applying to construction of carrying surface and type on its motion follow from formulation of a model.

The fluid flow is detailed likewise (ideal, incompressible, compressible, vortex wakes). In its general definition the problem of flapping wing is stated in the form of bonded hydroelasticity problem consisting of flexible carrying surface model, fluid model and interaction mechanism between carrying surface and flow.

Wing model can be represented by part of thin elastic shell of variable rigidity partially cantilever fixed on a separate part of its border the motion of which is given. In this case the carrying surface is represented by the middle surface of a shell, as it is generally agreed in the thin shell theory.

In many cases the assumption that carrying surface is ideally elastic is a far too strong idealization. For example, polymeric materials (natural and synthetic caoutchuc, amorphous polymers, composite materials, etc.) in broad range of loads and temperatures exhibit clearly defined viscous and elastic properties which may be closely described by integral relations of hereditary elasticity theory — Boltzmann – Volterra relations [1]. Because of this, we consider a shell made of hereditary elastic material in order to get closer approximation to reality. The carrying surface is assumed to be moving in incompressible ideal liquid at rest at infinity.

Let us set off a class of plane motion of the system "carrying surface — liquid". In this case the carrying surface is represented by an elastic thin profile moving in its plane. Given profile's motion including its deformations we can speak about the mechanized wing. A special feature of unsteady profile motion in incompressible ideal liquid is above all related to the circulation which assumes initiation of a vortex trace behind the moving profile.

Let us outline works on flapping wing theory that were carried out at the Mechanics and Mathematics Department of Moscow State University and Mathematics and Mechanics Department of St. Petersburg State University. In 30–40th, professor of MSU V. V. Golubev published series of works on flapping wing theory [2]. In particular, monograph "Lectures on Wing Theory", published in 1949, covers periodic motion of rigid profile with finite amplitude of swing. Here, for vortex trace author introduces vortex shedding of Carman vortex type.

In 1995, A. A. Zaitzev, the professor of MSU, published a book [3] summing up almost 20 years of work on carrying surface theory. The theory allows to calculate a flow of highly deforming carrying surface of complex form (wing, fin, sail, etc.). The author together with his disciples worked out models for fluttering normal and horizontal flight of a fly and dolphin's caudal fin. "But these models do not use the whole power of the constructed mathematical technique. Their development is in future."

Since 1980s, the problem of flapping elastic wing as a problem of hydroelasticity is being dealt with at the Department of Mathematics and Mechanics of St. Petersburg State University. Let us dwell on this investigations in greater detail.

The model of an elastic profile with the vortex trace behind the trailing edge. This model describes a wide class of hydrodynamic problems and makes it possible to investigate the hydrodynamical characteristics of an elastic profile. Let us to describe the model of an elastic profile.

We consider the unsteady arbitrary motion of a flexible thin profile encountering a gust in an incompressible flow. The dynamic model of the hydroelastic problem consists of three parts, namely, a flexible profile, a fluid flow and a flexible profile interacting with a fluid flow. The model of an profile interacting with an ideal fluid is defined by (i) an impermeability condition and (ii) a vortex trace, which is simulated by a line of a discontinuity in the velocity. The model considered here describes a wide class of hydrodynamic problems and makes it possible to investigate the hydrodynamical characteristics of a flexible profile, such as lifting force and moments.

An elastic body, as a thin carrying surface, can be modeled by an infinite plane profile (the plane problem), subsequently referred to as a thin flexible profile. Let the  $(x, y)$ -coordinate system be attached to the profile which is moving to the fixed  $(x_1, y_1)$ -system so that the following relations are valid:

$$x = x_1 - U_0 t - a, \quad y = y_1,$$

where  $2a$  is the width of the profile,  $U_0 = \text{const.}$  Using the two-dimensional theory of a thin elastic plate, we obtain the equation of the flexural vibration of the profile

$$D \left[ \frac{\partial^4 w(x, t)}{\partial x^4} - \int_0^t K(t - \tau) \frac{\partial^4 w(x, \tau)}{\partial x^4} d\tau \right] + m \frac{\partial^2 w}{\partial t^2} = p(x, t), \quad (1)$$

where  $w(x, t)$  is a bending flexure of profile,  $D$  is cylindrical rigidity,  $m$  is mass per unit wing area,  $K(t - \tau)$  is relaxation function of one-axis tension and  $p(x, t)$  is flow pressure per unit wing area. The boundary conditions are defined by the attachment method for the wing contour (a free wing, an aileron, etc.); the initial conditions determine the displacement and the velocity of the wing middle surface at any given time.

The model of a fluid flow is described as an ideal incompressible liquid at rest at infinity. An unsteady motion of the profile producing a vortex trace behind the trailing edge. The discontinuity in the velocity is defined by the integral equation

$$\int_0^{U_0 t} u_2(t) \sqrt{\frac{t - \tau + t_a}{t - \tau}} d\tau = - \int_{-a}^a \left[ -U_0 \frac{\partial w(x, t)}{\partial x} + \frac{\partial w(x, t)}{\partial t} \right] \sqrt{\frac{a - \xi}{a + \xi}} d\xi, \quad (2)$$

where  $t_a = U_0/2a$ .

The linearized Lagrange integral yields the pressure jump,  $p(x, t)$ , across the profile (lower surface minus upper surface pressures) in the moving coordinate system. For the unsteady motion of the elastic profile we can write a system of integro-partial differential equations for the deformation,  $w(x, t)$ , of the profile and the discontinuity in the velocity,  $u_2(t)$ , in the trace in the case of the gust  $V_g(x, t)$ :

$$\begin{aligned} & D \left[ \frac{\partial^4 w}{\partial x^4}(x, t) - \int_0^t K(t - \tau) \frac{\partial^4 w}{\partial x^4}(x, \tau) d\tau \right] + m \frac{\partial^2 w}{\partial t^2}(x, t) \\ & - \frac{2\rho}{\pi} \left\{ U_0 \int_{-a}^a \left( -U_0 \frac{\partial w}{\partial \xi}(\xi, t) + \frac{\partial w}{\partial t}(\xi, t) \right) L_1(\xi, x) d\xi \right. \\ & \left. - U_0 \int_{-a}^a \left( -U_0 \frac{\partial^2 w}{\partial \xi \partial t}(\xi, t) + \frac{\partial^2 w}{\partial t^2}(\xi, t) \right) L_2(\xi, x) d\xi \right\} \\ & - U_0 \sqrt{\frac{a+x}{a-x}} \int_0^{U_0 t} K_1(t - \tau) u_2(U_0 \tau) d\tau = 0, \\ & \int_{-a}^a \left( -U_0 \frac{\partial w}{\partial \xi}(\xi, t) + \frac{\partial w}{\partial t}(\xi, t) \right) L_0(\xi) d\xi + \int_0^{U_0 t} K_2(t - \tau) u_2(U_0 \tau) d\tau = 0. \end{aligned} \quad (3)$$

Kernel functions  $K_\alpha(t)$  and expressions for  $L_\alpha$  may be founded, for example, in [4]. System (3) is a set of integro-partial differential equations, which are generalizations of Fredholm and Volterra equations.

Vortex trace behind the profile and its hereditary elastic material accumulate and preserve information about preceding motions of the profile. Current deformations velocities of the profile's points are functionals of preceding motions of these points. In this sense one can speak about the memory of flow and material and, consequently, about their influence on characteristics of locked hydroelastic system. Let us go on to the flapping wing model proposed by V. V. Golubev [2]. In this model substitute rigid wing by a flexible profile.

Optimal control of the flapping fin propulsive agent was investigated using as an example the model problem of in-line motion of material point subject to hydrodynamic forces forming during the work of propulsive agent. Let us use the following coordinate systems:

$O_1x_1y_1z_1$  — fixed,

$Oxyz$  — moving, attached to the profile.

Consider  $z_1 = x_1 + iy_1$  on a fixed plane and  $z = x + iy$  on the moving one. The motion of the plane  $z$  relative to the fixed plane  $z_1$  is determined by the velocity  $v_0$  of pole  $O$  and angular velocity  $\bar{\Omega}$  of the fixed system's rotation,

$$v_0 = v\mathbf{i}_1 = U_0\mathbf{i} + V_0\mathbf{j},$$

$$q_0 = U_0 + iV_0,$$

$\varphi$  — angle between  $O_1x_1$  and  $Ox$  axes,

$$\Omega = \frac{d\varphi}{dt},$$

$$\bar{\Omega} = \Omega\mathbf{k} = \Omega\mathbf{k}_1.$$

Let us formulate the model problem. The point moves along the  $x_1$ -axis subject to hydrodynamic force  $\mathbf{F}$  and propulsive force owing to the Carman vortex shedding  $\mathbf{K} = K\mathbf{i}_1$ . The motion equation is

$$m \frac{d^2x}{dt^2} \mathbf{i}_1 = \mathbf{F} + \mathbf{K} + \mathbf{R}.$$

Here

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} = F_{x_1}\mathbf{i}_1 + F_{y_1}\mathbf{j}_1,$$

$$\mathbf{R} = R\mathbf{j}_1.$$

The motion equation projections on the axes of the fixed coordinate system have the form

$$\begin{aligned} (m - a_x) \frac{dv}{dt} &= b_x v + c_x + K, \\ a_y \frac{dv}{dt} + b_y v + c_y + R &= 0, \end{aligned} \quad (4)$$

here  $m$  is a point's mass,  $\rho$  is a fluid density,  $x_1$  and  $v = dx_1/dt$  are coordinate and velocity of a point,

$$\begin{aligned}
a_x &= -\lambda_x \cos^2 \varphi - \lambda_y \sin^2 \varphi + \lambda_{xy} \sin 2\varphi, \\
b_x &= -\dot{\lambda}_x \cos^2 \varphi - \dot{\lambda}_y \sin^2 \varphi + \dot{\lambda}_{xy} \sin 2\varphi \\
&\quad + \dot{\varphi} \{(\lambda_x - \lambda_y) \sin 2\varphi + \lambda_{xy} \cos 2\varphi\}, \\
c_x &= \dot{\varphi} \left( -\dot{\lambda}_{x\omega} \cos \varphi + \dot{\lambda}_{y\omega} \sin \varphi \right) + \ddot{\varphi} (-\lambda_{x\omega} \cos \varphi + \lambda_{y\omega} \sin \varphi) \\
&\quad + \dot{\varphi}^2 (\lambda_{x\omega} \sin \varphi + \lambda_{y\omega} \cos \varphi), \\
a_y &= -\frac{\lambda_x + \lambda_y}{2} \sin 2\varphi + \lambda_{xy} \cos 2\varphi, \\
b_y &= -\frac{\dot{\lambda}_x - \dot{\lambda}_y}{2} \sin 2\varphi + \dot{\lambda}_{xy} \cos 2\varphi \\
&\quad + \dot{\varphi} \{-(\lambda_x - \lambda_y) \cos 2\varphi - 2\lambda_{xy} \sin 2\varphi\}, \\
c_y &= \dot{\varphi} \left( \dot{\lambda}_{x\omega} \sin \varphi + \dot{\lambda}_{y\omega} \cos \varphi \right) - \ddot{\varphi} (\lambda_{x\omega} \sin \varphi + \lambda_{y\omega} \cos \varphi) \\
&\quad - \dot{\varphi}^2 (\lambda_{x\omega} \cos \varphi - \lambda_{y\omega} \sin \varphi).
\end{aligned}$$

Time dependent coefficients of attached masses of flexible profile  $\lambda_x$ ,  $\lambda_y$ ,  $\lambda_{xy}$ ,  $\lambda_{x\omega}$ ,  $\lambda_{y\omega}$  are determined in the following way.

Represent thin profile by a section of the moving plane  $z = x + iy$  attached to the profile:

$$z = x + iT(t) \sum_{k=1}^n a_k x^k, \quad 0 \leq x \leq 1, \quad (5)$$

here  $T(t)$  determines deformation of the profile-section, change in its curvature and  $\sum_{k=1}^n a_k x^k$  is a segment of Mac Laurin row of function setting the form of a section. Function of complex variable  $z = f(\zeta, t)$  mapping external with respect to section region of plane  $z$  to interior of unit circle with centre at zero of auxiliary plane  $\zeta$  may be written as

$$\begin{aligned}
f(\zeta, t) &= \frac{1}{4} \left( \frac{1}{\zeta} + 2 + \zeta \right) + iT(t) \left[ \sum_{m=1}^n \frac{a_m}{2^{2m}} \left( \frac{1}{\zeta} + 2 + \zeta \right)^m \right] \\
&= k_0 + T(t) \sum_{m=1}^n k_m \left( \frac{1}{\zeta^m} + \zeta^m \right).
\end{aligned}$$

Further let us use L. I. Sedov [5] method. Define rational part  $w_3(\zeta)$  of expression  $-if(\zeta, t)\bar{f}(\zeta, t)$ :

$$w_3(\zeta, t) = \sum_{m=1}^{2n} C_m \zeta^m;$$

$k_m$ ,  $C_m$  define the profile's outline.

Attached masses coefficients may be expressed through  $k_1$  and  $C_1$  according to the following formulae:

$$\begin{aligned}\lambda_x(t) &= 2\pi\rho(k_1\bar{k}_1 - k_1^2 - \bar{k}_1^2), \\ \lambda_y(t) &= 2\pi\rho(k_1\bar{k}_1 + k_1^2 + \bar{k}_1^2), \\ \lambda_{xy}(t) &= i\pi\rho(k_1^2 - \bar{k}_1^2), \\ \lambda_{x\omega}(t) &= -\pi\rho(k_1C_1 + \bar{k}_1\overline{C_1}), \\ \lambda_{y\omega}(t) &= -i\pi\rho(k_1C_1 - \bar{k}_1\overline{C_1}),\end{aligned}$$

where

$$\begin{aligned}k_0(t) &= \frac{1}{2} + iT(t) \sum_{k=1}^n 2^{-2k} \binom{2k}{k} a_k, \\ k_1(t) &= \frac{1}{4} + iT(t) \sum_{k=1}^n 2^{-2k} \binom{2k}{k+1} a_k, \\ k_m(t) &= iT(t) \sum_{k=1}^n 2^{-2k} \binom{2k}{k+m} a_k, \quad (m \leq k).\end{aligned}$$

Coefficient  $C_1$  is a quadratic form of  $k_\alpha(t)$  and  $\bar{k}_\beta(t)$ . Let us return to Eq. (4). Here

$$K = -\frac{\rho\gamma^2}{2\pi l} - \frac{\rho\gamma h}{l} \left( v \cos \varphi - \frac{\gamma}{l} \operatorname{th} \frac{\pi h}{l} \right)$$

is a mean in period force due to propulsive force of Carman vortex shedding;  
 $\gamma$  is absolute value of vortex density forming Carman shedding;  
 $h, l$  are vortex shedding parameters (assume  $h/l = 0.281$  for shedding stability);

$$\Gamma(t) = 2\pi i \left[ \sum_{m=1}^n m(\bar{q}_0 k_m - q_0 \bar{k}_m) + \dot{\varphi} \sum_{m=1}^{2n} m C_m \right] \quad (6)$$

is circulation of system "profile — vortex shedding" determined by Zhukovsky — Chaplign postulate;

$\gamma = |\Gamma(t)|$  at  $t$  corresponding to upper and lower positions of the profile.

Mathematical model of the problem consists of the first equation of system (4) and relation (6) determining circulation. In this model there are two controls:  $\varphi(t)$  determines turning of the moving coordinate system and  $T(t)$  determines deformation of the profile-section and change in its curvature. The controls enter into Eq. (4) in nonlinear way. Proposed mathematical model of the flexible flapping profile is non-trivial. The degree of its complexity allows to conduct experiments.

Mathematical experiment has revealed fundamental difference between stationary and nonstationary regimes (acceleration, braking). In the first case the motion is initiated by vortex shedding. Thus flexible profile generates vortex shedding.

In the second case propulsive force is created by nonstationary motion of flexible profile.

### ***References***

1. Rabotnov, Yu.N. (1977) Elements of Inherited Mechanics of a Rigid Body. Nauka, Moscow.
2. Golubev, V.V. (1949) Lectures on Wing Theory. GITTL, Moscow – Leningrad.
3. Zaitzev, A.A. (1995) Theory of Carrying Surface. Nauka, Moscow.
4. Blinkova, N.V. (1995) The General Form of Solution of the Problem of Inherited Dynamics of a Thin Flexible Profile in the Pre-sonic Incompressible Flow. Vestnik Sankt-Peterburgskogo Universiteta, Matematika, vol. 4 (N 22), pp. 56 – 60.
5. Sedov, L.I. (1980) Plane Problems of Hydrodynamics and Aerodynamics. Nauka, Moscow.

# INFORMATION IN MEDICAL SYSTEMS AND ITS USE FOR DISEASES DIAGNOSING AND PREVENTING

Yuri I. Musijchuk  
St. Petersburg Administration Public Health Committee  
30, Shkapina Str., 198095, St. Petersburg, Russia  
e-mail: igorkras@infopro.spb.su

**Abstract.** The information can be conditionally divided into several categories: information developed and delivered in the human body, information coming to humans from the environment and information about human beings. The medical science undergoes the revolution in information. Up-to-date systems of information support and automatic entering the information into data bases is to solve adequately many problems of preventing diseases and administering in public health.

**Keywords.** Medicine, information in the human body, information from the environment, diagnosing pathological states, informational disturbance.

Realizing and using the characteristics of information processes in medicine are of great importance for arranging the examining of the diseased and for taking administrative decisions when carrying out the program of stabilizing the population health. The information used by a physician either instinctively or scientifically can be conditionally divided into several categories:

- information developed and delivered in the human body;
- information coming to humans from the environment and used for making behavioral decisions;
- information about human beings and their environment used for making hypotheses about causes of unhealthy states, means of preventing diseases, methods of arranging efficient systems of rendering medical aid, etc.

The information providing regulations in human body is not practically recognized if the body functions normally. The main parameters of this information flow are studied by different parts of physiology and are mainly described by time-and-frequency characteristics. The frequency characteristics of this information specify the requirements set forth to designing diagnostic devices and systems, sensing devices for reading information, etc. The frequency of studying information flows when analyzing the functioning of organs and systems under various conditions by diagnosing tasks, but in all the cases the obligatory requirement is the correlation between the frequency of reading information and the rate of the process studied/ In this case the frequency characteristics of the devices should exclude misinterpretation or loss of the information.

In practice it turns out that in a number of cases only in-line reading of information allows to obtain sufficient data for analyzing automatic regulation systems in living human bodies [1]. A great number of methods used under conditions of loading do not allow to estimate the regulation process parameters. The example of such an investigation is «glucose test» in which case even 15-minute period of time do not allow to describe the regulation process. Investigating the regularities of the dynamics of physiological processes in human body is an important scientific task of rhythmology. The validity of efficient working in shifts, the de-

pendence of physiological processes upon rhythmological processes at the earth and in cosmos, seasonal and age variations of parameters in human bodies, peculiarities of administering drugs depending upon the phase state of a particular system - it is a far from total listing of questions arising in this field. These investigations are very laborious and expensive due to the need of repeated reading of information and consequently they are not always carried out with proper rate of studying parameters.

The importance of statistical approaches to analyzing physiological information under the conditions of different loading can be illustrated by the example of statistical evaluation of R-R intervals in ECG in which case the skew and excess of distribution are evidences of the transient process and provide grounds for the more correct working out of the experimental procedure or for making the decision about the time of the function recovery [2].

The physiological information is the basis of diagnosing many pathological states. As long ago as a quarter of a century its use for developing diagnosing systems allowed to work out diagnosing methods much more efficient than people can do it even if their teaching was purposeful [3].

The misinterpretation of information circulating inside the human body results in halucinosi feelings which indicates the beginning of a disease. Errors in standard use of information under conditions changed are rather common which also results in pathological control of liquid retention in the body when the stroke volume decreases in cases of blood loss (aimed in this case at life maintaining) start in mitral stenosis as well, resulting in undesirable liquid retention (edemae). The characteristic of a human being is filling memory gaps with some information which causes great difficulties in interpreting what the diseased tells about his or her state.

Increasing or decreasing impulse frequencies in the cell receptor system is also the ground of developing various diseases. Correcting such diseases is extremely difficult without using the laws of information processes in the body. Using some drugs is based on the principle of changing information delivery in one receptor or another.

Many hereditary or acquired states due to wrong retranslation of enzymatic proteins on matrices of membranes refer to typical informational disturbances. It is precisely these disturbances are responsible for carcinogenic and mutagenic effects of chemicals and some physical factors. In the body there is a protection system against such information disturbances. It identifies wrongly synthesized protein molecules or cells containing them in proper time and cuts them out. Practically all the immune processes are typical for informatics processes with a feedback the idea of which appeared due to investigating conditioned reflexes.

A lot of literature [4] is devoted to the problems of information entering human body from the environment, its volume and processing rate have been estimated for some human states. In general terms it is well known the importance of the excess and deficiency of such information for developing anxiety, especially under emergency conditions of human existence. Yet the modes of interpreting such information, the importance of general culture and the volume of information available needed for such interpreting still remain to be solved.

The importance of the volume and quality of information and the time of its processing when controlling production processes and transporting means is studied by ergonomics and labour physiology. The methods of the most rational visual or combined representation of informa-

tion on control panels in various presentative systems are being developed on the basis of the regularities available.

From the point of view of pathology there were described the so-called «information» neuroses when the lack of time for processing information or deficiency in information result in typical diseased state as neurosis, hypertensive disease, vegetative disfunctions.

The most important in this field are the questions of the significance of information in mass media on violence, ways of life unusual for the culture of the society, heroic style of criminal elements. The purpose of using information of such kind in the system of powerful countries with standing is unquestionable. It is precisely information processes assisted in rearranging our country to a large extent. Not without reason some investigators look at the future of the mankind in the context of a new information era replacing the agrarian and industrial ages [5].

He linguistic bases of informatics have been studied well enough and are purposefully used (both deliberately and by intuition) by modern political figures during elections and for concealing the true state of affairs. Using such methods should be finally limited by legislative acts on the basis of general convention processes and negotiations.

The influence of information on the general public is a matter of social psychologist, where as medical workers are interested only in the correlation between anxiety caused by information processes and the population health status, in the possibility of predicting adequate unfavourable responses and in preventing them with the aid of psychotherapy, drugs and other means.

The information about human being should be considered in two lines: (a) in the line of general biological erudition about human structure (constitution) and physiology (vast knowledge acquired in educational available) and (b) in the line of knowledge of a particular human being in view of the necessity of obtaining data about his work, family relations, interrelations with other people, about the disease prediction which can change his plans radically. The confidentiality of personal information is without doubt and it is protected by special legislative acts against divulging it to other persons. As for the question of informing a human of his own health and of probable unfavourable prediction, it rose greatly again in recent years which reflects democratic processes of the attitude to a person. Not long ago in our country it was conventional for medical ethics and deontology to conceal the unfavourable prediction of the disease course from the diseased. It was believed that such information would influence unfavourably on the disease course and would cause additional negative emotions. In contrast to Western countries, where any restriction of information on the disease is a ground for compensating the injury for the diseased, in our country there prevailed the practice of protecting the psychics of the diseased. The advantages and drawbacks of such approaches should be carefully considered with the aid of special investigations. It is not inconceivable that the home approach appeared long before socialistic changes in more human than the modern one based on purely financial relations. The questions of ethics and deontology of medical workers in present-day conditions require careful studying in view of new interrelations with the diseased. The highly moral professional ethics the carriers of which should be medical societies must be set against the system of inevitable rising of medical attendance in price and against corporativeness of medical workers.

The information coming to humans from the environment is predominantly verbal and its estimation greatly depends upon existing social. Ethnic, cultural «filters» created in the process of developing humans under social conditions. For everyday contacting the non-verbal information is of great importance (for example, human body pose during conversation, movements of hands, expression of eyes, etc.).

Some difficulties are connected with interpreting official information which is originally by physicians in polyclinics and hospitals. The quality of this information depends to a large extent upon the knowledge and skill of a physician, upon honest filling up by the corresponding medical certificates (for example, death and birth certificates, disease diagnoses in case histories, statistical notes about attendance of the medical institution, etc.). Unfortunately, the quality of gathering such information depends upon numerous factors which are difficult to be taken into account, namely: physician's fatigue, his mood, lack of time caused by the need of combining jobs due to the low labour payment, etc. Escaping these difficulties is in maximum providing of automatization of gathering information on human and presenting it to a physician in the generalized form, convenient for understanding. This would allow, on the one hand, to relieve a physician of doing routine work and to decrease the probability of high fatigue, and on the other hand - to facilitate the automatization of information gathering.

As to the statistics of population estimates, it is organized well enough in our country (there exists the strict system of registering population state acts), but the statistics of morbidity of all the levels is subjected to just criticism and censures. Russia turns to using new classifiers of diseases with great delay, being late for 10 years relative to other countries. At the same time using the International Classification of diseases, traumas and death causes requires significant preparation work to escape simple registering of syndromes and symptoms instead of nosological forms which is characteristic for the new revision of the classification.

The great drawback of the existing system of registration of taking medical aid is its binding with administrative regions. But the conditions of the population inhabitation do not have strict territorial correlation which requires special studies for estimating unfavourable exposure to environmental pollutants, especially in cases when these pollutions are not too considerable.

Increasing the quality of statistical materials will allow to work out the system of massive estimation of population, the qualities of the environment and dwelling, taking drugs, the time duration of reaching working places and other factors. Besides, it will provide the correct comparison between different conditions of life and morbidity which can be used as the basis of preventive procedures for non-infections diseases. The most important in this process is providing the population with true information. So that the majority of population would take the most rational made of life which would allow to preserve health and capacity for work as long as possible.

The medical science as any other modern field of knowledge undergoes the revolution in information. At the same time, precisely in this field the objective of which is the most complex subject of the environment we feel extreme retardation in information support both in examining the particular diseased and in receiving information on extremely complex multifactor processes of developing and populations. Without developing up-to-date systems of information support and automatic entering the information into data bases it is not possible to solve adequately many problems of preventing diseases and administering in public health.

## ***References***

1. M.V. Barcan, Yu.I. Musijchuk, A.V. Ravkin (1969) Methods of autoregulation theory in estimating arterial blood flow saturation with oxygen. The Journal of the Physiological Society of the USSR named after I.M.Sechenov, 55, 1250-1256.
2. Yu.I.Musijchuk, B.B.Pohodzej, I.A.Ravkin (1978) A new method of calculating the covariational function and its use for analyzing electrophysiological data. Human physiology, 4, 355-358.
3. I.L.Nazarov, V.I.Timohin, Yu.I.Musijchuk, Yu.G.Neimark (1972) On the use of the learning device for diagnosing the mitral heart defect. Experimental Surgery, # 2, 17-19.
4. R.M.Yusupov, V.P.Zabolotsky, V.P.Ivanov (1996) Human being in information space. Noology, Ecology of Noosphera, Health and Made on life.-SPb: «Nauka», 24-29.
5. L.Groff (1990) Informatic revolution: the tendencies of the global rearrangement. The problems of the theory and practice of administration, # 6, 28-30.

# KNOWLEDGE-BASED EVOKED POTENTIALS PROCESSING

E.Strakhovich,  
Institute for Informatic and Automatization of  
Russian Academy of Sciences, St.Petersburg  
e-mail:sev@iiias.spb.su

N.Ampilova,  
St.Petersburg State University  
e-mail: nba@nimm.spb.su

**Abstract.** Medical computer systems for neurosurgical patient's physiological monitoring include the analysis of biomedical signal, in particular, evoked potentials of difference modalities. The method of evoked potential processing based on the expert experience is described. The model of long-latent evoked potentials processing is created. It is implemented on IBM PC and designated to determine the auditory threshold. This system provides the high accuracy, speed and low volume of object code.

**Keywords.** Knowledge-based, biomedical records, visual perception.

**Introduction.** Patients physiological monitoring and life-threatening changes during and post-surgery are very important. The computer systems are designed to provide tools for acquiring, processing and displaying multiple types of neurophysiological data [5]. The evoked potentials analysis gives important information in such systems. An evoked potential (EP) in medicine is designated the bioelectrical signal, recorded from head surface of a man (or an animal) and obtained as the response on external action. As a rule the diagnosed record is defined as the averaging of a few signals (by the same external action) over the number of registrations. According to the action mode EP of auditory, visual and somatosensory modality are recognized. By the registration time EP may be classified as short-, average- and long-latent potentials.

Conventionally computer processing of EP followed two basic methods: using Fourier transform [1] and the structural approach to curve processing [2]. The last one requires large amount of revealable features to make the decision. Problems connected with the elimination of artifacts also appear.

We have developed the method [3,4], based on expert experience. As opposed to expert systems, which reconstruct the expert chain of arguments, we apply the obtained knowledge to creation of simple model.

**Conceptual model.** While working with the experts analyzing the EP records we have realized that they lean upon their visual perception of graphic representation of curves. Analyzing the curves the expert marks the areas, containing the most significant information. The mutual arrangement of the areas is of major importance. Regarding the to present model, the expert conclusion form may be as follows:

*"If on EP record, corresponding to action of force  $S_0$  in a time  $T_0$  the significant feature is located, then on action of force  $S_1 > S_0$  it must be located in a time  $T_1 < T_0$ ".*

The list of similar rules forms the expert knowledge block (EKB). The schema in Fig. 1 may present the interaction between model components

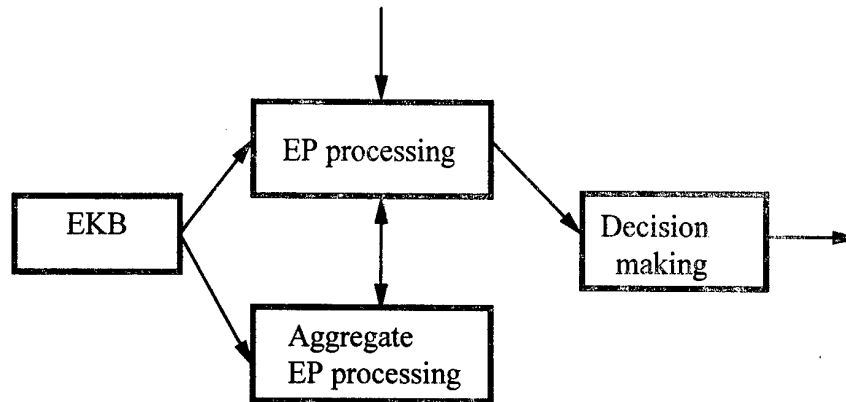


Fig. 1 Interaction between model components

The usage of EKB permits to set the constraints, which cut down the amount of information to be analyzed. As for the most part artifacts do not belong to the realizable knowledge structure, the problem does not appear.

Following this line of attack, we create the models short-latent and long-latent EP processing. The signals of this two types are distinguished as by registration time as amplitude-frequency characteristics. Hence their EKB are different.

**Formalization.** The bioelectrical signal, obtained as the response on external action, is recorded for a time  $T$  (registration time). This value depends on the external action type. Let  $N$  be a number of tests. We denote the magnitude of  $i$ -th external action as  $D_i$ , where  $1 \leq i \leq N$ . The obtained signal is recorded as a curve  $S_i$ . For any  $S_i$  we denote the most significant for expert areas as  $Q_{ij}$ , where  $1 \leq j \leq M$  and  $M$  depends on the signal type. The following equality holds

$$Q_{ij} \cap Q_{ik} = \Lambda, \quad j \neq k.$$

For any  $Q_{ij}$  we define a functional  $f(Q_{ij})$ . The point from  $\text{Int}(Q_{ij})$ , in which the functional  $f(Q_{ij})$  reaches its extreme value is denoted as  $q_{ij}$ . These points has a temporal character (latency time)  $t(q_{ij})$  - the time from beginning external action  $D_i$ . It should be noted that the area  $Q_{ij}$  may be chosen so that either the point  $q_{ij}$  exists and is unique either it is not there. So, the EKB may be constructed from the following conclusions:

$$\text{"If } \exists q_{ij}, q_{ik} \text{ and } (D_i > D_k), 1 \leq j \leq M \text{ then } t(q_{ij}) < t(q_{ik}). \text{"}$$

Hence, under our designations the problem of evoked potentials processing can be formulated as follows:

$$\text{"For given external action } D_i \text{ it takes to determine the points } q_{ij}, \text{ where } 1 \leq j \leq M. \text{"}$$

It is also possible to formulate another problems connected with EP processing. For example the task of auditory threshold determination will look like:

$$\text{"It takes to find the minimal } D_m, \text{ where } 1 \leq m \leq N, \text{ which is no more than pain action and such that for the corresponding curve } S_m \text{ all the points } q_{mj} (1 \leq j \leq M) \text{ are determined.} \text{"}$$

**Realization.** The both models were approved when the auditory EP were analyzed. The cipher signals, obtained at output of traditional medical equipment were used as input data. Model of long-latent EP processing was implemented on IBM PC and designed to determine the auditory threshold. It demonstrates high accuracy, speed and low volume of object code. The example of auditory long-latent EP processing is shown on Fig. 2

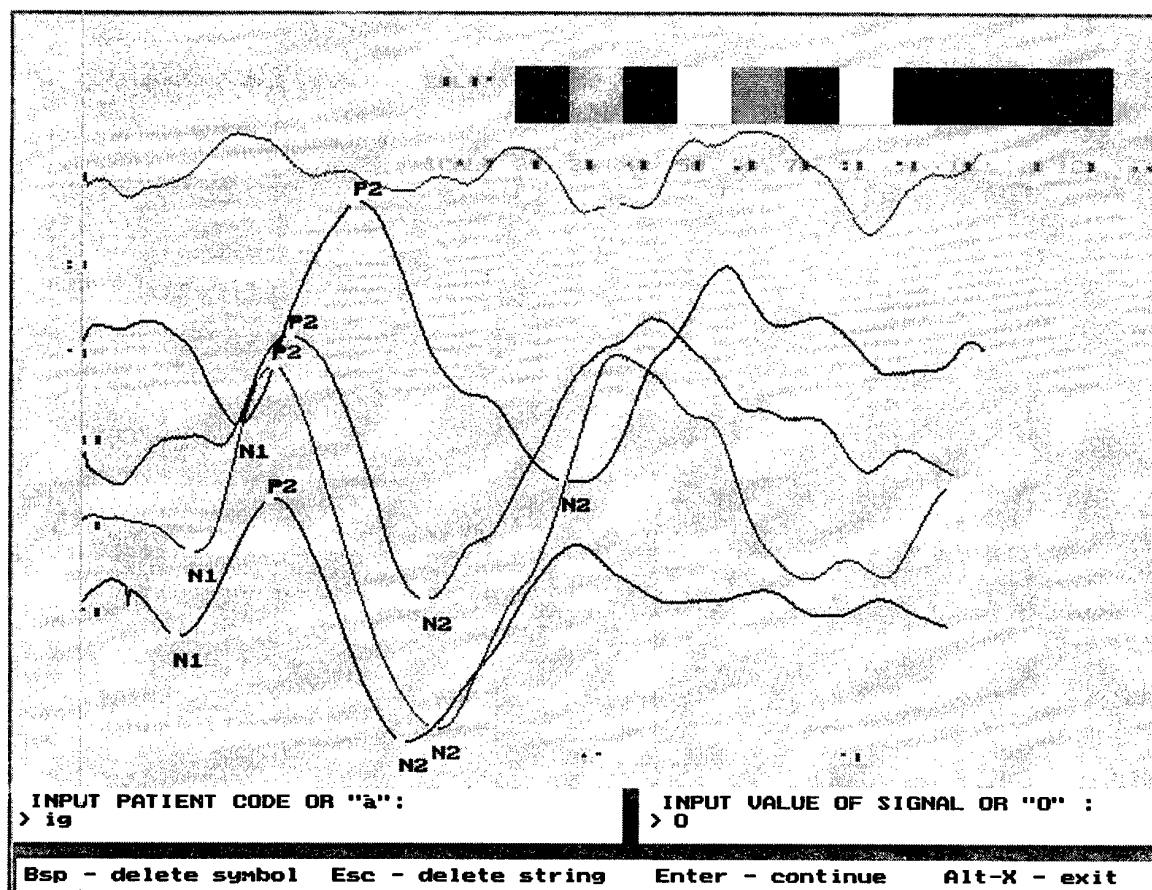


Fig. 2 Auditory long-latent EP processing result.

External action (in db) is specified on the left of curve corresponding to record of bioelectrical signal. Evoked potential for processing signal is defined by computed three points N1, P2, N2 ( $M = 3$ ). For solution of the problem of auditory threshold determination the minimal external action ( $D_m$ ) is determined as 90 db.

The system may be used in clinical investigations of patients, in the appraisal by experts of professional groups for screening, in surveying healthy people - an aging population, schoolchildren and so on.

**Conclusion.** The availability or absence points  $q_{ij}$  on analyzing curves enables to make decision about important physiological functions.

The described method may be applied to solve the problems, connected with the EP analysis of other modalities.

## ***References***

1. Vom Gehirn direkt in den Computer. *Microelektronik* 1995, v9, N1, p.52.
2. Kulanchev A., Kaplan A. Computer analysis systems of bioelectrical signals. *PC World* 1994 N2, p.132-137.
3. N.Ampilova, I.Bereznaya, E.Gurevich and E.Strakhovich. Image directed brainstem evoked potentials processing. 6th International Cleveland Clinic-Bethel Epilepsy Symposium, Bielefeld, Germany, March 23-26, 1995, p.78.
4. N.Ampilova, I.Bereznaya, K.Grachev, E.Gurevich and E.Strakhovich. The computer system, interpreting the long-latent evoked potentials records. Int. Conf. METROMED 95, S-Petersburg, June 19-22, 1995, p.218-220.
5. R.J.Scalabassi, D.Krieger, R.Simon, R.Lofink, Greg Gross and D.M.DeLauder. NeuroNet: Collaborative Intraoperative Guidance and Control. *IEEE Computer Graphics and Applications*. January, 1996, p.39-45.

# THE UNIFICATION OF THE INFORMATION AT PREPARATION OF THE REPORTS ABOUT HEALTH OF THE POPULATION

Igor A.Krasilnikov and Yuri I. Musijchuk  
St.Petersburg Administration Public Health Committee  
30, Shkapina Str., 198095, St.Petersburg, Russia  
e-mail: igorkras@infopro.spb.su

**Abstract.** The questions of standardization of statistical data considered in connection with to prepare «White book» of Russia Federation.

**Keywords.** Health of population, report, condition, parameter, unification.

**Introduction.** It is the good tradition, during the last years in Russia, to prepare and to issue so-called «White book» which is about a condition of the population health. The governmental report of the population health is preparing by the Russian Federation Ministry of the public health and based on the reports which are preparing by the subjects of the Russian Federation and on the central bodies of state statistics. The analysis of the reports from places shows that the comparison of materials of the separate reports meets large difficulties for the lack of precise standardization of a represented material, absence in a number of case an estimation of reliability of the items of the information included in the report. The that fact, that there is precisely no definition of the term «health» and the definition offered WHO requires rather large volume of the items of information not stipulated by the existing statistical reporting makes an effect for quality of the reports. The absence standards of the requirements to a represented material causes certain difficulties by comparison of parameters of health not only inside the country, but also with the countries abroad.

**Target meaning reception of the items of information about health of the population.** The parameters of health of the population cannot be considered as end in itself. They are necessary, first of all, from the point of view of forecasting labour resources in view of their number and quality (duration of able-bodied age, ability to teaching and etc.), planning of volumes of the medical help, structure of medical establishments, expenses on preventive maintenance of diseases, perfection of tax policy with the purpose of stimulation of manufactures and technologies, directed on preservation of health to a nation, substantiation of expenses of the separate citizens, family, manufacturers, territories and state on maintenance of health, at last, planning of activity of the insurance companies and etc.. Certainly, for each of listed cases it is necessary to use adequate parameters of health or morbidity of the population. The existing official statistics of public health services allows to receive only part of the information for the listed purposes.

**The international requirements to an estimation of health of the population.** The estimation of a condition of health of a nation or population can't be made on the single parameter, that requires acceptance of the decisions on creation of system of estimations. The experts WHO have allocated 9 groups of parameters, describing a condition of health of the population of cities [1]:

1. *Population: total number of citizens, age and sexual structure of the population in intervals in 5 years, percent of children in the age of 0-1; 1-14 and 5-14 years, percent of the oldest population is more senior 65,75 and 85 years, historical trends and the future population projections, ethnic origin of major population (it is important for cities with a high level of migration).*

2. *Health status: 2.1 Vital Statistics: birth rate and fertility, death rates, standardised mortality ratios (national standard) as a whole and for the separate reasons of death, perinatal mortality rate, maternal mortality rate, frequency of abortions, «years of life lost», «avoidable death; 2.2 parameters of morbidity: parameters of statistics of public health services, compulsory admission to hospital, morbidity of infectious diseases, measurement of sensation of health and well-being.*

3. **Lifestyles: smoking, use of alcohol, consumption of medicines, physical culture (exercise), feed (height and weight).**

4. **Housing: the characteristics of habitation (water supply, hot water, centralised heating, toilet, kitchen and etc.), density of the population (number of the inhabitants in the house, in a room, quantity toilet and etc.).**

5. *Socio-economic condition: formation(training), employment, incomes, crimes and violence, cultural life (presence of cinemas, sporting structures, exhibitions, museums, concert halls and etc.).*

6. **Physical environment: air quality, water quality, noise, radiation, open spaces, factors of anxiety: rodents, insects; food quality.**

7. **Inequality: the lowest socio economic classes, refugees, person with chronic physical disability, mentally ill people, people with learning difficulties, dependent from alcohol and medicines, homosexuals, homeless people.**

8. **Physical and social infrastructure: public and personal transport, means of the communications, urban renewal, planning of city, program of formation and training, financial maintenance and creation of working places, formation of groups of the communications.**

9. *Medical service and policies: immunisation, screenings, understanding of cost of disease, death; condition of statistics of diseases, planning of family, prevention stresses, formation sanitary education, control for an environment (smoking in public places, acts for restriction of consumption of alcohol, distribution of radioactive components, control of air and water).*

In the resulted list of parameters the italics allocates those from them, on which the existing domestic statistics has the information, assembled according to the statistical to rules or was published by other establishments. The tax of the information on some diseases, for example, infectious diseases in city is rather well adjusted. The tax of the information on the other directions needs the of the creation standartng methods of the tax and estimation of such information with the purpose of maintenance of reliability and comparability. Moreover the resulted list needs the future specification and detailed elaboration in view of national and ethnic peculiarities. For ex-

ample, alongside with a parameter «years of life lost» expediently to use a parameter «disability-adjusted life years».

The information Sources about a condition of the population health in St.Petersburg can be officially issued materials [2 - 4] and the results of the population-based Cancer Registry of St. Petersburg [5], which is preparing for input on diabetes, numerous development, spent by research institutes and groups [6, 7], as well.

The large meaning for the forecasting of the health condition is the precise representation about distribution of the factors of risk of this or that diseases, however the items of information, assembled rather correctly about them, is not enough in the domestic literature. The selection of the information for inclusion in the report on health of the population of city or region demands a careful estimation of reliability of the information represented in scientific development.

The report about a condition of health demands a participation in its drawing up of a plenty of establishments and organisations, owning the information on listed sections, that is why it must be prepared for and on behalf of administrations of the subjects of Russian Federation, it must be supported and financed by them. Such approach will allowed to consider the information on health as a basis for planning development of regions, for development of policy in the field of guards of health by the legislative bodies of authority to give the information to the population about a condition of health wider in hope for more wide circulation of a healthy image of life.

The report on health should contains the certain information, assembled and submitted on uniform techniques in all regions of Russia, that allowed to compare them and also must contains the information, specific to each region on the discretion of the composers of the report.

It is desirable, that the report contains the primary information in figures, allowing to receive any indirect parameters derivative the experts in the field of public health services, analytical part for the workers of administrative structures and mass media, and also the popular exposition of a material for wide sections of the population.

The basic requirement to represented materials is their reliability, and also correct concerning an opportunity of the comparison of information items which are representing the various groups of parameters. For example, a division into districts of pollution of environment should coincide with the region of the tax of information on morbidity; the distribution of the factors of diseases risk should be resulted for allocated of groups according to age in population and etc.. The items of information for territories with rather small number (a several hundred of thousands the person) should be in the certain order appreciated statistically and to contain confidence intervals of fluctuation of parameters, that is also certain for all the selective researches.

The certain unification is demanded by a choice of the standard age and sexual distribution of the population. The use of the advanced countries standards with a high level of health (Japan, USA) hardly is expedient for national statistics because of large distinctions in expenses on public health services and average duration of forthcoming life. Russia stands closer to the all-European standard, which can be used as reference for planning achievement on the first stage of develop-

ment. Comparisons inside the country is expediently conduct according to the national age standard, prospects of objectification which creates forthcoming the general census of the population.

Forecasting a condition of health requires(demands) representation of materials as dynamic lines, sufficient for construction of the forecasts. The minimum duration for such lines should be no less than 5 years, the comparisons only with the previous year can be used in exclusive cases. The preservation of continuity in ways of the tax of the information therefore remains to one of the most urgent questions. Planning of the long researches, creation of the registers in various directions will allow to decide arising problems of correct comparison of the information items.

One of the ways to increase the quality of the statistical information about a condition of health of the citizens is to use it for the estimation of activity of public health services establishments in the monitoring system of departmental, and extradepartmental quality, and also the application of it for the purposes of insurance public health services.

The territorial binding of the information according to the address or the name with use of computer geoinformation systems makes possible to create more modern ways of an interrelation estimation between conditions of living, pollution of an environment, and also to supply the multifactoral analysis of the health population reasons of deterioration, allowing to avoid the errors by the comparison of parameters only in the two-measure space.

Development of the modern means of electronic communication allows to hope that the creation of uniform information space in our country with deserving representation in it of the medical information, volumes and contents of which will allow to accept the administrative decisions at any levels according to the view of data on a condition of health of the population, its number and prospects of development. Thus acceptance of the decisions not only in the field of public health services, but also in other spheres, concerning to ability to live of the people and their well-being, should take into account the information on a condition of health and it forecasting estimation.

**Conclusion.** The co-ordinating role the standardization of the information tax about a condition of health of the population should belong to the public health services of Committee State Duma, Government and Ministry of public health services of Russian Federation, which should define list of the certain information, included in the national report on the population health of Russia.

For maintenance of the standardizing of the information tax on all groups of parameters, recommended by the international requirements, financing of scientific establishments is appropriate as with state, and local budgets and long-term planning of particular researches is necessary. The insurance companies, interested in such information, can be considered as the potential customers of the specialised information about an environment and health of the population, however the culture of insurance business in Russia does not allow in the nearest years to hope on rather volumetric and serious the orders from these companies.

Complexity of drawing up of the reports about health creations in regions of certain(determined) groups, coordinating efforts on planning of the collection of information, processing of the in-

formation and spelling of the report require.

**Reference:**

1. City Health Profiles. Guidance Document. A background document Health city technical Symposium (1994) Poznan, 22-23 September.
2. Public health services of St.-Petersburg in figures. Edit. V.G.Korukin, St.Petersburg, 1994.
3. The urban report on a condition of health of the population of St.-Petersburg in 1994. St.Petersburg Administration Public Health Committee, St.Petersburg, 1995.
4. St.-Petersburg - health of city. Edit. V.G.Korukin, St.Petersburg, , 1995.
5. Cancer in the population of St.Petersburg. Edit. V.M.Merabishvili, St.Petersburg, 1996.
6. Tihomirov S.M. (1990) The analysis of quota and factors of risk of formation of dependent behaviour. Urgent questions of formation of lifestyles, M.,176-177.
7. Aleksandrova N.I., Tavdidishvili T.S.(1990) A lifestyle, factors of risk, prevalence and current diseases of stomach of the children. (1990) HLS 90. An international conference «The healthy lifestyle», V.2, Novgorod, 193-194.

## **CHAPTER XII**

### ***ADAPTIVE CONTROL***

THIS CHAPTER INCLUDES PAPERS  
PRESENTED AT THE CONFERENCE SESSION:

**ADAPTIVE CONTROL**

Organized by: ***Prof. Alexander L. Fradkov.***

# ADAPTIVE FEEDFORWARD COMPENSATION OF STRUCTURAL VIBRATION IN SATELLITE-BASED LASER COMMUNICATION SYSTEMS

Victor A. Skormin, Donald J. Nicholson and Timothy E. Busch  
Watson School of Engineering, Electrical Engineering  
Binghamton University, Binghamton, NY 13902, USA  
Space Communication Branch, US Air Force Rome Laboratory  
Rome NY 13441-5700, USA  
e-mail: vskormin@binghamton.edu

**Abstract.** A technique, resulting in the significant reduction of negative effects of platform vibration on the beam positioning accuracy of satellite- or aircraft-based laser communication systems, is presented. It implies vibration monitoring and self-tuning feedforward compensation of vibration effects on beam position error. The developed technique is implemented in laboratory environment and is verified by computer simulation and experimentally. The suggested approach has the potential for implementation in many practical high precision positioning systems, affected by structural vibrations.

**Key words.** Space communication, laser, satellite, vibration rejection, position control.

**Introduction.** Laser technology can be successfully used in space communication. A properly positioned laser beam provides a highly efficient media for intersatellite, satellite-to-ground, satellite-to-air, and satellite-to-water communication. The words "properly positioned" represent the key condition for reliable communication. While the transmitting and receiving stations can be separated by a thousand mile distance and be engaged in a complex motion pattern at supersonic speed, the laser beam positioning constitutes a formidable engineering problem. The problem is further complicated by satellite jitter and aircraft vibration [1].

Ideally, laser beam generated by the transmitting station should be positioned in the center of the array of light-sensitive elements (ALSE) of the receiver. Platform vibration (jitter) of the transmitting station results in the oscillation of the laser beam, jitter of the receiving station causes the oscillation of its ALSE. Both phenomena lead to the noticeable fluctuation of the laser beam position on the ALSE of the receiver, and potentially may result in disrupting the communication session. Deviation of the beam position from the center of an ALSE can be accurately measured and provides an error signal for beam positioning control systems. These systems are responsible for the orientation of the coarse steering mirror (CSM) and the fine steering mirror (FSM) and are capable of reduction of the jitter-caused beam fluctuation. However, CSM and FSM operate at high frequencies, up to 5 KHz, and therefore the effectiveness of feedback control schemes is limited by the response time of the system hardware.

beam position error, reconstruction of the vibration effects on the beam position error, and results in a self-tuning feedforward compensation scheme, utilizing already existing control systems.

**The Compensation Principle.** Platform vibration in satellite communication systems is one of the most important conditions adversely affecting operation of a laser-based communication system. While aircraft vibration is well studied, satellite jitter is a far less known phenomenon. Internal cyclic moments created by gyroscopes, servo\_mechanisms, etc., thermal expansion/contraction of the platform body, and gravitational field effects create a constant source of satellite platform vibration. Other sources of vibration exist that are not cyclic in nature and create dynamic, unpredictable perturbations of the platform position. The dominance of any one source of vibration will depend entirely on the physical nature of the satellite and on the specifics of operation. A number of researchers represent satellite jitter by non-stationary "colored" noise under 1 KHz dominated by low frequencies with a number of distinct "spikes" on the higher side of the spectra.

Vibration of a satellite or an aircraft platform can be monitored by accelerometers. It is important that accelerometry cannot be affected by weightlessness. A miniature accelerometer, common in microprocessor-based vibration monitoring systems, has well-defined dynamic properties which can be expressed in the form of a transfer function.

In order to describe the arising problem mathematically, introduce the following notations:

- vector  $\mathbf{x}(t)=[x_1(t) \ x_2(t) \ x_3(t)]'$  represents three components of vibration,
- vector  $\mathbf{y}(t)=[y_1(t) \ y_2(t) \ y_3(t)]'$  is the signal recorded by three orthogonally-positioned accelerometers,
- vector  $\mathbf{e}(t)=[e_1(t) \ e_2(t)]'$  represents two components (elevation and azimuth) of the beam position error in the ALSE plane,
- $\mathbf{X}(s)$ ,  $\mathbf{Y}(s)$ , and  $\mathbf{E}(s)$  are Laplace transforms of the appropriate signals,
- diagonal transfer matrix  $\mathbf{A}(s)$  represents dynamic properties of accelerometers,
- transfer matrix  $\mathbf{G}(s)$  represents closed-loop dynamics of the existing beam steering control system,
- transfer matrix  $\mathbf{W}(s)$  relates beam position error to the platform vibration,
- variable  $t$  represents continuous time,  $s$  is Laplace variable, and  $'$  is the transpose symbol.

It can be seen that  $\mathbf{E}(s)=\mathbf{W}(s)\mathbf{X}(s)$  and  $\mathbf{Y}(s)=\mathbf{A}(s)\mathbf{X}(s)$ . By definition, a "perfect" feedforward compensation scheme has to satisfy the equation

$$\mathbf{E}(s)=\mathbf{W}(s)\mathbf{X}(s)+\mathbf{G}(s)\mathbf{U}(s)=0 \quad (1)$$

where  $\mathbf{U}(s)$  is the feedforward compensation signal, applied to the input of the existing control system. This signal can be defined on the basis of the vibration measurements as follows,

$$\mathbf{U}(s)=\mathbf{F}(s)\mathbf{Y}(s)=\mathbf{F}(s)\mathbf{A}(s)\mathbf{X}(s) \quad (2)$$

where  $\mathbf{F}(s)$  is a transfer matrix representing the feedforward controller. Finally,

$$\mathbf{E}(s)=[\mathbf{W}(s)+\mathbf{G}(s)\mathbf{F}(s)\mathbf{A}(s)]\mathbf{X}(s) \quad (3)$$

or in the frequency-domain,

$$\mathbf{E}(j\omega)=[\mathbf{W}(j\omega)+\mathbf{G}(j\omega)\mathbf{F}(j\omega)\mathbf{A}(j\omega)]\mathbf{X}(j\omega) \quad (4)$$

The effectiveness of the suggested technique can be explained by the fast response time of feedforward control schemes: feedforward control implies that the disturbance (i.e. the vibration), propagating along the mechanical path, is monitored and electrically compensated for *before* it affects the system output. In addition, the fast response time will be assured by the use of "fast" electrical compensation circuitry while the disturbance follows the "slow mechanical path". It should be emphasized that the suggested compensation technique is to supplement the existing control circuitry of CSM and FSM, which performs more functions than just jitter reduction.

Note that transfer matrix  $W(s)$  is unknown and time-dependent due to the varying environmental conditions and status of the mechanical structure; transfer matrix  $A(s)$  is also affected by environmental conditions (primarily by temperature). Therefore, it is unrealistic to expect that controller  $F(s)$  can be defined from the equation

$$W(s) + G(s)F(s)A(s) = 0 \quad (5)$$

and the "perfect" vibration compensation can be achieved.

The only opportunity for the definition of controller  $F(s)$  is based on the minimization of the quadratic expression for the "total power" of error  $e(t)$ ,

$$P = \text{tr}[E(j\omega)E^*(j\omega)] \quad (6)$$

with respect to parameters of a predefined transfer matrix  $F(s)$ . (Recall that  $\text{tr}[M]$  is the sum of diagonal elements of matrix  $M$ , and  $*$  is the symbol of conjugate.)

**Self-Tuning Compensation Scheme.** Although expression (6) is mathematically correct, the following is a much more useful expression

$$P(t) = \sigma_1^2(t) + \sigma_2^2(t) \quad (7)$$

which defines the "total power" of the error through variances of the elevation and azimuth components of the beam position error,  $\sigma_1^2(t)$  and  $\sigma_2^2(t)$ , which can be easily defined. Indeed,

$$\sigma_i^2(t) = T^{-1} \int_{t-T}^t e_i^2(\tau) d\tau, \quad i=1,2 \quad (8)$$

where  $T$  is some small time interval. Introduce a vector  $\alpha = [\alpha_1 \ \alpha_2 \ \dots \ \alpha_N]'$  representing adjustable parameters of the feedforward controller  $F(s)$ , i.e.  $F(s) = F(\alpha, s)$ . It could be seen that criterion (6) also depends on parameters  $\alpha$ , i.e.  $P = P(\alpha)$ . While an analytical expression  $P(\alpha)$  does not exist, this relationship can be defined numerically by introducing any desired set of parameters

$$\alpha(t) = [\alpha_1(t) \ \alpha_2(t) \ \dots \ \alpha_N(t)]'$$

and evaluating the corresponding  $P(t) = P[\alpha(t)]$  value as per (8) and (7). This situation facilitates the application of a numerical optimization procedure which minimizes criterion  $P(\alpha)$  iteratively thus providing the rejection of vibration-induced errors. The authors have successfully implemented direct optimization search (Simplex by Nelder and Mead) for the solution of this problem. It is quite important that the selected procedure is capable of automatic decrease and increase of the search step size which assures its ability to maintain the minimum error condition in spite of time-dependence of characteristics  $A(s)$  and  $W(s)$ .

It can be seen that the developed vibration compensation technique results in a direct self-tuning of the feedforward controller, i.e. it does not require identification or parameter estimation (tracking) of transfer matrices  $W(s)$  and  $A(s)$ . The optimization search procedure, running continuously, "protects" the feedforward controller from such conditions as temperature effect on accelerometers, changing characteristics of the mechanical vibration propagation path, etc.

**Implementation and Testing.** Satellite-based laser communication still is an experimental technology implemented in laboratory conditions. A prototype of such a system intended for optical intersatellite crosslinks was developed by Ball Aerospace Corporation and installed in the Space Communication Branch of Rome Laboratory. It consists of several terminals acting as transmitters and receivers, representing three satellites in low earth orbit, medium earth orbit, or geosynchronous earth orbit. Each terminal has an optical table on which electro-optic and electro-mechanic systems are mounted. An electro-optic system includes lasers, detectors, beam conditioners, ray optics, filters, and beam directors. Every electro-mechanic system consists of the computer, the computer interface, the fine steering subsystem, the coarse steering subsystem, and some auxiliary subsystems. Optical tables are mounted on special pneumatic suspensions and can be subjected to the desired vibration patterns.

In order to implement the developed feedforward compensation technique, one of the optical tables was equipped with a system of three orthogonally-positioned micro-accelerometers interfaced with a PC. A special PC-driven audio system was developed for the implementation of various vibration environments including satellite jitter.

The feedforward controller was defined by the transfer matrix

$$F(s) = \{\alpha_{ij}^1 + \alpha_{ij}^2/s + \alpha_{ij}^3 s, i=1,2, j=1,2,3\}$$

and implemented in the PC through a finite-difference code. It could be seen that its elements represent proportional-integral-derivative (PID) configurations which are known as flexible and robust. While dealing simultaneously with 18 variables of optimization ( $\alpha_{ij}^k$ ) constitutes a formidable task, the assumption was made that transfer matrix  $G(s)$  is close to diagonal and, therefore, variances  $\sigma_1^2(t)$  and  $\sigma_2^2(t)$  can be minimized one at a time. This assumption allows for dealing with two optimization problems with 9 variables each and facilitates parallel processing.

The testing results can be summarized as follows,

- The optimization procedure, working with 9 variables implemented in a 486 PC/33MHz converges within 3-5 sec from zero initial conditions. In the tracking regime, i.e. when the minimum point is found the procedure converges within a fraction of a second. It also should be noted that the procedure is suitable for parallel processing.

- The existing feedback control system is quite efficient at low frequencies, under 200 Hz. The effectiveness of the feedforward compensation scheme increases with the increase of frequency of vibration. Within the .8 - 1 KHz range it results in at least 30 db error reduction.

***Additional Applications.*** The developed vibration compensation approach can be successfully implemented in various technologies adversely affected by environmental structural vibrations. High precision multi-axis positioning systems and profilometers of various types present good examples of such technologies. Application of feedback control schemes for vibration reduction in positioning systems and profilometers results in unnecessary high control efforts and expensive system hardware. Use of special vibration "insulations" increases weight and cost of these systems. The self-tuning feedforward approach suggested herein does not eliminate the need for feedback control, but allows for a rational "distribution of tasks" thus leading to improved system operation and reduced cost and weight.

***Acknowledgement.*** The authors are grateful to Dr. Mark A. Tascillo (presently with Ford Motor Company) for his contribution to this research.

### ***References***

1. Dumas, R., Laurent, B. (1990) System Test Bed for Demonstration of the Optical Space Communications Feasibility. SPIE Vol. 1218, Free-Space Laser Communication Technologies
2. Skormin, V., Tascillo, M. and Nicholson, D. (1993) A Jitter Rejection Technique in a Satellite-Based Laser Communication Systems. Optical Engineering
3. Gleyzer, B., Skormin, V. (1992) Autotuning in Multi-Axis DSP-Based Servo Controllers. Motion Control, Vol. 3 No.6
4. Skormin, V.A., Busch, T.E. (1997) Demonstration of Jitter Rejection Technique for Free Space Laser Communication. IEEE Transactions on Aerospace and Electronic Systems

# A Revised Blind Equalization Algorithm Adaptive for MPSK Modulation

Hongchuan Yu, Nansheng Kang, Tinjian Fang  
Institute of Intelligent Machines Academia Sinica  
Hefei 230031, P. R. China  
e-mail: aleph@ustc.edu.cn

**ABSTRACT.** In this paper, a revised blind equalization algorithm adaptive for MPSK modulation is presented. Analysis and computer simulations indicate that it maintains the convergence performance and symbol error performance of the original blind equalization algorithm (the hybrid blind equalization algorithm), moreover, it produces less vestigial error after convergence and accelerates convergence speed.

**KEYWORDS.** Revised hybrid blind equalization algorithm, Equalizer, Cost function, Vestigial error, MSE curve

## § 1. Introduction

Blind equalization (BE) algorithm has been attentioned widely since it was raised. It is based on a priori knowledge about geometry or the constellation of the received signal without the transmission of a known training sequence. Up till now, a lot of BE algorithms have been reported, for example, Sato algorithm [1], Godard CMA algorithm [2], etc. All above were used generally in multilevel data transmission system (such as QAM). Studies indicated [3, 4] that the ill-convergence existed though they had concave characteristics. P. He reported [5] that the Hybrid BE algorithm was used to overcome the ill-convergence for MPSK modulation. But the large vestigial error still existed after convergence.

In this paper, the hybrid BE algorithm is revised. In § 2, the hybrid BE algorithm adaptive for MPSK modulation is analysed. In § 3, it is revised, so that it produces less vestigial error after convergence and accelerates convergence speed. The results of computer simulation and conclusion are given in § 4.

## § 2. Analysis of the Hybrid BE Algorithm adaptive for MPSK Modulation

### The Hybrid BE Algorithm

We consider a MPSK complex baseband data transmission system of the general model shown in Fig. 1. The transmitted baseband signal is of the form

$$I_k = a_k + jb_k = A \cdot \exp\{j\varphi_k\} \quad (1)$$

Let  $A=1$ ,  $\varphi_k = (2K+1)\pi/M$ ,  $K=0, 1, \dots, M-1$ . Denoting by  $\{X_k\}$  the channel output,  $\{Y_k\}$  the equalizer output,  $\{\hat{I}_k\}$  the decision device output. The equalizer structure is a  $N$ -tap FIR filter, its coefficient vector is  $C^T(K) = \{C_0, C_1, \dots, C_N\}$ , and sampling interval

is symbol interval  $T_b$ . The equalizer output  $\{Y_k\}$  can be written as

$$\{Y_k\} = C^T(K)X(K) \quad (2)$$

The input of equalizer is  $X^T(K) = \{X_k, X_{k-1}, \dots, X_{k-N}\}$ . The BE Equalizer attempts to adjust coefficient vector to achieve significant removal of ISI.

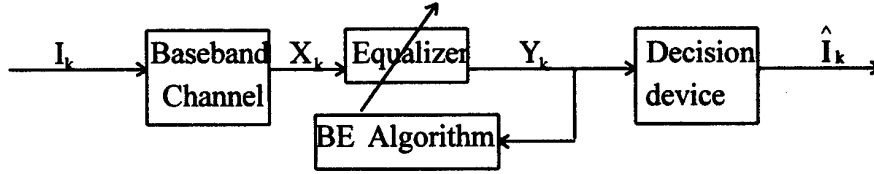


Fig.1 Adaptive blind channel equalization system

The hybrid BE algorithm is a compromise proposal, which gives consideration to both the classic LMS adaptive equalization algorithm and Godard CMA algorithm. For MPSK modulation, its cost function is of the form [5]

$$J(K) = \frac{1}{2}\lambda|Y_k - I_k|^2 + \frac{1}{2M}(1-\lambda)(|Y_k|^M + 1)^2 \quad (3)$$

where  $\lambda$  is a coefficient,  $0 < \lambda < 1$ . An iterative gradient search algorithm (i. e. the hybrid BE algorithm [5]) was established as

$$\begin{aligned} C(K+1) &= C(K) - \mu \frac{\partial J(K)}{\partial C(K)} \\ &= C(K) - \mu [\lambda(Y_k - I_k) + (1-\lambda)(|Y_k|^M + 1)Y_k|Y_k|^{M-2}]X^*(K) \end{aligned} \quad (4)$$

Denoting by " $*$ " complex conjugate,  $\mu$  the step size. In practice,  $\hat{I}_k$  will be used to replace  $I_k$ . As the hybrid BE algorithm's cost function  $J(K)$  involves instantaneous information of the transmitted signal, obviously, its symbol error performance is better than that of the Godard CMA algorithm.

#### Convergence Performance Analysis

Hessian matrix of the cost function  $J(K)$  can be expressed as

$$\begin{aligned} H(C) &= E \left\{ \frac{\partial^2 J(K)}{\partial C^2(K)} \right\} \\ &= E \{ [\lambda + (1-\lambda)[(2M-1)|Y_k|^{2M-2} + (M-1)|Y_k|^{M-2}]]X^*(K)X^T(K) \} \end{aligned} \quad (5)$$

Only if  $H(C)$  matrix is positive definite,  $J(K)$  can be a concave function about  $C(K)$ , meanwhile, the hybrid BE algorithm is convergent and has only one global minimum. Because  $E\{X^*(K)X^T(K)\}$  matrix is positive definite in Eq. (5), if

$$\lambda + (1-\lambda)[(2M-1)E\{|Y_k|^{2M-2}\} + (M-1)E\{|Y_k|^{M-2}\}] > 0 \quad (6),$$

$H(C)$  matrix must be positive definite. As  $0 < \lambda < 1$  and  $M \geq 2$ , inequality (6) is tenable. Thus, the hybrid BE algorithm can converge on the global minimum. When equalizer coefficient vector trajectory is captured by a local minimum, the fluctuation of the first term (i. e. decision directed error) far exceeds that of the second term in the cost function  $J(K)$ . So the

equalizer won't converge on the local minimum and continue to adjust coefficient vector  $C(K)$  to escape it. As soon as the trajectory is captured by the global minimum, the cost function  $J(K)$  will converge upon it. So that, the hybrid BE algorithm is able to overcome the ill-convergence for MPSK equalization.

In practice, because of time-variant channel, the signal to noise ratio of the received signal fluctuates frequently. In order to achieve quite good performance of convergence and symbol error, coefficient  $\lambda$  must be variable following with time-variant channel.  $\lambda$  can be expressed as [5]

$$\lambda = \frac{\sigma_1^2 E\{\omega_2^2(K)\}}{a\sigma_2^2 E\{\omega_1^2(K)\} + \sigma_1^2 E\{\omega_2^2(K)\}} \quad (7)$$

In Eq. (7),  $a$  is a constant, generally  $a=0.5$ ,

$$\begin{aligned} \omega_1^2(K) &= \frac{1}{2} |Y_K - I_K|^2 \\ \omega_2^2(K) &= \frac{1}{2M} |Y_K|^M + 1)^2 \\ \sigma_1^2 &= E\{\omega_1^2(K)\} \\ \sigma_2^2 &= E\{\omega_2^2(K)\} \end{aligned}$$

When the BE algorithm converges,  $\sigma_1^2, \sigma_2^2$  are constants. In Eq. (7), it is difficult to obtain the values of  $E\{\omega_1^2(K)\}, E\{\omega_2^2(K)\}$ . Generally, we obtain their estimates according to the weighting iterative gradient algorithm [6]

$$\sigma_1^2(K) = E\{\omega_1^2(K)\} = r\sigma_1^2(K-1) + (1-r)\omega_1^2(K) \quad (8)$$

$$\sigma_2^2(K) = E\{\omega_2^2(K)\} = r\sigma_2^2(K-1) + (1-r)\omega_2^2(K) \quad (9)$$

Where  $r$  is the weighting,  $0 < r < 1$ . It determines memory length and estimate precision of the BE algorithm. For slowly time-variant channel,  $0.9 < r < 1$ .

### Computer Simulation

Computer simulation [5] has shown that the convergence performance of this hybrid BE algorithm is similar to that of the Godard CMA algorithm and its symbol error performance close to the classic LMS algorithm. As Godard CMA algorithm, the hybrid BE algorithm has large vestigial error after convergence, which influences resisting noise performance of data transmission system. In order to decrease vestigial error, the hybrid BE algorithm for MPSK modulation is revised in following section.

### § 3. Revised Hybrid BE Algorithm For MPSK Modulation

Consider the error function in Eq. (4)

$$f(Y_K) = \lambda(Y_K - \hat{I}_K) + (1-\lambda)(|Y_K|^M + 1)Y_K|Y_K|^{M-2} \quad (10)$$

Analysis indicates that the vestigial error originate from  $|Y_K|^M$  (biased estimates of  $I_K^M$ ,  $I_K^M = \exp\{jM\phi_K\} = -1$ ). Thus, when the hybrid BE algorithm converges, we must restrain influence of the second term in Eq. (10) to decrease vestigial error.

Before convergence, as the fluctuation of the first term (i. e. decision directed error) far

exceeds that of the second term in Eq. (10), the coefficient  $\lambda$  is so small as to attenuate the influence of the first term. Because of  $\lambda \neq 0$ , the hybrid BE algorithm is able to overcome the ill-convergence. After convergence, in order to improve symbol error performance,  $\lambda$  is so big as to enhance the influence of the decision directed error. But the influence of the second term is still great, and increases vestigial error. If  $\lambda=1$ , the hybrid BE algorithm degrades into the classic LMS algorithm. Because the time-variant channel often causes the LMS algorithm divergence, much time must be spent to adjust equalizer. So, the second term in Eq. (10) must be attenuated as intense as possible after convergence.

Following the above analysis, it is convenient to replace  $f(Y_K)$  by

$$\hat{f}(Y_K) = \lambda(Y_K - \hat{I}_K) + (1-\lambda)(|Y_K|^M + 1)Y_K|Y_K|^{M-2}[q(Y_K - \hat{I}_K) + 1 - q] \quad (11)$$

In Eq. (11),  $q$  is flag function,

$$q = \begin{cases} 1, & |\bar{e}_D| < b_1, \text{ and } |e_D| < b_2, b_1 < b_2 \\ 0, & \text{others} \end{cases}$$

where the decision directed error  $e_D = Y_K - \hat{I}_K$ ,  $\bar{e}_D$  accounts for mean decision directed error of the preceding  $N$  points, and  $b_1, b_2$  are controlling parameters. In Eq. (8),  $\sigma_1(K)$  is an upper limit estimate of  $|\bar{e}_D|$ . When  $|\sigma_1^2(K) - \sigma_1^2(K-1)| < 0.01\sigma_1^2(K)$ , the revised BE algorithm maybe has converged. At this moment, let  $b_1 = 0.9\sigma_1(K)$ ,  $b_2 = \alpha \cdot b_1$ , and  $1.0 < \alpha < 1.5$ . The revised hybrid BE algorithm is described as following:

① In initial stage,  $e_D$  is very large and  $q=0$ , the equalizer attempts to adjust coefficient vector by the hybrid BE algorithm with iterative formula Eq. (4). Then, the revised BE algorithm can converge on global minimum.

② In initial convergence stage,  $e_D$  is attenuating and  $q=1$ , the error function is  $\hat{f}(Y_K)$ , and its iterative formula is of the form

$$C(K+1) = C(K) - \mu \hat{f}(Y_K) X^*(K) \quad (12)$$

Where the influence of the biased estimates is restrained, meanwhile, prompting convergence speed.

③ In further convergence stage, the error function  $\hat{f}(Y_K)$  is decreasing significantly, and determined mainly by  $e_D$ , so that the vestigial error tends to be minimum.

④ When channel distorts, the algorithm is diverging, and  $e_D$  is increasing rapidly, so that  $q=0$ . The revised BE algorithm is transformed into the hybrid BE algorithm. The equalizer returns to step ①.

#### § 4. Simulation and Conclusion

The minimum mean square error (MSE) cost function curves of the hybrid BE algorithm and the revised hybrid BE algorithm simulated by computer are shown in Fig. 2. The simulation conditions are:

① the channel model is a 3-tap FIR structure, its transfer function [5] is

$$H(Z) = 0.3 + 0.9Z^{-1} + 0.3Z^{-2},$$

② Input is 8PSK signal,

③the equalizer is 12—tap FIR filter, and its initial coefficient vector  $C^T(0) = (1.0, 0, 0, \dots, 0)$ ,

④ $\mu=0.005, r=0.95, \alpha=1.2, \sigma_2^2/\sigma_1^2=4.0$ .

In Fig. 2, obviously, MSE cost function of the revised BE algorithm is much decreased compared to that of the hybrid BE algorithm.

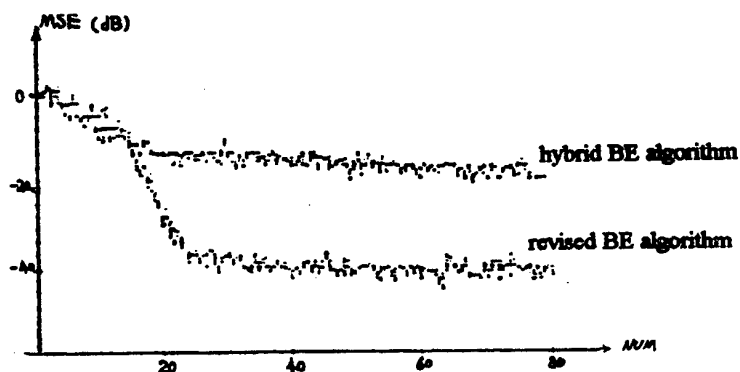


Fig. 2 MSE curves of the Revised Hybrid BE algorithm and the Hybrid BE algorithm

### References

1. Sato Y. (1975), A method of self—recovering equalization for multilevel amplitude modulation, IEEE Trans. on comm. , Vol. COM-23, No. 6, pp. 679-682
2. Godard D. N. (1980), Self-recovering equalization and carrier tracing in twodimensional data communication systems, IEEE Trans. on Comm. , Vol. COM-28, No. 11, pp. 1867-1875
3. Ding Z. , et al. (1991), Ill-convergence of Godard blind equalizers in data communication systems, IEEE Trans. on Comm. , Vol. COM—39, No. 9, pp. 1313-1327
4. Zhuang J. D. ,Zhu X. L. ,et al. (1992), ACTA Electronica Sinica, Vol. 20, No. 7, pp. 28-35
5. He P. (1995), Journal Of China Institute Of Comm. , Vol. 16, No. 2, pp. 97-102
6. Ye H. ,Wu B. X. ,et al. (1990), ACTA Electronica Sinica, Vol. 18, No. 7, pp. 63-69

# OUTPUT-FEEDBACK ADAPTIVE COMPENSATOR OF EXTERNAL DISTURBANCES FOR NONLINEAR SYSTEMS

A.Yu. Pogromsky

Inst. for Problems of Mech. Engg.

61, Bolshoy. V.O. 199178,

St. Petersburg, Russia

sasha@ccs.ipme.ru

V.O. Nikiforov

Inst. of Fine Mechanics and Optics

14, Sablinskaya, 197101,

St. Petersburg, Russia

nikifor@cyber.beam.ifmo.ru

**Abstract.** In this paper, we propose an adaptive compensator for an external inaccessible disturbance which may be presented as a finite sum of sinusoids of unknown frequencies, magnitudes and initial phases. The disturbance acts in span of control and the plant is assumed to be an exponentially passive nonlinear system. Under these assumptions it is shown how to design the adaptive compensator which does not depend on the parameters of the disturbance signal and on the system parameters, moreover, the compensator requires only output on-line measurements.

**Keywords.** Adaptive, nonlinear control, observers.

**1. Introduction.** In many control problems there is a requirement to suppress disturbances affecting the plant and the control objective is complete suppression of the effects of noise or external excitation. Among the most successful techniques dealing with the presense of disturbances we should refer to those based on the Internal Model Principle (IMP) [9, 7, 4]. The IMP means that the disturbances can be described as output of some external generator (or exosystem) and to solve the control problem one should consider the augmented plant including dynamics corresponding to the exosystem. According to the Principle the effects of the disturbances can be eliminated if the external generator model is suitably reduplicated in the feedback path of the closed loop control system.

In order to treat the external disturbance with an *a priori* uncertain waveform, Elliot and Goodwin [5] posed a problem of adaptive compensation of unknown disturbance generated by the linear exosystem of the known order but with unknown parameters. A few adaptive controllers of both the discrete and continuous-time type were designed in [5, 8, 1, 10]. However, in these papers the case of linear plants was only considered and the obtained results cannot be straightforward extended to the case of nonlinear ones.

In [11] there was proposed a state-feedback adaptive compensator of inaccessible disturbance for nonlinear SISO systems. It was assumed that the model of the plant is known and the whole state vector is accessible for measurement. Under these assumption the problem of tracking for the reference signal and disturbance cancellation was solved.

In this paper we will consider a class of nonlinear MIMO square systems with unknown parameters affected by the bounded disturbances generated by the linear exosystem of known order but with unknown parameters. Generators of such kind can describe, for example, the signals of the form of finite trigonometric series with unknown frequencies, magnitudes and initial phases. We propose the output adaptive controller which semiglobally stabilizes the system subject to the inaccessible disturbances of such kind.

2. *Preliminary.* Consider the nonlinear time-invariant affine in the control system:

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \quad (1)$$

where  $x(t) \in \mathbb{R}^n$  is the state,  $u(t) \in \mathbb{R}^m$  is the input which is assumed to be continuous and bounded function of time,  $y(t) \in \mathbb{R}^m$  is the output;  $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$  and the columns of the matrix  $g : \mathbb{R}^n \rightarrow \mathbb{R}^n \times \mathbb{R}^m$ , are smooth vector fields and  $h : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is a smooth vector function.

**Definition 1** [6] *System (1) is said to be  $C^r$ -exponentially passive,  $r \geq 0$  if there exist a  $C^r$ -smooth,  $r \geq 0$ , nonnegative function  $V : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$  (storage function) and a positive definite function  $S : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$ , such that  $V(x) = 0$  and the following relations*

$$V(x(t)) - V(x(0)) = \int_0^t (y(s)^T u(s) - S(x(s))) ds \quad (2)$$

$$\alpha_1 |x|^2 \leq V(x) \leq \alpha_2 |x|^2, \quad \alpha_3 |x|^2 \leq S(x) \quad (3)$$

hold for some positive numbers  $\alpha_1, \alpha_2, \alpha_3$ , for all  $0 \leq t < T_{u,x_0}$  and for all  $u \in C^0 \cap \mathcal{L}_\infty$  where  $T_{u,x_0}$  is the upper time limit for which the solution  $x(t)$  to (1) with input  $u$  and initial conditions  $x(0) = x_0$  exists.

It is clear that if  $V \in C^1$  then along any solution to (1) we have  $\dot{V}(x(t)) = y(t)^T u(t) - S(x(t))$  where  $x(t)$  is defined at least on some finite time interval.

3. *Problem statement.* Throughout this paper we consider the following problem. For the system

$$\begin{cases} \dot{x} = f(x) + g(x)(u + w) \\ y = h(x) \end{cases} \quad (4)$$

with the state  $x(t) \in \mathbb{R}^n$ , input  $u(t) \in \mathbb{R}^m$ , output  $y(t) \in \mathbb{R}^m$  and external disturbance  $w(t) \in \mathbb{R}^m$ ,  $f(0) = 0$ ,  $h(0) = 0$ , find an output dynamic smooth feedback which asymptotically stabilizes the system at the origin for all initial conditions  $x(0)$  from the given compact set  $\Omega$  and for any admissible disturbance  $w$ .

To solve the considered problem we accept the following assumptions:

**Assumption 1 Plant model hypotheses.**

System (4) is  $C^1$ -exponentially passive with respect to input  $v = u + w$  and output  $y$ .  $\square$

**Assumption 2 Disturbance model hypotheses**

i) The disturbance signal  $w$  is bounded and modelled as an output of the following homogeneous system (so-called exosystem)

$$\dot{\chi} = \Gamma \chi \quad (5)$$

$$w = C \chi \quad (6)$$

where  $\chi(t) \in \mathbb{R}^d$  is the state vector of the exosystem, a constant  $d \times d$  matrix  $\Gamma$  has all its eigenvalues on the imaginary axis and  $C$  is a constant matrix. Without loss of generality the pair  $(\Gamma, C)$  is assumed to be observable.

ii) The upper bound of the dynamic order  $d$  of the exosystem is known, but parameters of the matrices  $\Gamma$  and  $C$  are unknown.

iii) Neither disturbance  $w$  nor the state  $\chi$  are accessible to measurements.  $\square$

Assumption 1 is quite restrictive. It means that the system possesses some kind of internal stability property. This assumption will be satisfied by appropriate choice of (static) output feedback if the system (4) is  $\mathcal{C}^1$ -output feedback exponentially passive. It is shown in [6] that this assumption (with some additional conditions) is tantamount to the global exponential minimumphasesness and uniform relative degree  $(1, \dots, 1)^T$ .

As one can straightforward see, if Assumption 1 is satisfied then it is possible to stabilize the system by the VSS controller of the form  $u = -\gamma \text{sign} y$  provided that the constant  $\gamma$  is greater than the upper bound of the Euclidian norm of the disturbance signal. However in practice, controllers of such kind can cause *chattering*, so it is interesting to find a *smooth* control law which can also solve the same problem.

Assumption 2 means that we know *a priori* only a class of the possible external disturbances, but not their exact waveform. For example, the admissible disturbance can have the following form:

$$w(t) = (w_1(t), w_2(t), \dots, w_m(t))^T$$

where

$$w_i(t) = A_{i0} + \sum_{j=1}^{N_i} A_{ij} \sin(\omega_{ij}t + \phi_{ij}), \quad i = 1, \dots, m \quad (7)$$

and all information which we have is the upper bound of the number of independent frequencies. Clearly in this case one can take  $d = 2N + 1$ , where  $N$  is the number of nonequal frequencies. In order to overcome the disturbance waveform uncertainty, a new output adaptive regulator will be designed.

**4. Main result.** We begin with some preliminary mathematical results established by the following lemma. This lemma reformulates in an especially suitable form some results known in the theory of linear observers [13].

**Lemma 1** Along with (5)-(6) consider the following differential equation

$$\dot{\zeta} = G\zeta + Kw \quad (8)$$

where  $\zeta(t) \in \mathbb{R}^d$  is the state, and the pair  $(G, K)$  is controllable. Then for any  $d \times d$  Hurwitz matrix  $G$  there exists a constant matrix  $\theta \in \mathbb{R}^{m \times d}$  such that for a certain initial state  $\zeta(0)$  the signal  $w$  can be presented in the form

$$w(t) = \theta\zeta(t) \quad (9)$$

**Proof:** Define the vector  $e = M\chi - \zeta$ , where the  $d \times d$  matrix  $M$  obeys the following matrix equation:

$$M\Gamma - GM = KC$$

Since the pair  $(\Gamma, C)$  is observable and the pair  $(G, K)$  is controllable, the matrix  $M$  is unique and nonsingular [13]. Differentiating  $e$  with respect to time in view of (5) and (8) we obtain

$$\dot{e} = M\Gamma\lambda - G\zeta - Kw = Ge + (M\Gamma - GM - KC)\chi = Ge$$

with  $e(0) = M\lambda(0) - \zeta(0)$ . The latter equation means that  $e(t)$  decays exponentially and in particular, for  $M\lambda(0) = \zeta(0)$  we have  $e(t) \equiv 0$ . Assuming that  $M\lambda(0) = \zeta(0)$  and substituting  $\lambda = M^{-1}\zeta$  into (6) we obtain (9) with  $\theta = CM^{-1}$ . ■

Under the plant model assumptions accepted, the matrix  $\theta$  dependent on the parameters of the matrices  $\Gamma$  and  $C$  is unknown. Thus the lemma reduces the uncertainty of the signal  $w$  to the parametric uncertainty of a constant matrix  $\theta$  associated with the known "regressor"  $\zeta$ . This form is conventional for many tasks of identification, adaptive observation and control. However in the cases being of practical significance the disturbance  $w$  is not accessible to measurements and, therefore, the state variable filter (8) is not realizable. Thus an observer for the vector  $\zeta$  must be designed.

To solve the problem we impose the following assumption on the plants we shall deal with

**Assumption 3** *There exist matrices  $G_0 \in \mathbb{R}^{d \times d}$ ,  $K \in \mathbb{R}^{d \times m}$  and vector function  $\psi : \mathbb{R}^m \rightarrow \mathbb{R}^d$  such that  $G_0$  is Hurwitz, the pair  $(G_0, K)$  is controllable and  $\psi$  satisfies the following partial differential equation:*

$$\frac{\partial \psi}{\partial y} \frac{\partial h}{\partial x} g(x) = K$$

Let us demonstrate how to verify this assumption. In [3] it is shown that for any passive system under some mild assumptions  $\text{rank}[(\nabla h)^T g(0)] = m$ . We need more strong condition, namely  $\text{rank}[(\nabla h)^T g(x)] = m$  for all  $x \in \mathbb{R}^n$ . It is obvious that if this is the case then there exists solution  $\Psi(x) \in \mathbb{R}^{d \times m}$  to the following linear equation:  $\Psi(x)(\nabla h)^T g(x) = K$ . We require that this solution is a known function of measurable variables:  $\Psi(x) \equiv \Psi(y)$ . If, additionally, the following *integrability condition* is fulfilled

$$\frac{\partial \Psi_{ik}}{\partial y_j}(y) = \frac{\partial \Psi_{ij}}{\partial y_k}(y), \quad i = 1, \dots, d, \quad j, k = 1, \dots, m$$

where  $y_j$  is the  $j$ th entry of the vector  $y$  and  $\Psi_{ik}$  stands for the  $ik$ th element of the matrix  $\Psi$ , then there exists solution  $\psi(y)$  to the following partial differential equation  $\frac{\partial \psi}{\partial y}(y) = \Psi(y)$  and Assumption 3 is satisfied.

As one can notice Assumption 3 is formulated in the coordinate-dependent form, i.e. it can be checked if the system is written in some appropriate coordinate system. Nevertheless in this paper we will not look for the conditions under which the original system is equivalent via output feedback and differentiable coordinate transformation to the system which satisfies this assumption.

Let us introduce an estimate  $\hat{\zeta}$  of the state  $\zeta$  in the following form

$$\hat{\zeta} = \eta + \psi(y) \quad (10)$$

where the auxiliary vector  $\eta(t) \in \mathbb{R}^d$  is generated by the filter

$$\dot{\eta} = G\eta + G\psi(y) - Ku \quad (11)$$

where the matrix  $G$  is the same as in (8). In what follows we will use the following parametrization:  $G = \lambda G_0$  where  $\lambda \in \mathbb{R}^1$  is a positive number and  $G_0$  is Hurwitz matrix.

Introduce the main control loop

$$u = -\hat{\theta}\hat{\zeta} - \mu y, \quad (12)$$

where  $\mu \in \mathbb{R}^1$  is some number,  $\hat{\theta}(t) \in \mathbb{R}^{m \times d}$  and the parameter update algorithm is given by

$$\dot{\hat{\theta}} = \gamma R(y, \hat{\zeta}), \quad (13)$$

where  $\gamma \in \mathbb{R}^1$  is a positive number and the  $m \times d$  matrix  $R(y, \hat{\zeta})$  satisfies

$$R(y, \hat{\zeta}) = (\hat{\zeta}_1 y, \hat{\zeta}_2 y, \dots, \hat{\zeta}_d y), \quad (14)$$

Here  $\hat{\zeta}_i$  stands for the  $i$ th entry of the vector  $\hat{\zeta}$ . As one can notice matrix  $R(y, \hat{\zeta})$  is chosen to satisfy the equation  $\text{tr} [R(y, \hat{\zeta})^T A] = y^T A \hat{\zeta}$  for any  $m \times d$  matrix  $A$ .

Then the stability properties of the overall system are established by the following theorem.

**Theorem 1** Assume that Assumptions 1, 3, are satisfied.

Then for any compact sets  $\Omega \subset \mathbb{R}^n$ ,  $\mathcal{U}_1 \subset \mathbb{R}^d$ ,  $\mathcal{U}_2 \subset \mathbb{R}^{m \times d}$  and for any fixed disturbance  $w$  satisfying Assumption 2 there exist positive number  $\bar{\lambda} > 0$ , such that for any  $\lambda \geq \bar{\lambda}$  there exist numbers  $\bar{\mu}$ ,  $\bar{\gamma} > 0$  such that for any initial conditions  $x(0) \in \Omega$ ,  $\eta(0) \in \mathcal{U}_1$ ,  $\hat{\theta}(0) \in \mathcal{U}_2$  if  $\mu \geq \bar{\mu}$  and  $\gamma \geq \bar{\gamma}$  then all solutions of the overall system are bounded and the adaptive controller (10), (11), (12), (13) asymptotically stabilizes the system (4).

**Proof:** Introduce an auxiliary variable  $\delta(t) \in \mathbb{R}^d$  as follows:  $\delta = \zeta - \hat{\zeta} = \zeta - \eta - \psi(y)$ . Then according to Lemma 1 the disturbance signal can be rewritten in the following form:  $w(t) = \theta\delta(t) + \theta\hat{\zeta}(t)$ . Calculate time derivative of  $\delta$ :

$$\begin{aligned} \dot{\delta} &= G\zeta + Kw - G\eta - G\psi(y) + Ku - \frac{\partial\psi}{\partial y} \frac{\partial h}{\partial x}(x) (f(x) + g(x)(u + w)) \\ &= G\delta - \frac{\partial\psi}{\partial y} \frac{\partial h}{\partial x} f(x) = G\delta - a(x) \end{aligned}$$

where  $a(x) = \frac{\partial\psi}{\partial y} \frac{\partial h}{\partial x} f(x)$

Now rewrite the equation of the overall system:

$$\begin{cases} \dot{x} = f(x) + g(x)(\tilde{\theta}\hat{\zeta} + \theta\delta - \mu y) \\ \dot{\delta} = \lambda G_0 \delta - a(x) \\ \dot{\tilde{\theta}} = -\gamma R(y, \hat{\zeta}) \end{cases} \quad (15)$$

where  $\tilde{\theta} = \theta - \hat{\theta}$ .

Let positive definite matrix  $P$  satisfy the following Lyapunov equation

$$G_0^T P + P G_0 = -2I$$

Then consider the following Lyapunov function candidate:

$$W(x, \delta, \dot{\theta}) = V(x) + \frac{\delta^T P \delta}{2(1 + \delta(0)^T P \delta(0))} + \frac{\varepsilon \gamma^{-1} \text{tr} [\tilde{\theta}^T \tilde{\theta}]}{2(1 + \varepsilon \gamma^{-1} \text{tr} [\tilde{\theta}(0)^T \tilde{\theta}(0)])}$$

where the function  $V$  (storage function) is from the definition of exponential passivity and positive number  $\varepsilon$  is to be determined further.

Fix the set  $\bar{\Omega}$  of initial conditions which satisfy

$$V(x) + \frac{\delta^T P \delta}{2(1 + \delta^T P \delta)} + \frac{\varepsilon \gamma^{-1} \text{tr} [\dot{\theta}^T \dot{\theta}]}{2(1 + \varepsilon \gamma^{-1} \text{tr} [\dot{\theta}^T \dot{\theta}])} \leq 1 + D$$

for some positive constant  $D$ . Notice that the set  $\bar{\Omega}$  is not compact. However its projection on  $\{\delta = 0, \dot{\theta} = 0\}$  is compact and therefore in view of smoothness of the function  $a$  there exists a positive constant  $L$  such that, for all  $x \in \text{Pr}_{\{\delta=0, \dot{\theta}=0\}} \bar{\Omega} \subset \Omega_1 \subset \mathbb{R}^n$ , where  $\Omega_1 = \{x : V(x) \leq 1 + D\}$  we have  $|a(x)| \leq L|x|$ .

It should be also noticed that the constant  $D$  may be chosen such that  $\Omega \subset \Omega_1$  where  $\Omega$  is from the theorem statement. In the sequel, we will use the fact that the constant  $D$  can be chosen independently on  $\lambda$ , although  $\delta(0)$  and  $\dot{\theta}(0)$  depends on  $\lambda$  (it is seen from the proof of Lemma 1). Indeed, for any bounded  $\delta$  and  $\dot{\theta}$  it follows that

$$\frac{\delta^T P \delta}{2(1 + \delta^T P \delta)} < \frac{1}{2} \quad \text{and} \quad \frac{\varepsilon \gamma^{-1} \text{tr} [\dot{\theta}^T \dot{\theta}]}{2(1 + \varepsilon \gamma^{-1} \text{tr} [\dot{\theta}^T \dot{\theta}])} < \frac{1}{2}$$

and therefore the constant  $D$  can determine the upper bound for the initial value of the variable  $x$  (cf (3)).

Clearly,  $W$  is a radially unbounded function. Time derivative of  $W$  along the trajectories of (15) satisfies:

$$\begin{aligned} \dot{W} = & -S(x) + y^T (\hat{\theta} \hat{\zeta} + \theta \delta - \mu y) - \frac{\delta^T \lambda \delta}{1 + \delta(0)^T P \delta(0)} - \frac{\delta^T P a(x)}{1 + \delta(0)^T P \delta(0)} \\ & - \frac{\varepsilon \text{tr} [R(y, \hat{\zeta})^T \tilde{\theta}]}{(1 + \varepsilon \gamma^{-1} \text{tr} [\tilde{\theta}(0)^T \tilde{\theta}(0)])} \\ \leq & -\alpha_3 |x|^2 + y^T \theta \delta - \mu |y|^2 - \frac{\lambda |\delta|^2}{1 + \delta(0)^T P \delta(0)} - \frac{\delta^T P a(x)}{1 + \delta(0)^T P \delta(0)} \\ & - \frac{\varepsilon \text{tr} [R(y, \hat{\zeta})^T \tilde{\theta}]}{(1 + \varepsilon \gamma^{-1} \text{tr} [\tilde{\theta}(0)^T \tilde{\theta}(0)])} + y^T \hat{\theta} \hat{\zeta} \end{aligned}$$

Determine the numbers  $\bar{\lambda} > 0, \bar{\mu} > 0, \varepsilon > 0$  such that  $W$  is positive definite and  $\dot{W}$  is nonpositive as long as  $\lambda \geq \bar{\lambda}, \mu \geq \bar{\mu}$ . Let  $\bar{\lambda} = \lambda_1 + \lambda_2$ , where  $\lambda_1 > 0$  is to be chosen and  $\lambda_2 > 0$  is fixed, then in the set  $\bar{\Omega}$  we have

$$\dot{W}(x, \delta, \dot{\theta}) \leq W_1(x, \delta, \dot{\theta}) + W_2(x, \delta, \dot{\theta}) + W_3(x, \delta, \dot{\theta})$$

where

$$\begin{aligned}
W_1 &= -\alpha_3|x|^2 - \frac{\lambda_1|\delta|^2}{1 + \delta(0)^T P \delta(0)} + \frac{|\delta| \cdot |LP| \cdot |x|}{1 + \delta(0)^T P \delta(0)} \\
W_2 &= -\mu|y|^2 - \frac{\lambda_2|\delta|^2}{1 + \delta(0)^T P \delta(0)} + |y| \cdot |\theta| \cdot |\delta| \\
W_3 &= -\frac{\varepsilon \text{tr} [R(y, \hat{\zeta})^T \tilde{\theta}]}{(1 + \varepsilon \gamma^{-1} \text{tr} [\tilde{\theta}(0)^T \tilde{\theta}(0)])} + y^T \tilde{\theta} \hat{\zeta}
\end{aligned}$$

First we observe that if  $\lambda_1 \geq |LP|^2/(4\alpha_3)$  then  $W_1$  is negative definite. It is important that  $W_1$  is negative definite for all  $\delta(0)$  although  $\delta(0)$  depends on  $\lambda$ . Further for given  $\lambda_1, \lambda_2$  (they determine  $\bar{\lambda}$  and therefore  $\delta(0)$  and  $\theta$ ) it is possible to find  $\bar{\mu}$  such that  $W_2$  is negative definite. Consequently for any  $\lambda \geq \bar{\lambda}, \mu \geq \bar{\mu}$  it follows that  $W_1 + W_2 \leq 0$ .

The next step of the proof is to determine the number  $\varepsilon$ . We have already fixed numbers  $\bar{\lambda}$  and  $\bar{\mu}$ , therefore we determined compact set of initial conditions  $\tilde{\theta}(0)$  and  $\delta(0)$  which depends on  $\mathcal{U}_1, \mathcal{U}_2, \lambda$  and given disturbance signal  $w$  (recall that according to Lemma 1 matrix  $\theta$  and initial conditions  $\zeta(0)$  depend on matrix  $G$  and hence on  $\lambda$ ). Now let us find such  $\varepsilon$  that  $W_3(x, \delta, \tilde{\theta}) = 0$ . It is possible if  $\varepsilon (1 - \gamma^{-1} \text{tr} [\tilde{\theta}(0)^T \tilde{\theta}(0)]) = 1$ . This equation has positive solution if the following inequality is satisfied:  $\gamma \geq \bar{\gamma} = \text{tr} [\tilde{\theta}(0)^T \tilde{\theta}(0)]$ . Notice that this inequality can be satisfied since we have proved that  $\tilde{\theta}(0)$  is bounded and its bound depends on  $\lambda, \mathcal{U}_1, \mathcal{U}_2$  and  $\lambda$ , in turn, depends on  $\Omega$ .

Therefore we have shown that for the given initial conditions and given disturbance from the admissible class it is possible to construct such a positive definite Lyapunov function whose derivative is nonpositive as long as  $\lambda \geq \bar{\lambda}, \mu \geq \bar{\mu}, \gamma \geq \bar{\gamma}$ .

Then standard argument shows that all trajectories of the overall system are bounded and therefore exist on infinite time interval ( $W$  is radially unbounded) and,  $|x(t)| \rightarrow 0$  as  $t \rightarrow \infty$ , for all initial conditions  $\{x(0), \eta(0), \tilde{\theta}(0)\}$  from the set  $\Omega \times \mathcal{U}_1 \times \mathcal{U}_2$  and for the given disturbance signal  $w$ . ■

**5. Discussion.** In this paper the problem of output adaptive rejection of bounded external disturbances generated by the linear exosystem of the known order but with unknown parameters is solved for the class of nonlinear system which can be made passive by output feedback. Hence the problem of disturbance compensation can be solved in two steps. First one should find a passifying (output) feedback and then design the adaptive compensator. The systems which can be passified by an output (static) feedback have relative degree one. Since the relative degree is invariant under a static feedback the obtained result can not be directly extended to the case of arbitrary relative degree and this case is of our current research interest.

## References

1. M.E. Bonilla, J.A. Cheang and R. Lozano (1994) On the use of adaptive precompensators for rejecting output disturbances in non minimum phase SISO systems, *Prepr. 33rd IEEE Conf. Dec. and Control*, Florida, vol.2, pp.1183-1184.

2. C.I. Byrnes and A. Isidori (1991) Asymptotic stabilization of Minimum Phase non-linear systems, *IEEE Trans. Autom. Contr.*, 36(10), pp.1122-1136.
3. C.I. Byrnes, A. Isidori and J.C. Willems (1991) Passivity, feedback equivalence and the global stabilization of minimum-phase systems, *IEEE Trans. Automat. Contr.*, 36, pp. 1228-1240.
4. E.J. Davison (1976) The robust control of a servomechanism problem for linear time-invariant multivariable systems, *IEEE Trans. Autom. Contr.*, 21(1), pp.25-34.
5. H. Elliot and G.C. Goodwin (1984) Adaptive implementation of the internal model principle, *Proc. 23rd IEEE CDC*, pp.1292-1297.
6. A.L. Fradkov and D.J. Hill (1993) Exponential feedback passivity and stabilizability of nonlinear systems. *Technical Report EE9357*, University of Sydney, Australia.
7. B.A. Francis and W.M. Wonham (1975) The internal model principle for linear multivariable regulators, *Appl. Math. Opt.*, 2, pp.170-194.
8. Gang Feg and M. Palaniswami (1991) Unified treatment of internal model principle based adaptive control algorithms, *Int. J. Contr.*, 54(4), pp.883-901.
9. C.D. Johnson (1971) Accommodation of external disturbances in linear regulator and servomechanism problems, *IEEE Trans. Autom. Contr.*, 16(6), pp.635-644.
10. V.O. Nikiforov (1996) Adaptive servocompensation of input disturbances, *13th IFAC World Congress*, vol. K, pp. 175-180.
11. V.O. Nikiforov (1996) Adaptive nonlinear servocompensation of uncertain external disturbances, *Int. J. Control*, to appear.
12. Ya. Z. Tsypkin (1994) New classes of discrete periodic control systems, *Automatica i Telemekhanika*, 12, pp.76-92, (in Russian).
13. W.M. Wonham (1979) *Linear Multivariable Control: A Geometric Approach*, New York: Springer-Verlag.

# MAJORIZING FUNCTIONS APPROACH IN ADAPTIVE CONTROL DESIGN OF NONLINEAR SYSTEMS

V.V. Putov, I.G. Polushin

Department of Automatic Control Systems  
Saint-Petersburg Electrotechnical University

5, Professora Popova

St. Petersburg, 197376

e-mail: putov@sau.etu.spb.ru

**Abstract.** The new approach to design the direct adaptive schemes for control the nonlinear dynamical objects is considered. The relation is shown between majorizing functions algorithms and speed gradient algorithms.

**Keywords:** adaptive control, direct schemes, majorizing functions algorithm, speed gradient algorithm.

**Introduction.** Most of exist direct adaptive control schemes are based on assumption that only parametric uncertainties are allowable but all nonlinearities of object mathematical model are known exactly (see [1],[2]). In this paper we consider the adaptive control design approach (previously proposed in [4]) for nonlinear dynamical objects which admits not only parametric uncertainty but some structure uncertainty is allowed too. This means that some components of nonlinear dynamics which are unknown or too complex are ignored in control design procedure and replaced by specially designed functions which are "majorized" the ignored nonlinearity in some sense. Usually this functions is more simple than ignored components of nonlinear description of system. The "cost" of this simplification is the abandonment of asymptotic stability as an objective of control and replacement it by L-dissipativity (i.e. ultimate boundness of state trajectories [3]).

The structure of paper is as follows. In section 1 the majorizing functions algorithms are presented. The methods of simplification of such algorithms are considered in section 2 The relation between majorizing functions algorithms and speed-gradient algorithms is considered in section 3

**1 .Majorizing functions algorithms.** Majorizing functions based adaptive control algorithms were initially proposed in [4]. We will consider the system in the form

$$\dot{x} = A(x,t)x + B(x,t)u(t), \quad (1)$$

where  $x \in \mathbb{R}^n$ ,  $u \in \mathbb{R}^m$  is state and input of system (1) respectively,  $m < n$ ;  $A(x, t)$ ,  $B(x, t)$  are functional matrices of corresponding dimensions,  $B(x, t)$  is globally bounded uniformly on  $t$ . Furthermore we assume that matrix-function  $A(x, t)$  can be represent in the form

$$A(x, t) = \left\{ \sum_{r=1}^n \left[ \sum_{q=0}^{p_r} A_{qr}(x, t) f_{qr}^\theta \right] \right\}_{ij}, \quad i, j = \overline{1, n}, \quad (2)$$

where  $A_{qr}(x, t)$  are globally bounded matrix-functions,  $f_{qr}(x)$  are continuous possibly unbounded scalar functions called growth functions and the following condition is valid:

$$\lim_{|x_r| \rightarrow +\infty} \frac{|f_{q+1, r}^\theta(x_r)|}{|f_{qr}^\theta(x_r)|} = +\infty.$$

Hence index  $q$  is introduced for separation of the components of matrix-function  $A(x, t)$  with different growth rate while argument tends to infinity,  $p_r$  is a number of components with different growth rate on  $x_r$ . We consider the problem of tracking on program trajectory produced by model

$$\dot{x}_M(t) = A_M x_M(t) + B_M u^0(t), \quad (3)$$

where  $(A_M, B_M)$  - is controllable pair;  $A_M$  - Hurwitz matrix,  $u^0(t)$  - program trajectory,  $\|u^0(t)\| \leq \text{const}$  for any  $t$ .

Control objective is defined by relationship

$$\lim_{t \rightarrow \infty} \|x(t; t_0, x_0, u) - x_M(t; t_0, u^0)\| < D,$$

i.e. tracking with ultimately bounded error (dissipativity in sense of Levinson) is assumed as a goal of control.

Consider the adaptive control law:

$$u(t) = u(t)_A + u^0(t),$$

$$u_A(t) = \left\{ \sum_{q=0}^p \left[ \sum_{r=1}^n K_{qr}^A(t) f_{qr}(x_r) \right] \right\} x + K_B(t) u^0(t), \quad (4a)$$

$$\dot{K}_{qr}^A(t) = -f_{qr}(x_r) \Gamma_{qr}^A B_M^T P e x^T - \Lambda_{qr}^A K_{qr}^A(t), \quad (4b)$$

$$\dot{K}_B(t) = -\Gamma_B B_M^T P e u^{0T}(t) - \Lambda_B K_B(t); \quad r = \overline{1, n}, q = \overline{0, p_r},$$

where  $K_{qr}^A(t) \in R^{m \times n}$ ,  $K_B(t) \in R^{m \times m}$  - matrices of adjustable parameters  
 $\Gamma_{qr}^A, \Lambda_{qr}^A, \Gamma_B, \Lambda_B \in R^{m \times m}$  are symmetric positive defined (typically diagonal) gain  
matrices,  $P = P^T > 0$  is unique solution of the matrix Lyapunov's equation

$$A_M^T P + P A_M = -G,$$

for some  $G = G^T > 0$ . Functions  $f_{qr}(x_r)$  are arbitrary scalar smooth functions which is  
related with growth functions  $f_{qr}^\theta(x_r)$  by the following inequalities

$$\left| f_{qr}^\theta(x_r) / f_{qr}(x_r) \right| \leq a_{qr}, \quad r = \overline{1, n}, q = \overline{1, p_r} \quad (5)$$

when  $|x_r| \leq b_{qr}$  for some numbers  $a_{qr}, b_{qr} > 0$ .

Functions  $f_{qr}(x_r)$  are named majorizing functions for a growth functions  $f_{qr}^\theta(x_r)$   
and the control law (4a) (4b) is called adaptive control algorithm with majorizing  
functions.

The problem of analytical foundations of adaptive control algorithms with  
majorizing functions are considered in [5] and solved under some additional assumptions  
which is valid for number types of systems important in application. In general case  
the problem is open.

## 2. Simplification of algorithms with majorizing functions.

Obviously that in general the adaptive control algorithms with majorizing  
functions (4) are simpler than speed gradient ones [1] because in the last case exact  
description of matrices-functions  $A(x, t)$  and  $B(x, t)$  is exploited. On the other hand  
algorithm (4a, b) has the following properties:

a) nonlinear structure of globally bounded matrices  $A(x, t)$ ,  $B(x, t)$  elements  
is ignored completely;

b) exact nonlinear functions  $f_{qr}^\theta(x_r)$  of system's description (1) are replaced by  
approximate (and simpler) majorizing functions  $f_{qr}(x_r)$ .

For example majorizing functions may be selected as ascending power functions

$$x_r^k, r = \overline{1, n}, k \in R^1$$

which has real powers and corresponds majorizing conditions (5).

It is shown in [5] that further significant simplification of algorithms (4a) and (4b)  
is possible. In this case only function with highest growth rate on each  $x_r$  are included  
in algorithm. Simplified majorizing functions algorithms are:

$$u_A(t) = K_A(t) \text{diag}\{f_q(x_r)\}x + K_B(t)u^0(t) \quad (4c)$$

$$K_A(t) = -\Gamma_A B_M^T P e x^T \text{diag}\{f_q(x_r)\} - \Lambda_A K_A(t),$$

$$K_B(t) = -\Gamma_B B_M^T P e u^{0T}(t) - \Lambda_B K_B(t). \quad (4d)$$

$$r = \overline{1, n}; q \geq p_r ; ;$$

where

$$\text{diag}\{f_q(x_r)\} = \text{diag}\{f_q(x_1), f_q(x_2), \dots, f_q(x_n)\}.$$

### 3. Relation between majorizing functions algorithms and speed gradient algorithms.

In this section we will show that if accessibility condition is weakened then algorithms (4) can be obtained by direct application of speed gradient technique [1]. Consider the control objective functional

$$Q(t) = 1/2 e(t)^T P e(t), \quad (6)$$

where  $P \in \mathbb{R}^{n \times n}$  is symmetric positive defined matrix. The time derivative of (6) with respect to (1), (3) and (4) is

$$\begin{aligned} \frac{dQ}{dt} = \omega(x, \theta, t) = & e(t)^T P \left\{ A_M e + B(x, t) \left[ B^+(x, t) \left( \sum_{q=0}^p \sum_{r=1}^n \tilde{A}_{qr}(x, t) f_{qr}(x_r) \right) x + \right. \right. \\ & \left. \left. + B^+(x, t) (B(x, t) - B_M) u^0 + \left( \sum_{q=0}^p \sum_{r=1}^n K_{qr}^A(t) f_{qr}(x_r) \right) x + K_B(t) u^0(t) \right] \right\}, \end{aligned}$$

where  $B^+$  is quasi-inverse matrix for  $B$ . We have

$$\text{grad } K_{A_{qr}} \omega(x, \theta, t) = f_{qr}(x_r) B^T(x, t) P e x^T,$$

$$\text{grad } K_B \omega(x, \theta, t) = B^T(x, t) P e u^{0T}$$

Then the speed-gradient control algorithm [1] is obtained by expressions

$$K_{A_{qr}}(t) = -f_{qr}(x_r) \Gamma_{A_{qr}} B^T(x, t) P e x^T - \Lambda_{A_{qr}} K_{A_{qr}}(t),$$

$$K_B(t) = -\Gamma_B B^T(x, t) P e u^{0T}(t) - \Lambda_B K_B(t), \quad q = \overline{0, p}, \quad r = \overline{1, n}. \quad (7)$$

Algorithm (7) can not be realized, because matrix  $B(x, t)$  is unknown. It was shown in [5] that if matrix  $B(x, t)$  is replaced by model matrix  $B_M$  then under some additional conditions algorithm (7) is still operational. This replacement leads to identity of algorithms (46) and (7).

It is a matter of direct verification to prove that all conditions of feasibility the SG-algorithms are fulfilled except so called accessibility condition [1]. We will

demonstrate that it is possible to formulate the new condition of this type, which asserts for the algorithm (4). Really, substituting (3) in (1) we obtain

$$\dot{e} = A_M e + \sigma(x, t, u^0) + B(x, t) u_a(t), \quad (8)$$

where

$$\sigma(x, t) = [A(x, t) - A_M]x + [B(x, t) - B_M]u^0(t). \quad (9)$$

Right part of (8) can be transformed by quasi-inverse matrix function  $B^+(x, t)$  into

$$\dot{e} = A_M e + B(x, t)[B^+(x, t)\sigma(x, t, u) + u_a(t)].$$

The transformation given above is correct only if following expression is true

$$B(x, t)B^+(x, t)\sigma(x, t, u^0) = \sigma(x, t, u^0),$$

for all  $x, t, u^0$ . This condition is equal to following matching condition for functional matrix [4,5]

$$(B(x, t)B^+(x, t) - I_n)(A(x, t) - A_M) = 0,$$

$$B(x, t)B^+(x, t) - I_n)B_M = 0.$$

Let us mention following facts. First, if the decomposition (2) of  $A(x, t)$  contains functions with positive growth rate then constant matrix  $A_M$  may be decomposed (possibly not in unique way) by the same growth rate functions  $f_{qr}^\theta(x_r)$  as the matrix  $A(x, t)$

$$A_M = \sum_{q=0}^p \left[ \sum_{r=1}^n A_{qr}^M(x, t) f_{qr}^\theta(x_r) \right],$$

where  $A_{qr}^M(x, t)$  matrices which is bounded at least outside some compact in state space.

It is obvious that

$$A(x, t) - A_M = \sum_{q=0}^p \left[ \sum_{r=1}^n (A_{qr}(x, t) - A_{qr}^M(x, t)) f_{qr}^\theta(x_r) \right]. \quad (10)$$

Furthermore it follows from (5) that there exist scalar functions  $\alpha_{qr}(x_r)$  s. t.

$$f_{qr}^\theta(x_r) = f_{qr}(x_r) \alpha_{qr}(x_r), \quad (11)$$

where  $f_{qr}(x_r)$  are majorizing functions from algorithm (4a) and (4b), and

$$|\alpha_{qr}(x_r)| \leq a_{qr} \quad \text{when } |x_r| \geq \eta_0,$$

where  $a_{qr}, \eta_0$  are positive constants. Substituting (11) into (10), we can write following decomposition of matrix function  $\tilde{A}(x, t) = A(x, t) - A_M$

$$\tilde{A}(x,t) = \sum_{q=0}^p \left[ \sum_{r=1}^n (A_{qr}(x,t) - A_{qr}^M(x,t)) \alpha_{qr}(x_r) f_{qr}(x_r) \right] =$$

$$\sum_{q=0}^p \left[ \sum_{r=1}^n \tilde{A}_{qr}(x,t) f_{qr}(x_r) \right],$$

where

$$\tilde{A}_{qr}(x,t) = (A_{qr}(x,t) - A_{qr}^M(x,t)) \alpha_{qr}(x_r). \quad (12)$$

All functional matrices  $\tilde{A}_{qr}(x,t)$  ( $q = \overline{0,p}, r = \overline{1,n}$ ) are bounded at least outside some compact in state space. This follows from boundness of all components included in expression (12). By substitution (4a) into equation (8) and using last decomposition of matrix  $\tilde{A}(x,t)$  we obtain

$$\dot{e} = A_M e + B(x,t) \left( \sum_{q=0}^p \sum_{r=1}^n [B^+(x,t) \tilde{A}_{qr}(x,t) + K_{qr}^A(t) f_{qr}(x_r)] \right) x +$$

$$+ [B^+(x,t)(B(x,t) - B_M) + K_B(t)] u^0. \quad (13)$$

Considering the expression (13) we can conclude that there are "ideal" values of adjustable parameters which are functions of state variables and time s.t.

$$K_{qr}^{A*}(x,t) = -B^+(x,t) \tilde{A}_{qr}(x,t), \quad (14)$$

$$K_B^*(x,t) = -B^+(x,t)(B(x,t) - B_M). \quad (15)$$

If matrixes of adjustable parameters  $K_{qr}^A(t)$  and  $K_B(t)$  are equal to corresponding "ideal" values (14), (15) then all terms of right part of (13) (except the first one) canceled and we obtain the closed loop system equation as

$$\dot{e} = A_M e.$$

Its trivial solution is asymptotically stable by choice of matrix  $A_M$ . Consequently we obtain, that following condition is true for algorithm (4a,b).

**Condition 1.** There exists vector-function  $\Theta^*: \mathbb{R}^n \times \mathbb{R}^+ \rightarrow \mathbb{R}^m$  which is bounded outside of some compact and for some continuous scalar function  $\rho: \mathbb{R} \rightarrow \mathbb{R}$ , that  $\rho(0)=0$  and  $\lim_{Q \rightarrow \infty} \rho(Q) = \infty$ , and for some constant  $\beta$  the inequality

$$\omega(x, \Theta^*(x, t), t) \leq -\rho(Q) + \beta$$

is true.

**Conclusions.** In this paper we consider the new approach to design the direct adaptive schemes for control the nonlinear dynamical objects. This approach can be considered as an effective method for systematic simplification of known adaptive control algorithms.

It is believed that this approach enables to reduce the distance between theoretical results and practical implementations of adaptive control schemes.

### **References**

1. Fradkov A.L. Adaptive control in complex systems: direct methods. - M.: Nauka, 1990. (in Russian).
2. Kanellakopoulos I., Kokotovich P.V., Morse A.S. Adaptive feedback linearization of nonlinear systems // Foundations of adaptive control. Berlin: Springer-Verlag, 1991, pp.311-346.
3. Levinson N. Transformation theory of non-linear differential equations of the second order. Ann. Math. V.45, N.4 1944, pp.723-737.
4. Putov V.V. Adaptive stabilization of cascade mechanical systems //Avtomatizaciya issledovaniy i proektirovaniya system upravleniya.- L. LGU, 1987.- P. 136-145. (in Russian)
5. Putov V.V. Methods of adaptive control design for nonlinear dynamically objects with functional-parametric uncertainty of model. Sc.D. Thesis. Saint-Petersburg, 1993.
6. Polushin I.G. Adaptive control design for nonlinear mechanical object. Ph.D.Thesis:- Saint-Petersburg, 1995. (in Russian).
7. Gantmaher F.R.. The theory of matrices. M.:Nauka,1988. (in Russian)
8. Bortsov Y.A., Polyahov N.D., Putov V.V.. Electromechanical systems with adaptive and modal control, SPb, Energoatomizdat, 1984. (in Russian).

# ADAPTIVE CONTROL OF NONLINEAR BUSINESS-CYCLE MODELS

B.R.Andrievsky	E.G.Dymkin	A.L.Fradkov
Baltic State	St.Petersburg State	Inst. for Probl. of
Technical Univ.	Technical Univ.	Mechan. Engg.
1, 1-st Krasnoarmeyskaya,	29 Polytechnicheskaya,	61 Bolshoy ave V.O.,
St. Petersburg, 198005,	St.Petersburg, 195251,	St. Petersburg,199178,
Russia	Russia	Russia
andr@ccs.ipme.ru		alf@ccs.ipme.ru

**Abstract.** This work is the first attempt to apply methods of nonlinear control to chaotic systems having economical interpretation. The problem of investments control aimed at increasing predictability of a business-cycle evolution is discussed. The applicability of shunt method of output feedback adaptive control for Metzler business-cycle model has been demonstrated.

**Keywords.** nonlinear control, adaptive, business-cycle.

**1. Introduction.** Business-cycle theory was one of the first to appear as a separate branch of macroeconomics. Moreover, one can assume it to be elder, than the latter. First business-cycle models date back to the nineteenth century. The most attention was paid to this problem in the interwar period, when national economy of most of the countries had been succeeded by crisis. It was then, that various models of business-cycles formed a real *theory*.

First business-cycle models generated smooth and regular trajectories of system variables. Evidently, linear models could have given no result, but dampening or explosive oscillations with constant frequencies. In fact, no economist could believe a business cycle to converge to a fixed point or to diverge with no limits. Therefore, those, who aimed at developing a new business-cycle model, had to test themselves by trying to fit real economic data from one hand, and by providing suitable economic reasons for a new-made model.

Nonlinear dynamical business-cycle models make an important part of the "New Classical" theory, which differs greatly from the ancient one by using much of calculus. Thus, one should use appropriate methods of stability theory in order to find out, whether a modelled economy is stable or not and to discover most of its dynamical properties. One of the most simple business-cycle models, involving nonlinear high-order differential equations, is a continuous-time version of the Metzler model.

Basic assumptions, which lie under this model are rather reasonable. The output of a firm (its net income) is supposed to be proportional to the difference between desired and actual inventory stocks. The actual inventory stock changes, following demand/supply ratio in the market. Note, that one cannot choose the desired level freely - it should be proportional to *expected net income*. This is the second, but not least, important supposition. Thirdly, the expected value of net income is calculated with respect to dynamics of changes of actual output (extrapolative expectations). Finally, we suppose the variations of actual level of inventories to be equal to the difference between actual savings and investment. Formally, these assumptions can be written as follows [4]:

$$\dot{Y} = \beta(\hat{Q} - Q) \quad (1)$$

$$\dot{\hat{Q}} = k\hat{Y} \quad (2)$$

$$\dot{Y} = Y + C_1\dot{Y} + C_2\ddot{Y} \quad (3)$$

$$\dot{Q} = S(Y) - I(Y) \quad (4)$$

where  $Y$  denotes actual firms' output;  $\hat{Y}$  is its expected value;  $Q$  is actual level of inventories;  $\hat{Q}$  is its desired level;  $S(Y), I(Y)$  are savings and investment functions respectively;  $\beta \in [0, 1]$  is a constant reaction coefficient;  $k > 0$  is a proportionality (accelerator) coefficient;  $C_1, C_2 > 0$  are expectation (sensitivity) coefficients;

Using simple substitutions this model can be reduced to the following third order differential equation:

$$\ddot{Y} + A_1\dot{Y} + A_2Y = A_3(S(Y) - I(Y)) \quad (5)$$

with  $A_1, A_2, A_3$  - positive constants, depending on previously mentioned  $\beta, k, C_1, C_2$ .

The behaviour of solutions of this equation was studied in [1]. The supplementary supposition concerned the shape of  $S(Y) - I(Y)$ . It is supposed to be a unimodal function of  $Y$ , for example - a function in the logistic form:

$$S(Y) - I(Y) = Y(C - Y) \quad (6)$$

In this case solution  $Y$  of (5) may exhibit chaotic behavior, which is very sensitive to variations of parameters and initial conditions. Thus, one cannot predict the evolution of a business-cycle. Therefore the problem arises to find a strategy of exogenous financial intervention making the business-cycle evolution predictable. In other words, one should design the control algorithm, which makes the system, described by (5), follow the desired trajectory. Let us describe this problem more formally.

*2. Problem statement.* Consider a problem of control for a business-cycle, described by equation (5). The controlled system model may be written as follows:

$$\ddot{Y} + A_1\dot{Y} + A_2Y = A_3(S(Y) - I(Y) + U(Y)) \quad (7)$$

where  $U(Y)$  is a control action, interpreted as exogenous investment.

The goal of an investor is to make a business-cycle predictable, i.e. to obtain a kind of periodic solution to equation (5). Evidently, we should impose some restrictions to the absolute value of  $U(Y)$ , as a big investment will undoubtedly break the structure.

Note, that one cannot be assured of knowing exact values of  $A_1$  and  $A_2$ . Moreover, the control law should be robust to changes in parameters and should achieve the goal even when they vary slightly in time. Therefore, one should use an adaptive control approach suggesting algorithms with adjustable parameters.

Another restriction to be imposed to the control law concerns system output. The only measurable model variable is  $Y$  itself - net income of a firm. Thus, the control law being designed should depend only on  $Y(t)$ .

After having posed the problem of stabilization itself, let us describe the method of adaptive control, which is in our opinion the most appropriate.

*3. Shunt method.* The method of parallel feedforward compensators or shunts (see bibliography in [2]) is based on introducing additional filter (shunt) in parallel with the controlled system model to correct its dynamical properties and to make it more appropriate for control. Describe briefly the version of shunt adaptive controller, proposed in [2] and further simplified in [3].

Consider nonlinear affine in control model

$$\dot{x} = f(x) + g(x)u, \quad y = h(x), \quad (8)$$

where  $x \in \mathcal{R}^n$  is state vector,  $u \in \mathcal{R}^1$  is input,  $y \in \mathcal{R}^1$  is output,  $f, g, h$  are smooth functions such that  $f(0) = 0, h(0) = 0$ , i.e. origin  $x = 0$  is equilibrium of free system  $\dot{x} = f(x)$ .

If system (8) has relative degree  $r$  in the open set  $\mathcal{D}$ , (for definitions of relative degree and other notions of nonlinear control theory see [7]) then there exists smooth nonsingular coordinate change  $z = \Phi(x)$ ,  $x \in \mathcal{D}$ , such that system (8) model has in the new coordinates normal form

$$\begin{aligned} \dot{z}_i &= z_{i+1}, \quad i = 1, \dots, r-1, \\ \dot{z}_r &= a(z) + b(z)u, \\ \dot{\bar{z}} &= q(z), \quad y = z_1, \end{aligned} \quad (9)$$

where  $b(z) \neq 0$  in  $\Phi(\mathcal{D})$ .  $z = (\hat{z}, \bar{z})$ ,  $\hat{z} = (z_1, \dots, z_r) \in \mathcal{R}^r$ ,  $\bar{z} \in \mathcal{R}^{n-r}$ .

Introduce *shunt system* (parallel feedforward compensator) as follows

$$(p+1)^{r-1}\eta = \kappa\epsilon(p\epsilon+1)^{r-2}(b(z)u + a_0(y)), \quad (10)$$

with

$$a_0(y) = a(z) - A(p)y \quad (11)$$

where  $A(p)$  is a polynomial of degree  $r-1$ ,  $\eta$  is auxiliary variable,  $\kappa > 0$ ,  $\epsilon > 0$  and consider augmented plant model described by equations (1), (10) and output equation  $y_a = y + \eta$ . If additional structural information about plant nonlinearity  $a(z)$  is available then it may be taken into account when choosing the adequate structure of controller. Suppose that  $a(z)$  has the following form

$$a(z) = a_0(y) + A(p)y + \sum_{i=1}^l \theta_i a_i(y), \quad (12)$$

where  $a_i(y)$  are known (measured) functions,  $\theta_i$  are unknown constant coefficients. Then the structure of adaptive controller can be taken as follows:

$$u = -\frac{1}{b(y)} \left[ Ky_a + a_0(y) + \bar{a}(t) + \sum_{i=1}^l \hat{\theta}_i \bar{a}_i(t) \right], \quad (13)$$

where  $k, \hat{\theta}_i$  are adjustable parameters ( $k$  is controller gain, while  $\hat{\theta}_i$  are estimates of unknown parameters  $\theta_i$ ) and functions  $\bar{a}(t), \bar{a}_i(t)$  are the outputs of  $l+1$  identical filters:

$$\begin{aligned} G(p)\bar{a}(t) &= \tilde{A}(p)y, \\ G(p)\bar{a}_i(t) &= a(y(t)) \quad i = 1, \dots, l. \end{aligned} \quad (14)$$

where  $G(p) = 1 + \kappa\epsilon(\epsilon p + 1)^{r-2}(r-1)$  is a Hurwitz polynomial of degree  $(r-1)$ . The adaptation algorithms are obtained

$$\begin{aligned} \dot{K} &= \gamma_0 y_a^2, \quad \gamma_0 > 0, \\ \dot{\hat{\theta}}_i &= \gamma_i y_a \bar{a}_i(t), \quad \gamma_i > 0, \quad i = 1, \dots, l. \end{aligned} \quad (15)$$

The applicability conditions of the proposed algorithm are given in [3] as follows:

**Theorem 1** Let plant model (9) have structure (12) and the following conditions be valid:



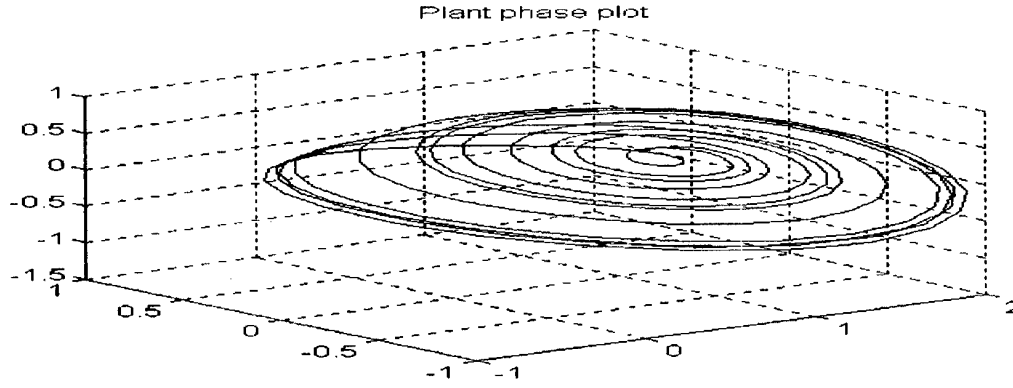


Figure 2: Uncontrolled plant phase plot.

Subtracting (18) from (17) obtain error equation:

$$(p+1)^3 e = \theta_1 a_1 + \theta_2 a_2 + f + A_3 U \quad (19)$$

where  $a_1 = p^2 Y$ ;  $a_2 = pY$ ;

$f = 3p^2 e + A_1^* p^2 Y_* + 3pe + e + A_2^* pY_* + A_3(S(Y) - I(Y) - A_3^*(S(Y_*) - I(Y_*)))$ . To design adaptive controller of Section 3 note, that in our case the plant order  $n = 3$  and relative degree  $r = 3$ , i.e. there are no zero dynamics.

Introduce shunt system as follows:

$$(p+1)^2 \eta = \kappa \varepsilon (\varepsilon p + 1) A_3 U \quad (20)$$

Multiplying (20) by  $(p+1)$  and adding to (19), we obtain the following augmented error equation:

$$(p+1)^3 e_a = G(p) A_3 U + \theta_1 a_1 + \theta_2 a_2 + f \quad (21)$$

where  $G(p) = 1 + \kappa \varepsilon (\varepsilon p + 1)(p+1)$ . To choose  $\kappa$  and  $\varepsilon$  such, that  $G(\lambda)$  is a Hurwitz polynomial and introduce filtered signals note, that  $\deg G(\lambda) = 2$  and therefore it is Hurwitz for any  $\kappa > 0$ ,  $\varepsilon > 0$ .

$$\begin{cases} G(p) \bar{a}_i = a_i, & i = 1, 2 \\ G(p) \bar{f} = f \end{cases} \quad (22)$$

Then augmented error equation (5) can be rewritten as follows:

$$(p+1)^3 e_a = G(p) [A_3 U + \theta_1 \bar{a}_1 + \theta_2 \bar{a}_2 + \bar{f}] \quad (23)$$

It is clear, that the goal  $e_a \rightarrow 0$  can be achieved, if we choose the controller in the form

$$U = -\frac{1}{A_3} [K e_a + \hat{\theta}_1 \bar{a}_1 + \hat{\theta}_2 \bar{a}_2 + \bar{f}] \quad (24)$$

and take  $\hat{\theta}_i = \theta_i$ ,  $i = 1, 2$ ,  $K > 0$ .

Since  $\theta_i$  are unknown, we make  $\hat{\theta}_i$  adjustable and use adaptation algorithm, described in the previous Section.

$$\begin{cases} \dot{\hat{\theta}}_i = \gamma e_a \bar{a}_i, & i = 1, 2 \\ \dot{\hat{K}} = \gamma_0 e_a^2 \end{cases} \quad (25)$$

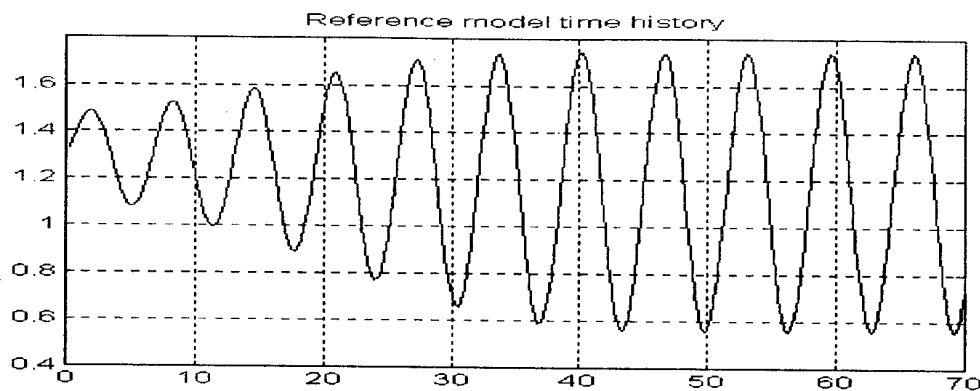


Figure 3: Reference model time history.

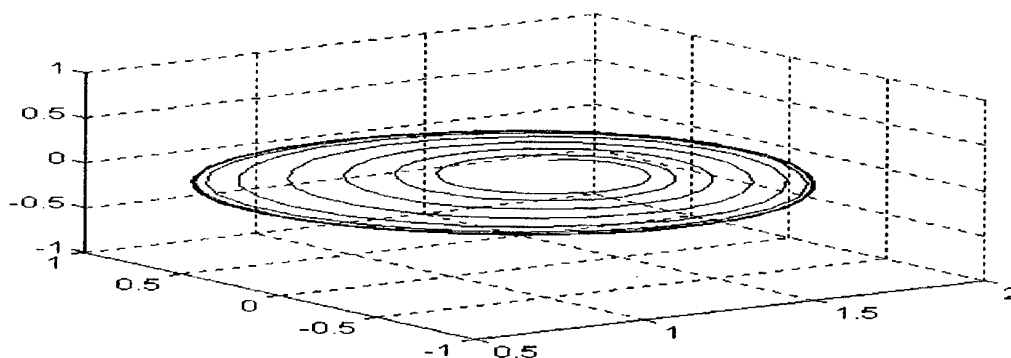


Figure 4: Reference model phase plot

Equations (20), (22), (24), (25) describe the adaptive controller of total dynamic order  $2 + 6 + 3 = 11$ . According to Section 3, it ensures the control law  $Y(t) - Y_*(t) \rightarrow 0$ , if  $\kappa > 0$ ,  $\varepsilon > 0$  and functions  $Y_*(t)$ ,  $\dot{Y}_*(t)$ ,  $\dots$ ,  $Y_*(t)$ .

*Simulation results.* Uncontrolled plant has parameters  $A_1 = 0.4$ ,  $A_2 = 0.95$ ,  $A_3 = 0.6$ ,  $C = 1.3$ ,  $b = 1$  and the initial conditions  $Y(0) = 1.4$ ,  $\dot{Y}(0) = 0.05$ ,  $\ddot{Y}(0) = 0.15$ , corresponding to chaotic behavior (Fig.1, Fig.2). Reference signal is generated by the reference model, also described by equation (2). It has stable limit cycle (Fig.3, Fig.4) for parameter values  $A_1^* = 0.475$ ,  $A_2^* = 0.9$ ,  $A_3^* = 0.45$ ,  $b^* = 1$ ,  $C = 1.3$ . The reference model initial conditions are taken as:  $Y_*(0) = 1.3$ ,  $\dot{Y}_*(0) = 0.1$ ,  $\ddot{Y}_*(0) = 0.1$  Fig. 5 shows the controlled plant behavior when the proposed adaptive controller (24), (25) is used. Parameters of the filters are:  $\varepsilon = 0.2$ ,  $\mu = 1$  and the adaptation gains are  $\gamma_0 = \gamma = 5.0$ . The initial conditions of the adjustable parameters are taken as:  $K(0) = 5.0$ ,  $\hat{\theta}_1 = \hat{\theta}_2 = 0$ . Simulations show that the tracking error approaches zero, while the limit values of adjustable parameters may differ from the true plant parameter values. The time history of adjustable parameters is shown on the Fig. 6. Fig 7. demonstrates some "robust" properties of the adaptive controller (24), (25). This simulation result shows that the difference of 100% in the nominal value of  $b$  in the plant equation results in the magnitude of the reference error about 4% from the magnitude of the reference signal.

*Conclusion.* This work is the first attempt to apply methods of nonlinear control to chaotic systems having economical interpretation. It demonstrated that the well-known

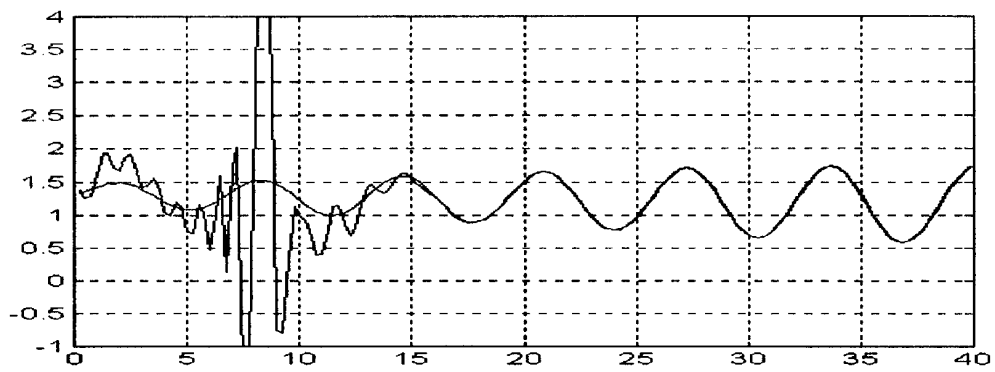


Figure 5: Tracking for the reference signal

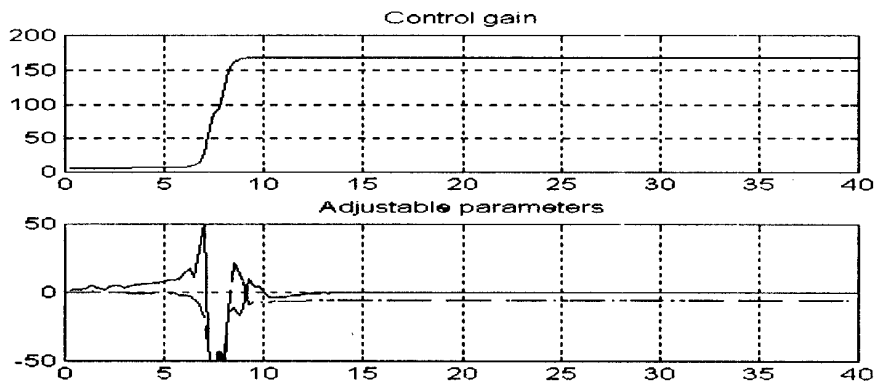


Figure 6: Control gain  $K(t)$  and the adjustable parameters  $\hat{\theta}_1(t), \hat{\theta}_2(t)$  time history

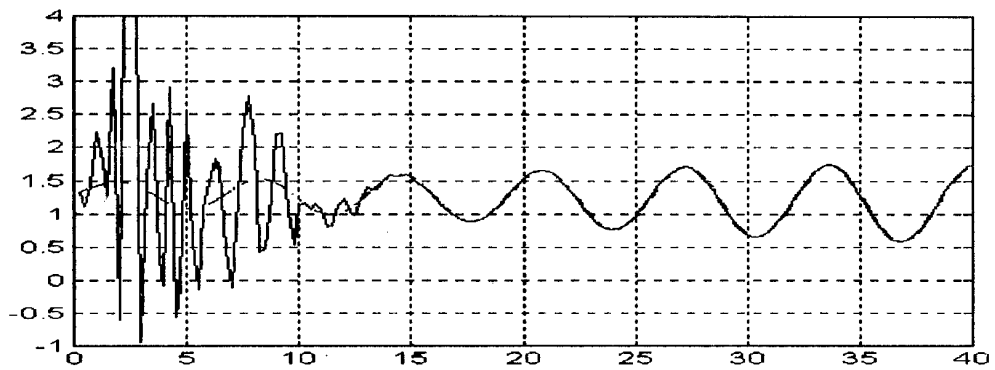


Figure 7: Tracking for the reference signal with  $b = 2$

problem of modifying system behavior from chaotic to periodic has economical meaning - it corresponds to increasing predictability of a business-cycle. It is quite natural to interpret exogenous investments as control actions when stabilizing a business-cycle. It has been shown hereabove, that shunt method of output feedback adaptive control is applicable to the Metzler model of business cycles.

Future research should investigate more detailed and realistic models.

*References.* 1. H.-W. Lorenz. *Complexity in deterministic, nonlinear business-cycle models - foundations, empirical evidence and predictability.* in Nonlinear dynamics in economics and social sciences, Lecture notes in economics and mathematical systems, v.399, pp. 17-52, Springer-Verlag.

2. Fradkov, A.L. Shunt output feedback adaptive controller for nonlinear plants. In: *Preprints of 13th IFAC World Congress*, San-Francisco, 1996.

3. Fradkov, A.L., Druzhinina, M.V., Reduced order shunt nonlinear adaptive controller, 1996, *submitted to ECC'97*.

4. Gandolfo, G., *Economic Dynamics: methods and models*. 2nd ed., Amsterdam, North Holland, 1983.

5. Metzler, L.A., *Nature and stability of inventory cycles*. Review of economic studies, **23**, pp. 113-129.

6. Creedy, J., Martin, V.L., *Chaos and non-linear models in economics*. Ed. Elgar, 1994.

7. Isidori, A., *Nonlinear control systems.*, 2nd edition. Springer-Verlag. 1989.

# A SUCCESSIVE DAMPING METHOD FOR CONTROL PROBLEMS BY INCOMPLETE FEEDBACK \*

Nikolai E. Kirin and Aleksei N. Kirin  
St. Petersburg State University  
Department of Applied Mathematics and Processes of Control  
St. Petersburg, Russia

**Abstract.** An optimal damping method application for control problems by incomplete feedback is under consideration. The method was proposed by V.I. Zubov [1] as a successive approximation method in constructing of an optimal programmed control, where the control is a function of time, a synthesis of control by complete feedback, where the control is a function of time and phase state. Below this method modification is determined for cases of incomplete feedback and perturbation affect. The method is illustrated on stabilization problem for a linear discrete system received through approximation of the lag-time system.

**Keywords.** Feedback, optimal control, iterative methods, the Isaacs-Bellman equation, lag-time systems.

## 1. The terminal control problem.

Let us consider a problem of trajectory bundle control for the system

$$\dot{x} = f(x, u, t), \quad x(0) = x_0 \in X_0 \in \mathcal{R}^n \quad (1)$$

in the finite interval of  $[0, T]$ , where  $x = x(t)$  is a state argument,  $u = u(t)$  is an admissible control being described below. The purpose of control is to minimize the functional

$$\lambda(u, x_0) \equiv \varphi(x(T, u, x_0, 0)) \Rightarrow \min_u \quad (2)$$

for each  $x_0 \in X_0$  or

$$\lambda(u) \equiv \max_{x_0} \rho(x_0) \lambda(u, x_0) \Rightarrow \min_u, \quad x = x_0 \in X_0. \quad (3)$$

Here  $x(t) = x(t, u, x_0, 0)$ ,  $t \in [0, T]$  is a system (1) solution with initial condition  $x(0) = x_0$ , where  $\varphi(\cdot), \rho(\cdot)$  are given functions in  $\mathcal{R}^n$ .

\*) This research is conducted within the research grant № 95-01-00355 from the Russian Foundation for Fundamental (basic) Research.

Let us the feedback is given on system (1) solutions (trajectories)

$$y(t)=g(x(t),t), \quad t \in [0, T], \quad (4)$$

where  $g : \mathbb{R}^n \times [0, T] \rightarrow \mathbb{R}^m$  is a known function. For every moment  $t$  the control signal  $u=u(t)$  in the system (1) can be formed by certain (for this time) values  $g(\tau)$ ,  $\tau \in [0, t]$  i.e.

$$u(t)=u(t, y(\tau) \big|_{\tau=0}^t). \quad (5)$$

Here  $u(\cdot)$  is a functional transformation. The transformation  $u(\cdot)$  is called an admissible control if the closed by the relations (4),(5) for every  $x_0 \in X_0$  system (1) has the unique extendable in  $[0, T]$  solution  $x=x(t, u, x_0, 0)$ ,  $t \in [0, T]$ . The class of admissible controls is denoted by  $U$ .

## 2. The descent method in search problem of control by feedback signals.

In the capacity of admissible controls we will consider the functions (a control without memory)

$$u=u(t, y(t)) \quad (6)$$

The set of such controls we denote  $U_t$ .

Following the optimal damping method idea [1], we will construct the sequence of controls  $u_p(t, y)$ ,  $p=1, 2, \dots$ , where the functional (2) is decreased for every  $x_0 \in X_0$ . A method step consists of the function  $V=V_p(x, t)$  constructing as a solution of Cauchy problem

$$\begin{aligned} \partial V_p / \partial t + \partial V_p / \partial t \cdot f(x, u_p(t, g(x, t)), t) &\equiv W_p(x, u_p, t) = 0, \\ V_p(x, T) &= \varphi(x), \quad (x, t) \in S = \mathbb{R}^n \times [0, T] \end{aligned} \quad (7)$$

under the given control  $u_p(\cdot)$  for the step  $p$  and constructing the following control under the condition

$$u_{p+1}(t, y) = \arg \min_u \max_{g(x, t)=y} W_p(x, u, t), \quad (y, t) \in \mathbb{R}^m \times [0, T] \quad (8)$$

Here we consider the minimum by  $u \in \Omega$ , where  $\Omega$  is a set of control signal admissible values. We omitted the problems of existing the given minimax by regarding them to be positive solved. If the function  $u_{p+1}(\cdot)$  constructed by the operation (8) to be an admissible control then on the strength of (7),(8) we obtain

$$dV_p/dt = W_p(x(t), u_{p+1}(t, y(t), t)) \leq 0 \quad (9)$$

on system (1) trajectories. Therefore,

$$\lambda(u_{p+1}, x_0) = V_p(x_0, 0) + \int_0^T W dt \leq \lambda(u_p, x_0)$$

In the last inequality we exploited the equality which implied from the equation (7). The inequality (10) is strict for those points  $x_0$ , where trajectories of the system (1) begin for  $u=u_{p+1}(\cdot)$  and the trajectories have intervals the inequality (9) proves to be strict. Thus, the inequality (10) specifies the step (7),(8) of the given optimal damping method as a descent step in the problem (2).

From the inequality it follows the similar inequality (10)

$$V_{p+1}(x,t) \leq V_p(x,t), \quad t \in [0, T], \quad x \in X(t) \subset \mathfrak{R}^n,$$

which serves a base in approximation process convergence analysis.

A stationary point of the iteration process (7),(8) will be a pair of the functions  $(v_p, u_p)$ , where  $u_p(t, y) = u_{p+1}(t, y)$  i.e.

$$\min_u \max_{g(x,t)=y} \{ \partial V_p / \partial t + \partial V_p / \partial x \cdot f(x, u, t) \} = 0 \quad (11)$$

is fulfilled on opened sets in the system (1) trajectories bundle for  $u=u_p(\cdot)$ . If the solution of Cauchy problem (7) exists then the relation (11) is a necessary condition for the control  $u=u_p(\cdot)$  (in class  $U_t$ ) to be optimal in the problem (2). As is known, a sufficient condition for the control  $u_p(\cdot)$  to be optimal in class  $U_p$  is as follows: the inequality

$$w_p(x, u(t, g(x, t)), t) \geq 0$$

fulfills under any control  $u(t, y) = u$  in the layer  $S = \mathfrak{R}^n \times [0, T]$ .

For the problem (3) the descent by the functional  $\lambda(u)$  can be extended. For that the maximum problem (8) is substituted for the following. It is required to determine

$$\max \{ w_p(x, u, t) : x \in G_\varepsilon(t, y) \}, \quad (12)$$

where

$$G_\varepsilon(t, y) = \{ x : g(x, t) = y, V_p(x, t) \geq \lambda(u_p) - \varepsilon \}$$

Here  $\varepsilon > 0$ . The set  $G_\varepsilon(t, y)$  includes those points  $x$  from system  $g(x, t) = y$  solutions, which lie on  $\varepsilon$ -extremals of the problem, in particular, the extremals which determined the value  $\lambda(u_p)$  as a function (3) maximum on  $X_0$ .

For the sufficient small  $\varepsilon > 0$  a solution of the maximum problem (8) will be directed to decreasing of the functionals values  $\lambda(u, x_0)$  in a neighborhood of the extremals by  $x_0$  for  $u = u_p(\cdot)$ . The strict inequality (9) in the neighborhood of separate intervals of each extremals will guarantee a descent for the functionals  $\lambda(\cdot) : \lambda(u_{p+1}) < \lambda(u_p)$ . This process stationary point will be the control  $u_p$  if the equality

$$\lim_{\varepsilon \rightarrow +0} \min_u \max_{x \in G_\varepsilon(t, y)} W_p(x, u, t) = 0 \quad (13)$$

fulfills on the system (1) trajectories bundle for  $u = u_p(\cdot)$ . The relation (13) on opened sets in the bundle will be similarly the necessary condition (11) for control  $u = u_p(\cdot)$  extremality in the problem (3). For the function  $V_p$  the equalities (11),(13) are similar to the Isaacs-Bellman ones.

And now we extend the control (6) class and introduce a transformation operation of signal (4) values  $y(\tau)$ ,  $\tau < t$  by integrating the system

$$r(\tau)=k(\tau,y(\tau),r(\tau),u(\tau),\tau), \quad r(0)=r_0 \quad (14)$$

We will form the control (a control with memory)

$$u(t)=u(t,y(t),r(t))$$

regarding that functions  $u=u(t,y,r)$  class decision ensures control (15) admissibility. Then one can realize the descent procedure through the given scheme in paragraph 2. For that it is sufficient to supplement the system (1) by the system (14) and make some renaming :  $(x,r) \rightarrow x^*, (y,r) \rightarrow y^*, (g,r) \rightarrow g^*, (f,k) \rightarrow f^*, (u,k) \rightarrow u^*$ .

The given approach makes possible to consider perturbations in the relations (1),(4):  $f=f(x,u,t,\xi), g=g(x,t,\eta)$ . In this case for the interior problem (8) the maximum search is added for variables  $\xi, \eta$  in bounds of the problem formulation [2].

### 3. Search of a stabilized control in linear discrete systems and lag-time systems.

For the stationary controllable systems of differential equations with a lag-time argument

$$\dot{x}(t)=\Phi_0 x(t)+\int_{-h}^0 d\Phi(\tau)x(t+\tau)+Hu(t), \quad x(t) \in \mathbb{R}^n, \quad t \geq 0 \quad (15)$$

we will construct a discrete approximation of order  $o(\Delta^2)$  by transferring to an integral equivalent of the system (1) and making use of a quadrature formula. We obtain the many-dimensional system

$$z_{k+1}=Pz_k+Qu_k, \quad k=0,1,2,\dots, \quad (16)$$

where  $z_k=(x^T(k\Delta), x^T((k-1)\Delta), \dots, x^T((k-(J-1))\Delta))$ ,  $u_k=u(k\Delta)$  and  $\Delta$  is a step of digitization,  $\Delta=h/(J-1)>0$ ,  $P=P(\Delta)$ ,  $Q=Q(\Delta)$ . Let us consider the problem of stabilized control constructing, i.e. we determine a matrix, which substitution from the formula (4) to (16) for the system (16)

$$u_k=Mz_k \quad (17)$$

supplies asymptotic stability of the trivial solution. To decide the problem we use the optimal damping method and the method of parameter extension. Let us introduce parameter  $\alpha$  in the system

$$z_{k+1}=\alpha(Pz_k+Qu_k), \quad \alpha \in [0,1] \quad (18)$$

and consider the criterion of optimal control (17) in the system (18) by minimum of the quadratic functional

$$\lambda(z_0, u) = \sum_{j=0}^{\infty} \sigma(z_j, u_j), \quad (19)$$

where  $\sigma(z_j, u_j) = z_j^T A z_j + z_j^T B u_j + u_j^T B^T z_j + u_j^T C u_j$  is a positive definite quadratic form of variables  $(z_j, u_j)$ . An optimal control approximations under fixed  $\alpha$  one can construct by the optimal damping method if the initial stabilized control to be known. For  $\alpha=0$  and small  $\alpha$  that control is  $u_k \equiv 0$  (i.e.  $M \equiv 0$ ). A step of the method is as follows: for a determined admissible (stabilized) control  $u_k = M_1 z_k$  criterion representation in the view of quadratic form  $z_0^T \Theta z_0 = V_1(z_0)$  is obtained, and new control  $u_k = M_2 z_k$  is defined by minimum of function  $V_1$  difference condition on system (18) trajectories. The minimum achieves under

$$M = M_2 = -(C + \alpha^2 Q^T \Theta_1 Q)^{-1} (B^T + \alpha^2 Q^T \Theta_1 P) \equiv M(\Theta_1)$$

By the control  $u_k = M_2 z_k$  one determines representation of criterion (19) values  $\lambda(z_0, u_2) = z_0^T \Theta_2 z_0$  etc. Such process converges in a sense of the limits  $\lim M_1 = M^*$ ,  $\lim \Theta_p = \Theta^*$ ,  $p \rightarrow \infty$  existing which define an optimal by a criterion (6) minimum control  $u_k = M^* z_k$  for the system (18) and a minimum value of the criterion  $\lambda(z_0, u^*) = z_0^T \Theta^* z_0$ . Since we are interested in system (16) stabilization the process is repeated for a finite monotone sequence of parameter  $\alpha$  values  $0 = \alpha_1 < \alpha_2 < \dots < \alpha_n = 1$ . Moreover, the iteration for every  $\alpha_s$  is terminated by the criterion which guarantees the obtained stabilized control for  $\alpha = \alpha_s$  to be stabilized for some  $\alpha = \alpha_{s+1} > \alpha_s$ . Additionally, it is desirable to ensure a possibly large difference  $d\alpha = \alpha_{s+1} - \alpha_s$ . It is proved if the initial system (15) to be stabilized by linear control

$$u(t) = \int_0^h M(\tau) x(t-\tau) d\tau \quad (20)$$

then described above process of parameter  $\alpha$  correction is finite and gives through the optimal damping method arbitrarily precise approximation of the optimal control in class of the quadratic criterion controls (20), where the sum (19) is an integral Riemann sum (by points  $t_k = k\Delta$ ,  $k=0, 1, 2, \dots$ ).

In case the matrix  $M(\Theta)$  at (20) is not arbitrarily and it has a given structure  $M(\Theta) = L(\Theta) G^*$  ( $G^*$  is a given one,  $L(\Theta)$  is a required one) the control (17) of the discrete system (18) is substituted to the control

$$u_k = \Gamma y_k, \quad y_k = D z_k \quad (21)$$

Here  $D$  - is a given matrix defined by the matrix  $G^*$  when transferring from the system (1) to the system (16) and  $\Gamma$  is constructed by the optimal damping method so that the signal  $u = u_k$  is a minimum in the following problem :

$$\min_u \max_{Dz=y} (\alpha^2 (Pz + Qu)^T \Theta (Pz + Qu) - z^T \Theta z + \sigma(z, u) = 0, \quad y = y_k \in D\mathcal{R}^n.$$

In this formula  $\Theta$  is a matrix of the quadratic form  $z_0^T \Theta z_0 = \lambda(z_0, u^*)$ , where  $u^*$  is a control of (21) received at the previous step of the optimal damping method.

Note that if the signal of instantaneous feedback in the system (16) to be defined by the equality  $y_k = G z_k$  then, in special case, one can assume  $D = G$  (a control of type (16)). However, such signal might be insufficient for the system (16) to be stabilized in class of the controls

(21). In general case for optimal control search the matrix  $D$  should be discovered as a maximum rank matrix among solutions of the observability matrix equation

$$DP^\tau = \sum_{j=0}^{\tau-1} F_j G P^{\tau-j},$$

where  $F_j$ ,  $\tau$  is an arbitrarily selected matrices and a number ( $0 < \tau \leq nJ$ ). In this case the equalities (21) are substituted by relations [3] (a control of type (15))

$$\begin{aligned} u_k &= \Gamma^* y_k^*, & y_k^* &= \sum_{j=0}^{\tau} F_j y_{k-j} - \sum_{j=1}^{\tau} \Psi_j Q u_{k-j}, \\ \Psi_j &= \Psi_{j-1} P + F_{j-1} G, & \Psi_0 &= D + F_0 G, & j &= 1, \tau. \end{aligned}$$

The proposed computing procedure of stabilization and optimal stabilized control approximation were approved at concrete systems.

### References

1. Zubov, V. I., (1975) Lectures on optimal control theory. Nauka. Moscow.
2. Kirin, N. E., (1993) Estimation and control methods in dynamic systems. State Univ.' P.H. St. Petersburg.
3. Kirin, N. E., A. N. Kirin. (1996) A method of sequential approximations in the control problem by incomplete feedback for linear discrete systems and lag-time systems. V.I.N.I.T.I. № 1082-B96 03.04.96. Moscow.

# CONTROL OF OSCILLATIONS IN ELECTROMECHANICAL SYSTEM

Ilja I. Blekhman and Kira S. Yakimova  
Corporation MEKHANOB, R,  
8 Linia, V.O., 21,  
St. Petersburg, RUSSIA

Olga P. Tomchina and Olga L. Nagibina  
St. Petersburg Machine Building Institute,  
Polustrovskii avenue, 14  
St. Petersburg, RUSSIA

**Abstract.** The control problem for non-linear oscillatory systems with unmodelled drive is addressed and a series of speed-gradient-like algorithms are proposed. The controller design is based on reduced 2nd order pendulum-like plant model and 1st order drive model. Another novelty of the paper is the choice of controller gain based on minimization of drive windings heating. The proposed algorithms are simple and easy for implementation.

**Keywords.** Non-linear control, adaptive, oscillations.

**Introduction.** In the most commonly used vibration technological machines the s called inertial vibroactuator is used for excitation of the working body. Such a vibroactuator includes the unbalanced rotor (or pendulum), which is driven by the induction motor. The rotor axis is usually disposed horizontally and the centre of gravity of the rotor takes the lowest possible position when at rest. In this case the electrical motor should satisfy the following two requirements:

- 1) It should allow to raise the centre of gravity to the upright position.
- 2) Its power should be sufficient to overcome the losses caused by the friction in the bearings as well as to provide the additional useful power necessary for technological purposes.

In many machines (particularly in the heavy ones) the first requirement is crucial for the choosing the motor because it is more restrictive.

The purpose of this paper is to demonstrate that using the proper control algorithm allows to significantly reduce the power necessary to meet the first requirement. Therefore it gives the possibility to reduce the nominal power of the driving motor. Four control algorithms for the start-up mode have been analysed. It has been shown that using the proposed algorithms allows to replace the first requirement by the milder one preventing the overheating of the motor.

Pendulum-like mechanisms are widely used as parts of various mechanical systems, e.g. in vibration equipment [1-5]. However the drive dynamics were not taken into account in existing publications. The drive dynamics are significant when we need to use low-power motor for driving heavy mechanism.

In this paper the problem of rotational control of pendulum-like mechanism in presence of drive dynamics. Different control strategies are analysed based on speed-gradient algorithm [6] and allowing to speed the heavy pendulum up to the desired energy level. The control power required to achieve the desired speed up time is evaluated using both Lyapunov functions. The design is based on using first order drive model. Restrictions of motor heating are taken into account too.

**Control goals.** We consider the problem of organizing rotational mode of the pendulum turning around the horizontal axis. Since such a mode should retain for along time the problem solving can be split into two stages.

At the first stage the pendulum should be made swinging until it achieves the upright position. At the second stage the pendulum should rotate in certain direction and speed up until it achieves the desired average angular velocity or the desired energy level. It is clear that required controlling torques may be different at different stages. Indeed, at the first stage external torque should increase the energy of system for pendulum could achieve the upright position. On the other hand at the second stage the external energy should be spent only to compensate losses (mechanical friction, losses of the heating the drive and so on).

The control goal at the first stage can be expressed as follows: find control law  $u(t) = U(y(t))$  ensuring of  $t_* > t_0$  such that

$$H_M(t_*) > H_M^* \quad (1)$$

where  $H_M(t_*)$  is the part of total system energy corresponding to the mechanical energy,  $H_M^*$  is the potential mechanical energy, corresponding to the upright position of pendulum.

At the second stage we introduce the following control goal

$$H_M^* < H_M(t) \leq H_M^* + \Delta H \quad (2)$$

where  $\Delta H$  is determined by the desired average velocity of rotation.

Additionally the control strategy should ensure not only constraints (1) and (2) but also heating constraint

$$I_{eq} \leq I_n \quad (3)$$

where  $I_{eq}$ ,  $I_n$  are equivalent and nominal values of motor current which can be found, e.g. by method of equivalent current.

**Design of control algorithm.** The electromechanical system (EMS) may be described by Lagrange-Maxwell equations:

$$A(x)\ddot{x} + A_1(x, \dot{x}, q, \dot{q}) = P(x, \dot{x}, q, \dot{q}, t)$$

$$L(q)\ddot{q} + L_1(q, \dot{q}) = E(x, q, \dot{q}, t) \quad (4)$$

where  $x, \dot{x} \in R^n$  are vectors of generalized co-ordinates and generalised velocities of mechanical part,  $q, \dot{q} \in R^m$  are vectors of generalized electrical charges and currents in independent loops of drives electrical circuits.

The model (4) can be further transformed by means of separation of motions procedure. The rigid mechanical model forms subsystem of slow motions  $S_0$ . The elasticity model of mechanical part together with electrical part form the subsystem of fast motions.

For purposes of controller design we separate the dominating part of the system corresponding to one-degree-of-freedom oscillatory mechanical part and first order drive dynamics. Therefore the approximate model of EMS looks as follows:

$$\ddot{q} + \omega_0^2 \sin q = k_M I \quad (5)$$

$$T\dot{I} + I = k_e u \quad (6)$$

where  $q$  is angular co-ordinate of mechanical part,  $\omega_0^2 = mgl / J$ ,  $k_M$  are parameters of mechanical part,  $I$  is drive current,  $u$  is control signal (drive voltage),  $T$ ,  $k_e$  are drive parameters (values of all parameters are positive).

Therefore the controlled plant (5), (6) is non-linear cascade system from the control point of view. The extensive research was carried out on control of non-linear cascade systems, starting with [10, 11]. However the previous publications were devoted to the regulation and tracking problems, while our goal is excitation of rotational motions of given energy, i.e. the desired system trajectory is not prespecified.

To solve the problem we use the speed-gradient approach described in [5-7]. The important advantage of this approach is that it ensures achievement of the arbitrary energy level of conservative system with arbitrary small controller gain. It allows to spend the control energy in steady-state mode only for overcoming the friction. Hence the heating losses will be minimized.

1. Depending on possibilities of available sensors and actuators the following control algorithms are proposed to use

$$u(t) = \begin{cases} -\gamma, & \text{if } \dot{q} > \varepsilon, H \leq H_* + \varepsilon \\ \gamma, & \text{if } \dot{q} < -\varepsilon, H \leq H_* + \varepsilon \\ 0, & \text{else} \end{cases} \quad (7)$$

where  $\varepsilon > 0$  is small threshold introduced to organize autostart.

## 2. Directed relay algorithm

$$u(t) = \begin{cases} -\gamma, & \text{if } \dot{q} > \varepsilon, q < 0, H \leq H_* + \varepsilon \\ \gamma, & \text{if } \dot{q} < -\varepsilon, q > 0, H \leq H_* + \varepsilon \\ 0, & \text{else} \end{cases} \quad (8)$$

According to the algorithm (7) the control action is applied only when the pendulum is moving down. It allows to further decrease the heat losses.

## 3. Linear and linear-relay speed-gradient algorithms:

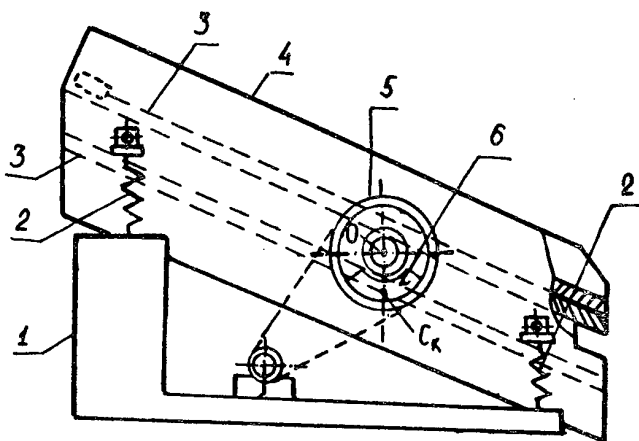
$$u(t) = -\gamma(H - H_*)\dot{q} \quad (9)$$

$$u(t) = -\gamma \text{sign}[(H - H_*)\dot{q}] \quad (10)$$

Algorithms (9) and (10) provide decreasing control value as far as system trajectory approaches either the goal set or equilibrium.

The theoretical investigation is performed and condition of the goal achievement are established based on analysis of energy-based SG-algorithms in Hamiltonian systems [7] and singular perturbations analysis of [8].

**Simulation results.** The simulation based comparative study of four control strategies (7), (8), (9) and (10) was done. Simulation was carried out in two stages. At the first stage the performance of controller with reduced plant model was analysed. At the second stage more detailed drive model was used, the heat losses criterion was investigated.



The constructive kinematic scheme of inertia sieve bolter is shown in Fig. 1, where 1 - is bearer frame, 2 is- shock-absorber, 3 - is sieve, 4 - is box, 5 - is sheaf, 6 - is unbalance.

Fig. 1 The inertia sieve bolter

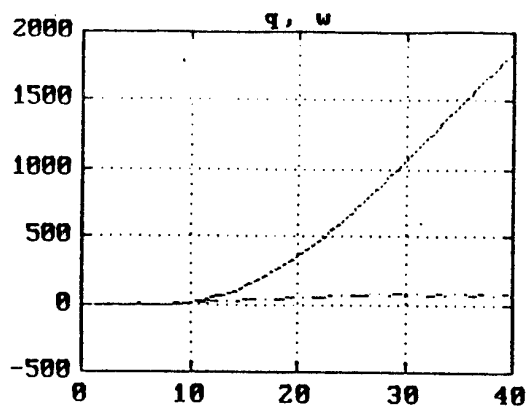


Fig. 2 Time history of coordinates.

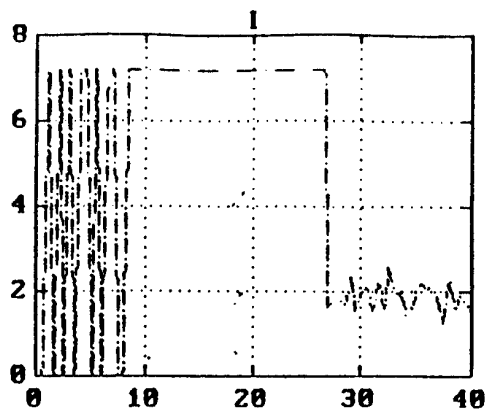


Fig. 4 Time history of current  $I(t)$ .

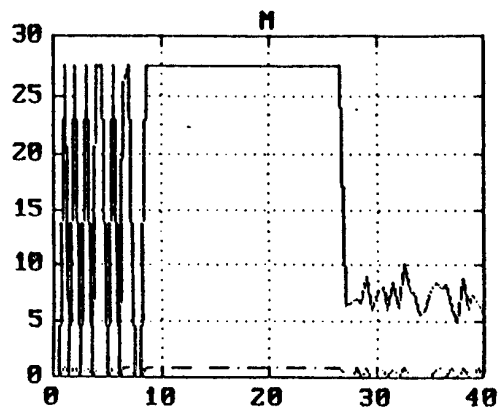


Fig. 3 Time history of torque  $M(t)$ .

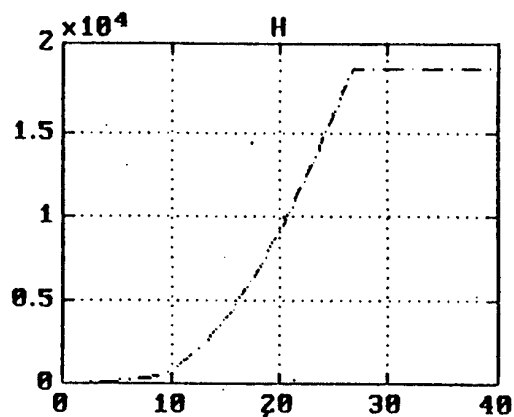


Fig. 5 Time history of energy  $H(t)$ .

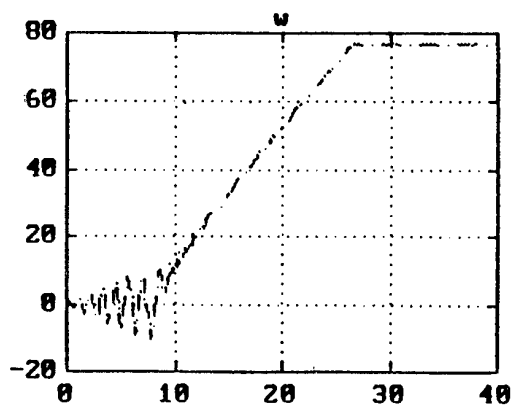


Fig. 6 Time history of velocity  $w(t)$ .

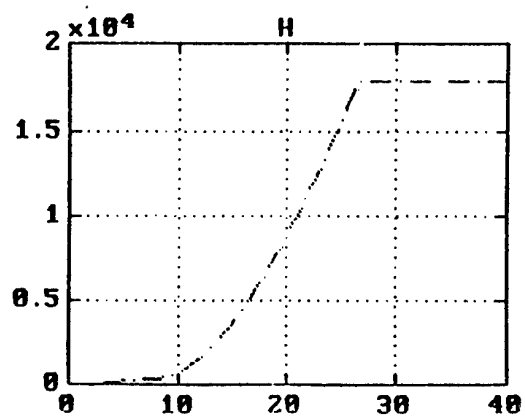


Fig. 7 Time history of energy  $H(t)$ .

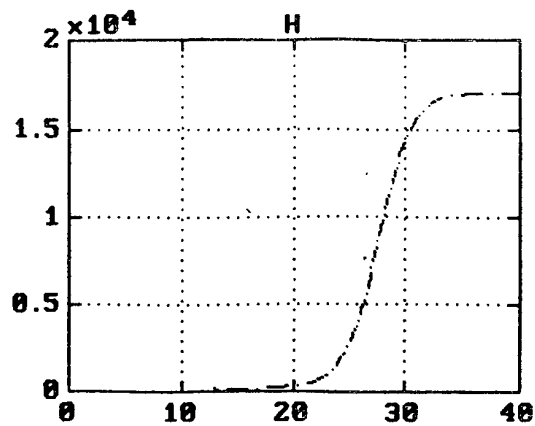
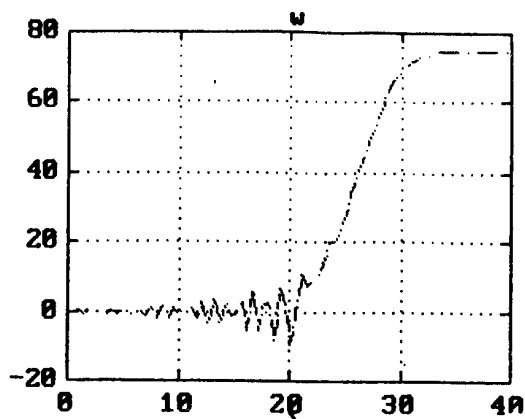


Fig. 8 Time history of velocity  $w(t)$ . Fig. 9 Time history of energy  $H(t)$ .

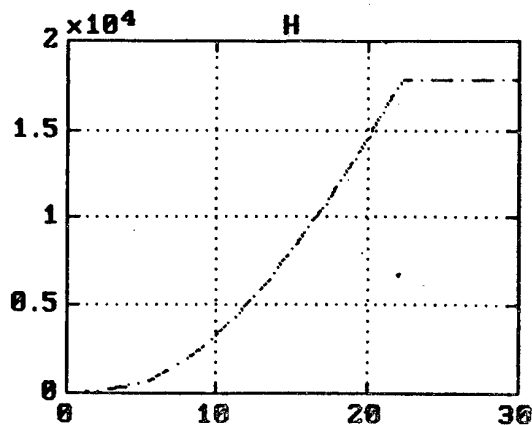
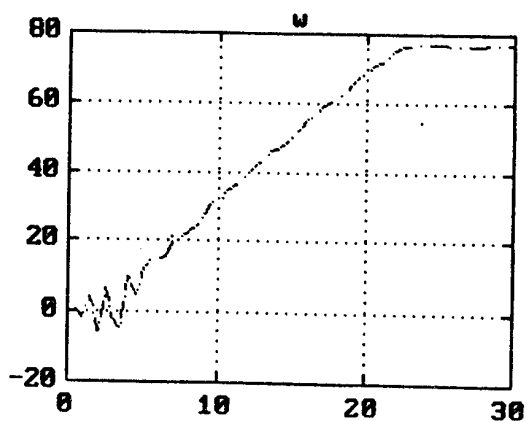


Fig. 10 Time history of velocity  $w(t)$ . Fig. 11 Time history of energy  $H(t)$ .

Stage 2. To evaluate constraints on the drive windings temperature the criterion of equivalent current is used.

Table 1 Results Of Equivalent Currents For Different Strategies

	strategy			
	(7)	(8)	(9)	(10)
$I_e$	7.04	6.9	7.8	7.71
$t_k$	27	27	33	22

The equation of motion for pendulum is following

$$J\ddot{\varphi} + k\dot{\varphi} + mgl \sin \varphi = M \quad (11)$$

State vector for this system is  $x = [\varphi, \omega]^T$ , where  $\omega = \dot{\varphi}$  is the angular velocity. It should think of mean, that  $|\omega(t)|$  is limited by technical parameters of motor-pendulum system.

Stage 1. The time history of angular co-ordinate  $\varphi(t)$ , velocity  $\omega(t)$ , mechanical torque  $M(t)$ , current  $I(t)$  and total energy of system  $H(t)$  for controller (7) are shown in Fig. 2-5. Figures 6-11 exhibit time history of  $\omega(t)$  and  $H(t)$  for controllers (8), (9), (10) respectively.

As it follows from Fig. 2-11 the best results are obtained when the algorithm (10) is used.

The results of the equivalent current for control strategies (7), (8), (9) and (10) are given in Table 1, where  $t_k$  is the transient time.

Using the proposed control laws the pendulum can be put into the upright position. The power of motor does not exceed the level required for overcoming the friction in the bearings. However in practice the saving of the motor power amounts to not greater than 20%. The reason is that the energy consumption in the working mode is sent both for overcoming the friction in the bearings and for transportation of the material along sieve.

The useful energy for material transportation looks as follows:

$$N = [1 - 3]Wt * L * Q * v \quad (12)$$

where  $L = 5m$  is length of sieve,  $Q = 263.89kg / s$  is productivity of sieve,  $v = 0.3m / s$  is speed of material moving along the sieves (in our case  $N=8.8$  kWt).

**Conclusions.** The control problem for non-linear oscillatory systems with unmodelled drive dynamics is addressed and a series of speed-gradient-like algorithms are proposed. The performance of the control system was examined by means of computer simulations which have shown satisfactory transients. Another novelty of the paper is the choice of controller gain based on minimization of drive windings heating. Algorithms are easy for implementation. More complicated solutions may be suggested based on dynamic non-linear dumping [9].

## References

1. Blekhman, I. I. (1971). Synchronization of dynamic systems. Nauka, Moskow.
2. Beards, C. F. (1988). Vibration and control systems. Elis Horwood, Chichester.
3. Fradkov, A. L., O. P. Tomchina, O. L. Nagibina (1995). Swing Control of Rotating Pendulum. Proc. of 3rd IEEE Mediterranean Control Conference, Limassol, Vol. 1, pp. 347-351.

4. Astrom, K. J., K. Furuta. Swinging up a pendulum by energy control. 13th Congress of IFAC, V.E., pp. 37-42.
5. Fradkov, A. L., P. Yu. Guzenko, D. J. Hill, A. Y. Pogromsky (1995). Speed-gradient control and passivity of non-linear oscillations. Proc. of the 3th IFAC Symposium on Non-linear Control Systems, Tahoe-Sity, Vol. 2, pp. 655-659.
6. Fradkov, A. L. (1990). Adaptive control in large-scale systems. Nauka, Moscow.
7. Fradkov, A. L. (1996). Swinging control of non-linear oscillations. International Journal of Control, Vol. 64, n. 5.
8. Fradkov, A. L. (1987). Adaptive control systems design for non-linear singularly perturbed objects. Automatika: Telemekhanika. J, 6, 100-110.
9. Krstic, M., J. Sun, P. V. Kokotovic' (1996). Robust Control of Non-linear systems with Input Unmodelled Dynamics. IEEE Trans. on Automatic Control, Vol. 4, N 6.
10. Ortega, R. (1991). On global stabilization of cascaded non-linear systems. Automatica.
11. Saberi, A., P. V. Kokotovic', H. J. Sussmann (1990). Global stabilization of partially linear composite systems. SIAM J. Contr. Optimiz., Vol. 28, pp. 1491-1503.

# OPTIMAL UNIVERSAL REGULATORS IN THE PROBLEM OF DAMPING OF FORCED OSCILLATION

Anton S. Shiriaev

Saint Petersburg State University

Mail: P. O. Box 141 St. Petersburg 198904 Russia

Email: anton@bobka.stud.pu.ru

**Abstract.** This paper considers a forced oscillations damping problem in linear systems subjected to either harmonic disturbances or singular stochastic disturbances. The regulator is required to minimize a quadratic cost function with *any* disturbances from the given class. The main results give a sufficient conditions for the existence of a suitable linear stabilizing regulator.

**Keywords.** Linear system, optimal control, harmonic disturbance damping, singular stochastic disturbance damping.

**Introduction.** The problem of damping of forced oscillations is of great practical importance. Consider this problem in the form of the following optimization one

$$dx(t)/dt = Ax(t) + bu(t) + f^0\phi_1(t), \quad (1)$$

$$y(t) = c^*x(t) + g^0\phi_2(t), \quad (2)$$

$$\Phi(x(\cdot), u(\cdot)) = \overline{\lim}_{T \rightarrow \infty} \frac{1}{T} \int_0^T \mathcal{G}[x(t), u(t)] dt \rightarrow \min, \quad (3)$$

where  $0 \leq t < \infty$ ,  $x(t) \in R^n$ ,  $u(t) \in R^m$ ,  $y(t) \in R^k$ ,  $\phi_1(t) \in R^{l_1}$ ,  $\phi_2(t) \in R^{l_2}$ ,  $\mathcal{G}(x, u) = x^*Gx + 2x^*gu + u^*\Gamma u$  is a real quadratic form,  $A, b, c, f^0, g^0, G = G^*, g, \Gamma = \Gamma^* > 0$  are known real matrices,  $\det c^*c \neq 0$ . It is assumed that the pair  $(A, b)$  is stabilizable and the pair  $(A, c^*)$  is detectable. The external disturbances  $\phi_1(t), \phi_2(t)$  are harmonic functions

$$\phi_1(t) = \sum_{j=1}^{N_1} \phi_j^{(1)} e^{i\omega_j^{(1)}t}, \quad \phi_2(t) = \sum_{j=1}^{N_2} \phi_j^{(2)} e^{i\omega_j^{(2)}t} \quad (4)$$

with known frequencies  $\Omega_1 = \{\omega_j^{(1)}\}_{j=1}^{N_1}$ ,  $\Omega_2 = \{\omega_j^{(2)}\}_{j=1}^{N_2}$  and with unknown complex vector amplitudes  $F_1 = \{\phi_j^{(1)}\}_{j=1}^{N_1}$ ,  $F_2 = \{\phi_j^{(2)}\}_{j=1}^{N_2}$

The objective is to find the set of physically realizable stabilizing regulators with input  $y(t)$  and output  $u(t)$  which are optimal in the problem (1), (2), (3) with any harmonic disturbances in the form of (4) and which do not depend on the initial conditions  $x(0) = a$  and the complex vector amplitudes  $F_1, F_2$ .

In the case  $g^0 = 0$ ,  $y(t) = c^*x(t)$ , the problem (1), (2), (3) was considered in [1-2]. It turned out that under the natural assumptions the infimum in the optimization problem (1), (2), (3) for the wide class of all admissible processes coincides with the infimum in the class of linear physically realizable stabilizing regulators of the form

$$\alpha(p)u = \beta(p)y, \quad (p = d/dt), \quad (5)$$

where  $\alpha(\lambda), \beta(\lambda) — m \times m$  and  $m \times k$  matrix polynomials (m.p.). Moreover, under the additional assumption

$$k \geq l_1, \quad (6)$$

i.e. the dimension of disturbance vector  $\phi_1(t)$  is not more than the dimension of output vector  $y(t)$ , there exists an regulator (5) which is optimal in the problem (1)-(3) for an arbitrary amplitudes set  $F_1$  and does not depend on  $F_1$  (universal in the class  $F_1$ ).

In this paper it is shown that in the case  $g^0 \neq 0$  in the problem (1), (2), (3) under the same assumptions with changing (6) to

$$k \geq \max\{l_1, l_2\}, \quad \Omega_1 \cap \Omega_2 = \emptyset \quad (7)$$

$$k \geq l_1 + l_2, \quad \Omega_1 \cap \Omega_2 \neq \emptyset \quad (8)$$

also there exists the optimal universal in the class  $\{F_1, F_2\}$  regulator of form (5).

The second part of this paper is devoted to closely related stochastic optimization problem. It will be assumed that the control system has the form (1), (2) and external disturbances  $\phi_1(t), \phi_2(t)$  are stationary random processes representable by the stochastic integrals

$$\phi_1(t) = \int_{-\infty}^{+\infty} e^{i\theta t} W_1(i\theta) d\zeta_1(\theta), \quad \phi_2(t) = \int_{-\infty}^{+\infty} e^{i\theta t} W_2(i\theta) d\zeta_2(\theta), \quad (9)$$

where  $W_1(i\theta), W_2(i\theta)$  are  $l_1 \times l_1, l_2 \times l_2$  matrices,  $W_1(i\theta), W_2(i\theta) \in \mathcal{L}_2(-\infty, +\infty)$  and  $\zeta_1(\theta) \in R^{l_1}, \zeta_2(\theta) \in R^{l_2}$  are independent stochastic processes with uncorrelated increments

$$\begin{aligned} E d\zeta_1(\theta) &= 0, \quad E d\zeta_1(\theta_1) d\zeta_1(\theta_2)^* = \delta(\theta_1 - \theta_2) I_{l_1} d\theta_1 d\theta_2, \\ E d\zeta_2(\theta) &= 0, \quad E d\zeta_2(\theta_1) d\zeta_2(\theta_2)^* = \delta(\theta_1 - \theta_2) I_{l_2} d\theta_1 d\theta_2. \end{aligned} \quad (10)$$

Here  $E$  is the symbol of the expectation,  $I_l$  is the  $l$ -dimensioned identity matrix and  $\delta(\theta)$  is the Dirac delta-function. Let

$$S_1(\theta) = W_1(i\theta) W_1(i\theta)^*, \quad S_2(\theta) = W_2(i\theta) W_2(i\theta)^* \quad (11)$$

be the spectral densities of the processes  $\phi_1(t), \phi_2(t)$  and suppose that  $S_1(\theta), S_2(\theta)$  are unknown but their upper bounds  $\sigma_1(\theta), \sigma_2(\theta)$  are known, i. e.

$$S_1(\theta) \leq \sigma_1(\theta) I_{l_1}, \quad S_2(\theta) \leq \sigma_2(\theta) I_{l_2}, \quad (12)$$

and that scalar functions  $\sigma_1(\theta), \sigma_2(\theta)$  rapidly decrease to zero

$$\int_{-\infty}^{+\infty} \frac{\ln \sigma_i(\theta)}{1 + \theta^2} d\theta = -\infty, \quad \int_{-\infty}^{+\infty} |\theta|^N \sigma_i(\theta) d\theta < +\infty, \quad \forall N, \quad i = 1, 2. \quad (13)$$

Denote  $\mathcal{P}_\phi$  the set of all pair external disturbances satisfying relations (9)-(13).

We will consider two sets  $\mathcal{N}, \mathcal{L}$  of admissible processes  $(x(\cdot), u(\cdot))$ . The set  $\mathcal{N}$  is determined by the following condition: the stochastic process  $(x(\cdot), u(\cdot))$  belongs to  $\mathcal{N}$  if  $E|x(0)|^2 < \infty, E|u(t)|^2 < \infty$  and the equation (1) holds along with the 'stability' assumption

$$1/t E|x(t)|^2 \rightarrow 0 \quad t \rightarrow \infty. \quad (14)$$

The same notation  $\mathcal{N}$  we keep to the family of regulators providing the set of admissible processes  $\mathcal{N}$  as the solution of the closed-loop system. Particularly,  $\mathcal{N}$  includes nonlinear regulators and even physically nonrealizable regulators.

The set  $\mathcal{L}$  contains the processes  $(x(\cdot), u(\cdot))$  generated by the equations (1), (2) and by the linear stabilizing regulator (5), where  $\alpha(\lambda)$ ,  $\beta(\lambda)$  are  $m \times m$ ,  $m \times k$  matrix polynomials. The differentiation is understood in the sense of the theory of generated random processes. It is assumed that the initial condition of (1), (2), (5) are with the finite second moments and not dependent from  $\phi_1, \phi_2$ .

The performance index of the optimization problem has the form

$$\Phi(x(\cdot), u(\cdot)) = \overline{\lim}_{T \rightarrow \infty} \frac{1}{T} E \int_0^T \mathcal{G}(x(t), u(t)) dt, \quad (15)$$

where  $\mathcal{G}(x, u) = x^* G x + 2x^* g u + u^* \Gamma u$  is given quadratic form. An admissible regulator (5) providing the infimum of the functional (15) over the set of the process  $\mathcal{N}$  is called an optimal regulator. In the case when  $\inf_{\mathcal{N}} \Phi$  is not attained in  $\mathcal{L}$  we will consider an  $\varepsilon$ -optimal regulator (5), i. e.

$$\Phi(x(\cdot), u(\cdot)) \leq \inf \Phi + \varepsilon, \quad (16)$$

where  $\varepsilon$  is positive number. The  $\varepsilon$ -optimal regulator (5) is said to be universal in the class  $\mathcal{P}_\phi$  if the inequality (16) is fulfilled for any  $(\phi_1, \phi_2) \in P_\phi$ .

In a manner like [3] we show that usually the optimal stabilizing regulator (5) does not exist. At the same time we show that under the natural assumptions and with the additional assumptions

$$k \geq \max\{l_1, l_2\}, \quad \text{supp } \sigma_1(\theta) \cap \text{supp } \sigma_2(\theta) = \emptyset \quad (17)$$

$$k \geq l_1 + l_2, \quad \text{supp } \sigma_1(\theta) \cap \text{supp } \sigma_2(\theta) \neq \emptyset \quad (18)$$

for any  $\varepsilon > 0$  there exists the  $\varepsilon$ -optimal universal in  $\mathcal{P}_\phi$  regulator (5).

## References

1. Yakubovich, V.A., *Doklady Akademii Nauk*, vol. 337, 1994, pp.323-327; English trans. in *Phys. Dokl.* vol. 39, 1994.
2. Yakubovich, V.A., *Universal Regulators in LQ Optimization Problem*, In Proceedings of the 3d European Control Conference, pp.53-69, september 1995.
3. Yakubovich, V.A., *Doklady Akademii Nauk*, vol. 338, N 1, 1994, pp. 19-24.

# PROJECTIVE OBSERVABILITY OF DYNAMICAL SYSTEMS.

Yuri V. Bolotin, Svetlana N. Morgunova.  
Faculty of Mathematics and Mechanics,  
Department of Applied Mechanics and Mathematics,  
Moscow Lomonosov State University, 119899, Russia.  
e-mail: ybolotin@nw.math.msu.su  
e-mail: smorgun@nw.math.msu.su

## *Abstract.*

The trajectory estimation problem for linear dynamic systems with bearing only measurements is considered. By treating bearing only measurements as projective, the problem can be reduced to that of total least squares (TLS). The difficulty in applying TLS methods lies in frequent unobservability of the trajectories. In this paper the definition of the projective observability is suggested and discussed from the geometrical standpoint. Some connections with hamiltonian mechanics and applications to TLS algorithms are presented.

## *Keywords.*

Bearing only estimation, observability, maximum likelihood, least squares problem.

## *Introduction.*

Bearing only trajectory estimation is one of the central problems in aerospace navigation and control. With plane observations of the coordinate vector  $\bar{r} = (r_1, r_2)$ , bearing only measurements are given by the angle  $\arctan(r_1 / r_2)$ , or alternatively by a unit vector  $\bar{r} / \|\bar{r}\|$ , i.e. a point on the unit circle  $S^1$ . In spatial dynamics they are given by a unit vector in three dimensional space, i.e. a point on the unit sphere  $S^2$ . In multidimensional case with measurement  $z \in R^m$  they can be identified with a point on the unit sphere  $S^{m-1}$ . Neglecting the direction of  $z$ , a bearing only measurement can be treated as a point in the projective space  $RP^{m-1}$ . We shall refer to the above two representations as the spherical and the projective ones, respectively.

The bearing only estimation problem is nonlinear. For given dynamics, trajectory estimation is usually done with the spherical representation by the extended Kalman filter (EKF), with versions varying mainly in the linearization scheme [1]. The drawback of EKF is that it unexpectedly fails in poor observability cases.

A different approach was first suggested in [2], where the projective measurements were used to reduce the problem to that of ordinary least squares (LS). Unfortunately, the LS estimates are biased with low signal to noise ratio (SNR). A generalization of the LS algorithm is to reduce the problem to restricted total least squares (TLS) [3]. Under not strict statistical assumptions the TLS solution is unbiased and consistent in the observable case [4]. However, in many important aerospace applications, in particular when the observer doesn't move, the aircraft trajectories are unobservable, and the TLS algorithm becomes degenerate [4].

The goal of this paper is to develop the TLS approach for the latter case. Special definitions of projective observability and of observation subspace are introduced. Sufficient observability conditions are derived in linear algebraic terms, and connections of necessary conditions with

hamiltonian mechanics are established. The TLS problem is posed and solved as that of best fitting the observation subspace.

Bearing only observability analysis is usually done under special restrictions on the system dynamics, often with  $m = 2, 3$  [5]. In this paper the general linear systems are considered. It seems that the general approach not only widens the possible application area, but makes the technique more straightforward.

The symbol  $I$  denotes the unit matrix, for a matrix  $A$  the superscript  $A^*$  denotes a transposed matrix, for a linear space  $V$  superscript  $V^*$  denotes the dual space, i.e. the space of linear forms on  $V$ . The symbol  $\wedge$  denotes outer product of two vectors. The set of smooth functions with values in  $X$ , on an interval  $0 \leq t \leq T$ , is denoted by  $X_T$ . In this paper the complex case is considered. Reduction of most results to the real case is straightforward.

### ***Projective observability.***

Let  $G$  be a linear system in the  $n$  - dimensional complex state space  $X \ni x$  with  $m$  - dimensional linear observations  $Z \ni z$

$$\dot{x} = Ax, \quad z = Cx \quad (1)$$

Here  $A, C$  are constant matrices with dimensions  $n \times n, m \times m$ , respectively. The trajectories  $x(t) \in X_T$  are considered. The system is called linearly observable if the observations  $z(t) \in Z_T$  uniquely determine the trajectory  $x(t) \in X_T$ . A well known linear observability condition is that the matrix

$$(A^* - \lambda I \quad C^*) \quad (2)$$

must be of maximum rank for all complex  $\lambda$ .

**Definition 1.** The measurements  $z'(t) \in Z_T, z(t) \in Z_T$  are projectively equivalent if  $z'(t) = \lambda(t)z(t)$  for some complex  $\lambda(t)$ . The trajectory  $x(t) \in X_T$  is projectively observable if for any other trajectory  $x'(t) \in X_T$  with projectively equivalent observations  $x'(t) = \mu(t)x(t)$  for some complex  $\mu(t)$ . The observation subspace  $\Pi_z \subset X_T$  for the data  $z(t) \in Z_T$  is the set of trajectories with measurements projectively equivalent to  $z(t)$ .

The system  $G$  is projectively observable if it doesn't allow projectively unobservable trajectories.

Obviously, for given observations  $z(t)$  the observation subspace  $\Pi_z$  is the linear subspace determined by the linear equation  $z(t) \wedge Cx(t) \equiv 0$ . The last formula generalizes the so called pseudo measurement form of representing bearing only data [2].

Projective observability of a system implies linear observability, but not vice versa. A specific property of projective measurements is that the same system can allow both observable and unobservable trajectories.

### ***Mechanical systems.***

Let first consider the special case of mechanical systems with coordinates  $q = (q_1, \dots, q_k)$ ,  $k = n/2$  of the form

$$\ddot{q} + 2\Gamma\dot{q} + Kq = 0, \quad z = q, \quad (3)$$

where  $\Gamma$  is an antisymmetric matrix of gyroscopic forces, and  $K$  is a symmetric matrix of elastic forces.

**Proposition 1.**

A mechanical system (3) is projectively unobservable iff a coordinate transformation in  $q$  decomposes it to a set of subsystems of the following types:

- (1) Harmonic oscillator;
- (2) Several harmonic oscillators with equal frequencies, possibly linked by gyroscopic forces.

**Proof.** With a transformation to a rotating coordinate system  $q = \exp(-\Gamma t)r$ , we obtain

$$\ddot{r} + \exp(\Gamma t)(K - \Gamma^2)\exp(-\Gamma t)r = 0, \quad z = \exp(-\Gamma t)r. \quad (4)$$

If the system is projectively unobservable, there exist two trajectories  $r(t)$ ,  $r'(t)$ , such that  $r'(t) = \lambda(t)r(t)$ ,  $\lambda(t) \neq \text{const}$ . Then (4) yields  $d/dt(\lambda(t)r(t)) = 1/2\ddot{\lambda}r$ , i.e. the vector  $r(t)$  doesn't rotate in space. Therefore, the matrix  $\exp(\Gamma t)(K - \Gamma^2)\exp(-\Gamma t)$  has a constant eigenvector  $r_0$ , so that the matrices  $K$ ,  $\exp \Gamma t$  commute on some common invariant subspace. The latter is possible if either there exists an eigenspace of  $K$  where  $\Gamma$  degenerates, or there exists an eigenspace of  $\Gamma$  where  $K$  acts as a scalar, q.e.d.

**Example 1.** A harmonic oscillator with one degree of freedom  $\ddot{q} + q = 0$ ,  $z = q$  is projectively unobservable. Indeed, the phase  $\phi$  of the solution  $q(t) = a \exp(it + \phi)$  can't be recovered.

**Example 2.** A pair of harmonic oscillators  $\ddot{q} + 2\Gamma\dot{q} + Kq = 0$ ,  $q = (q_1, q_2)$ ,  $z = q$  is projectively observable, if the matrix  $\Gamma$  is not degenerate, and the frequencies of  $K$  are different.

**Example 3.** Consider the spatial motion of a charged particle with position  $z = \bar{r} = (r_1, r_2, r_3)$  and velocity  $\bar{v} = (v_1, v_2, v_3)$  in the electromagnetic field with constant magnetic inductance  $\bar{B}$  and in some potential field with strength  $\bar{E}(\bar{r}) = K\bar{r}$ , where  $K$  is a symmetric matrix. The field  $\bar{B}$  induces gyroscopic forces  $\bar{v} \times \bar{B}$  with a nondegenerate invariant space  $V$  orthogonal to  $\bar{B}$ . Thus the system is projectively unobservable iff  $\bar{B}$  is an eigenvector of  $K$ .

**Example 4.** In typical bearing only aerospace applications the trajectory of a spacecraft is modeled by the equation  $\ddot{\bar{r}} = 0$ . Since the gyroscopic forces are absent, the system is projectively unobservable. The family of projectively indistinguishable trajectories can be written as  $\bar{r}(t) = (a + bt)\bar{r}_0$ , where  $a, b$  are arbitrary scalars.

**Observability conditions.**

Let  $X^*$ ,  $Z^*$  denote the dual spaces for  $X$ ,  $Z$ , respectively, and  $G^*$  be the dual to  $G$  control system with state space vector  $x^* \in X^*$  and control  $z^* \in Z^*$

$$\dot{x}^* = A^*x^* + C^*z^*. \quad (5)$$

Let  $\Lambda^2 X$ ,  $\Lambda^2 Z$ ,  $\Lambda^2 X^*$ ,  $\Lambda^2 Z^*$  be the linear spaces of outer products  $x \wedge x'$ ,  $z \wedge z'$ ,  $x^* \wedge x'^*$ ,  $z^* \wedge z'^*$  respectively. The flow on  $X$  determined by  $G$  induces a linear system  $\Lambda^2 G$  with state vector  $\xi \in \Lambda^2 X$  and observations  $\eta \in \Lambda^2 Z$  as

$$\dot{\xi} = \tilde{A}\xi, \quad \eta = \tilde{C}\xi \quad (6)$$

where

$$\begin{aligned} \tilde{A}(x \wedge x') &= Ax \wedge x' + x \wedge Ax' \\ \tilde{C}(x \wedge x') &= Cx \wedge Cx'. \end{aligned}$$

Similarly, the flow induces a linear control system  $\Lambda^2 G^*$  with state space vector  $\xi^* \in \Lambda^2 X^*$  and control  $\eta^* \in \Lambda^2 Z^*$  as

$$\dot{\xi}^* = \tilde{A}^* \xi^* + \tilde{C}^* \eta^* \quad (7)$$

where

$$\begin{aligned} \tilde{A}^*(x^* \wedge x'^*) &= A^* x^* \wedge x'^* + x^* \wedge A^* x'^* \\ \tilde{C}^*(z^* \wedge z'^*) &= C^* z^* \wedge C^* z'^* \end{aligned}$$

The linear observability of  $\Lambda^2 G$  is equivalent to linear controllability of  $\Lambda^2 G^*$ . Moreover, if for the trajectories  $x(t)$ ,  $x'(t)$  of  $G$  the observations are projectively equivalent, then  $z(t) \wedge z'(t) \equiv 0$ . If  $\Lambda^2 G$  is linearly observable, then  $G$  is projectively observable. [6]

### Proposition 2.

Let  $\Omega \in \Lambda^2 X$  be the linearly unobservable subspace of  $\Lambda^2 G$ . Then the observation subspace  $\Pi_z$  for the data  $z(t) = Cx(t)$  is spanned by the trajectories  $x'(t)$  such that  $x(0) \wedge x'(0) \in \Omega$ . [6]

A system  $G$  with  $\Lambda^2 G$  linearly observable will be called  $\Lambda$ -observable. The reduction of the problem to that of  $\Lambda$ -observability makes the problem linear. In [6] the  $\Lambda$ -observability is said to be equivalent to projective observability. We shall show that this is generally not true.

### Proposition 3.

A mechanical system (3) is  $\Lambda$ -unobservable.

**Proof.** Let  $p_i^*$ ,  $q_i^*$  be the canonical impulses and coordinates in the dual to (3) mechanical system  $G^*$ . The symplectic form for  $G^*$  can be written as  $\omega = dp_1^* \wedge dq_1^* + \dots + dp_k^* \wedge dq_k^*$ . Since the flow induced by  $G^*$  is hamiltonian,  $\omega$  is invariant to the flow, i.e.  $\tilde{A}\omega = 0$ . On the other hand,  $\omega = 0$  on the subspace  $p^* = 0$ . With duality transformation, the last condition can be written as  $\tilde{C}\omega = 0$ . The equalities  $\tilde{A}\omega = 0$ ,  $\tilde{C}\omega = 0$  contradict the linear observability test (2) for  $\Lambda^2 G$ , q.e.d.

**Definition 2.** The system (1) is called near mechanical, if a coordinate transformation  $(p, q) = \exp(\lambda t)x$ , where  $\lambda$  is a complex constant, reduces it to the form (3).

### Proposition 4.

For a  $\Lambda$ -unobservable system  $G$  there exists a coordinate transformation  $Fx = (p \ q \ w)$  such that the subspace  $V = \{w \equiv 0\}$  is an invariant of  $G$ , and on  $V$  the system is near mechanical.

**Proof.** If  $\Lambda^2 G$  is linearly unobservable, then  $\tilde{A}\omega = \lambda\omega$ ,  $\tilde{C}\omega = 0$  for some complex  $\lambda$  and  $\omega \in \Lambda^2 X$ . With a coordinate transformation  $x = \exp(\lambda t)y$ , and notation  $A' = A - \lambda I$ , the  $\Lambda$ -unobservability conditions can be written as  $\tilde{A}'\omega = 0$ ,  $\tilde{C}\omega = 0$ . Let consider  $\omega \in \Lambda^2 X$  as a 2-form on  $X^*$ . Since  $\tilde{A}'\omega = 0$ , the form is invariant to the flow induced by  $A'$ , i.e. this flow is hamiltonian with respect to the symplectic structure  $\omega$  on the maximum subspace  $V^* \in X^*$  where  $\omega$  is not degenerate. The equality  $\tilde{C}\omega = 0$  can be rewritten as  $\omega(C^*u^*, C^*v^*) = 0$  for all  $u^*, v^* \in Z^*$ , i.e.  $\omega = 0$  on  $C^*Z^*$ . Thus  $C^*Z^*$  is a lagrangian subspace of  $V^*$ . By the Darboux theorem there exists a coordinate transformation  $Fx = (p \ q \ w)$  reducing  $\omega$  to the canonical form  $\omega = dp_1^* \wedge dq_1^* + \dots + dp_k^* \wedge dq_k^*$ , and reducing  $C$  to the form  $(C_1 \ 0 \ C_3)$ . The system  $G^*$  can then be written as

$$\begin{aligned} \dot{q}^* &= \frac{\partial H^*}{\partial p^*} + C_1^* z^*, & \dot{p}^* &= -\frac{\partial H^*}{\partial q^*} \\ \dot{w}^* &= A_1^* q^* + A_2^* p^* + A_3^* w^* + C_3^* z^* \end{aligned}$$

where  $H^*(p^*, q^*)$  is a quadratic energy function. With notation  $H(p, q) = -H^*(q, -p)$ ,  $G$  can be written as

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p} + A_1 w, & \dot{p} &= -\frac{\partial H}{\partial q} + A_2 w, \\ \dot{w} &= A_3 w, & z &= C_1 q + C_3 w. \end{aligned} \quad (8)$$

The equality  $w \equiv 0$  determines a mechanical subsystem in (8) of the form (3), q.e.d.

From the proposition 4 follows:

#### Theorem 1.

Suppose  $G$  doesn't contain near mechanical projectively observable subsystems. Then  $G$  is projectively observable iff it is  $\Lambda$ -observable.

To check for mechanical subsystems in a  $\Lambda$ -unobservable system  $G$  the latter must first be decomposed as described in proposition 4 for all linearly unobservable eigen pairs of  $\Lambda^2 G$ . This can be done by the standard QZ algorithm. If all the mechanical subsystems are projectively unobservable as described in proposition 1, i.e. have special block structure with block dimensions less than 4, the system is projectively unobservable.

In some cases equivalence of  $\Lambda$ -observability and projective observability is obvious. For example, when all the sums  $\lambda_i + \lambda_j$  are different, where  $\lambda_1, \dots, \lambda_n$  are the eigenvalues of  $A$ .

#### A TLS estimation algorithm.

Let us now consider the noisy estimation problem, with  $z / \|z\| \in S^{m-1}$  being the exact spherical data, and  $e = z / \|z\| - \eta$  being the noisy measurements. Here  $\eta$  is a time independent random variable with some mean square error  $\sigma$  and probability density  $p(\sigma)$ . Following the idea of [2], the projective data is rewritten in the pseudo measurement form  $(e + \eta) \wedge Cx = 0$ . The pseudo measurement is a point in  $\Lambda^2 G$ , i.e. linear space with dimensions  $n(n-1)/2$ .

Let  $\Phi$  be the class of  $k$  - dimensional subspaces  $\Gamma_k \subset X_T \times Z_T$ , such that for each pair  $(x(t), \eta(t)) \in \Gamma_k$  the dynamic and measurement equations are satisfied:

$$(e + \eta) \wedge Cx(t) = 0, \quad x(t) = \exp(At)x(0). \quad (9)$$

**Definition 3.** The order  $k$  maximum likelihood estimation problem  $MLE(k)$  is to find a subspace  $\bar{\Gamma}_k \in \Phi$ , maximizing the likelihood function:

$$\int_{\Gamma_k \in \Phi} p(\eta) d\eta \rightarrow \max \quad (10)$$

The projection of  $\bar{\Gamma}_k$  on  $X_T$  is an estimate of the observation subspace  $\bar{\Pi}_z$ . In the gaussian noise model the  $MLE(k)$  problem is to find  $k$  minimum norm orthogonal error functions  $\eta^{(i)}(t)$ ,  $1 \leq i \leq k$ , satisfying (9). The last problem can be reduced to that of total least squares as follows.

Let  $\Lambda_e(t)$  denote the operator  $\Lambda_e : Z \rightarrow \Lambda^2 Z$ , acting as  $\Lambda_e(t)z(t) = e(t) \wedge z(t)$ , and let denote  $\Psi(t) = C \exp(At)$ . Next, let us introduce an arbitrary operator  $\Delta(t) : Z \rightarrow \Lambda^2 Z$ , or equivalently a matrix with dimensions  $m \times m(m-1)/2$ . With notation  $\Delta(t) = \Lambda_\eta(t)$  the equations (9), (10) can be rewritten in the matrix form

$$(\Lambda_e(t) + \Delta(t))\Psi(t)x(0) = 0, \quad \int_0^T \|\Delta(t)\|_F^2 dt \rightarrow \min \quad (11)$$

This is exactly a TLS problem, except that  $\Delta(t)$  is of special structure. However,  $\Delta(t)$  can be assumed arbitrary, since the minimum in (11) is reached on  $\Delta(t)$  of the form  $\Delta(t) = \eta \wedge z$ , for some  $\eta \in Z$ . This yields the following result.

### Theorem 2.

The  $MLE(k)$  approximation of the observation subspace with gaussian noise is given by the  $k$  solutions  $x^{(i)}(0)$ ,  $\Delta^{(i)}(t)$  of the TLS problem (11) with orthogonal minimum norm perturbations  $\Delta^{(i)}(t)$ .

An outline of a numerical algorithm for solving the TLS problem is given below. In practice it is done in discrete time  $t = 0, \dots, T$ , with  $T \gg n$ .

- (1) Perform QR - decomposition  $\Psi = QR$ , and denote  $\Lambda' =$  first  $n$  columns of  $\Lambda_e Q$ .
- (2) Perform SVD of  $\Lambda' = U \Sigma V^*$ , and select the  $k$  smallest singular values  $\sigma_{n-k} > \sigma_{n-k+1} \geq \dots \geq \sigma_n$ .
- (3) Denote  $W$  the last  $k$  columns of  $V$ , and obtain the basis for  $x(0)$  in  $\bar{\Pi}_z$  as the columns of  $R^{-1}W$ .

For data with  $\dim \Pi_z = k$  and without noise the last  $k$  singular values are exactly zeros, while  $\sigma_{n-k} > 0$ . When the noise is asymptotically small, or the number of observations is great,  $\sigma_{n-k} \gg \sigma_{n-k+1}$ , and the dimensions of  $\Pi_z$  can be estimated correctly. Since the right singular vectors are estimated consistently [3], this proves that the above algorithm provides an asymptotically consistent estimate of  $\Pi_z$ .

## *Conclusions.*

Treating the bearing only measurements as projective the trajectory estimation problem can be reduced to the linear 'error in variables' form, and both the observability analysis and data processing can be almost always done with linear algebra. The only possible obstacle to sufficiency of linear analysis can be caused by complex mechanical structure of the system.\*

## *References.*

1. Song, T.L., J.L. Speyer (1985). A stochastic analysis of a modified gain extended Kalman filter with applications to estimation with bearing only measurements. *IEEE Transactions on Automatic Control*, Vol. AC-30, No. 10, pp.940 - 949.
2. Stansfield, R.G. (1947). Statistical theory of DF fixing. *Journal of IEE*, 14, Pt III A, 15, pp. 762 - 770.
3. Van Haffel, S., H.Zha (1991). The restricted total least squares problem: formulation, algorithm and properties. *Matrix Anal. Appl.* Vol. 12, No. 3, pp. 292 - 309.
4. Bolotin, Yu.V. (1994). A TLS approach to angle-only trajectory estimation. *IEEE Workshop on real time computing*, Prague.
5. Payne, A.N. (1989). Observability problem for bearing-only tracking. *Int. J. Control*, Vol.49, No. 3, pp. 761 - 768.
6. Ovseevich, A.I. (1994). Singularities of attainable sets and observability problems (in Russian). The 5 Russian Workshop on mathematical navigation methods. M., MGU.

---

\* This work is supported by RFFI grant 95-01-00218.

# MODELING, CONTROL AND LABORATORY EXPERIMENTS WITH OSCILLATORY MECHANICAL SYSTEM

V.A.Konoplev,  
Institute of Problems of Mechanical Engineering,  
61, Bolshoy Ave., V.O., St.Petersburg, Russia.  
A.P.Konjukhov,  
Baltic State Technical University,  
1, 1<sup>st</sup> Krasnoarmeyskaya st., St. Petersburg, Russia

**Abstract.** In the article a problem of mathematical models construction for oscillatory mechanical systems (OMS) with complex (chaotic) dynamics is considered. The mathematical model of system is constructed with application of the aggregative method. The device for OMS control with use of the computer is described. Verification of the obtained model on the basis of laboratory experiments results is made. Pulse-number algorithm of OMS control is offered and checked with use of laboratory set-up.

**Key words:** oscillatory mechanical system, control algorithm, laboratory experiments, aggregative method of models building, mathematical model, numerical experiment.

**Controlled plant.** The controlled plant is the oscillatory mechanical system (OMS) (see Figure 1) which consists of the double links pendulum with displaced centres of parts weight and sliding support of an external link. An external part of system is a metal ring with located on it a massive ball and two cylindrical magnets. The ball and magnets displace the centre of weight of ring. Magnets, in addition, provide transfer of control efforts to both links. On an outside surface of a ring two opposite directed half-axes, ensuring its support on two flat platforms with terminators of a course, are located. System is fixed on the massive basis, in centre of which the inductive sensor of the first link zero state and electromagnet, transmitting control efforts on cylindrical magnet are located.

The second part is presented by two cylindrical loads with magnets which established on axis symmetric concerning its centre. This part rotates inside an external ring with an axis of rotation fixed on it with the help of cylindrical hinges. These hinges are on an axis which is turned by 45 degrees with respect to the axis of an external ring rotation.

The specified mechanical system has three degrees of freedom:

1. Parallel carry of an external ring on flat support of the basis;
2. Rotation of an external ring with the above-stated half-axes;
3. Rotation of the second link of a rather external ring.

On the specified mechanical system the following control efforts are apply:

1. Influence of an electromagnet of the basis on one of magnets, established on an external ring;

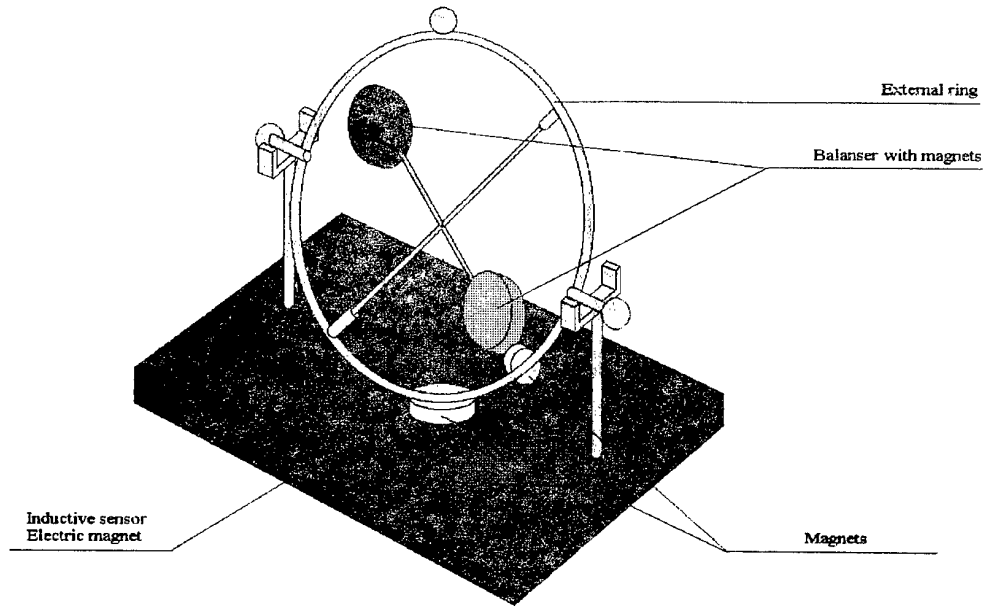


Figure 1. Oscillatory mechanical system

2. Influence of the second magnet, established on an external ring on magnets of second (internal) link.

Measuring value is an interval of time between transitions of an external ring magnet above an electromagnet of the basis.

**Mathematical model of mechanical system.** The equations of a mechanical system motion are written in aggregative form as follows [5,6]:

$$A(q)\ddot{q} + B(q, \dot{q}) = SG + p + u, \quad (2)$$

$$A(q) = SAS^T, B(q, \dot{q}) = S\Phi AS^T + SAS^{T*},$$

Where  $S - (3 \times 12)$  - structural matrix of system;  $A = \text{diag}(A_1^1, A_2^2)$  - block-diagonal  $(12 \times 12)$ - matrix of inertia with  $(6 \times 6)$ - blocks of a kind  $A_1^1 = \sum_{s=1}^{10} L_{1s}^1 \theta_{1s}^{1s} L_{1s}^{1,T}$ ,  $A_2^2 = \sum_{s=1}^4 L_{2s}^2 \theta_{2s}^{2s} L_{2s}^{2,T}$ , where  $\theta_{LS}^{LS}$  - inertia matrix of an  $S^{\text{th}}$  body in the system of coordinate which connected to it,  $L_{LS}^L$  - matrix of transformation from system of coordinates  $E_L$  to coordinate system which connected to a body  $E_S$ ;  $\Phi = (\Phi_1^{0,1}, \Phi_2^{0,2})$  - block-diagonal  $(12 \times 12)$  - matrix of quasivelocities with  $(6 \times 6)$  - blocks on a main diagonal,

$$\Phi_L^{0,L} = \begin{Bmatrix} \langle \omega_L^0 \rangle^L & 0 \\ \langle v_L^0 \rangle^L & \langle \omega_L^0 \rangle^L \end{Bmatrix},$$

Where  $L=1, 2$ ;  $\langle * \rangle$  - mark of a skew-symmetric matrix;  $v_L^{0L}, \omega_L^{0L}$  - elements of a quasivelocities vector  $V = \|V_1^{01}, V_2^{02}\| = S^T \dot{q}$ ,  $V_L^{0L} = \|v_L^{0L}, \omega_L^{0L}\|$ ;  $G - (3 \times 1)$  - vector of

gravitation forces dynamic screws;  $p - (3 \times 1)$ - column of friction efforts on axes of system mobility;  $u - (3 \times 1)$ - column of magnets interactions from which first element is control effort.

The dependence of magnets interaction force amplitude was approximated by density-of-Gauss-distribution-like equation

$$F_{mi} = A_{mi} \exp\left(-\frac{(\theta_s^i)^2}{\sigma_{mi}^2}\right), i = 1, 2 \quad (3)$$

Where  $\theta_s^i$  - corner of  $i^{\text{th}}$  body rotation.

The numerical meanings of factors  $A_{mi}$  and  $\sigma_{mi}$  were determined with use of experimental data and looks as follows  $A_{m1} = 10, A_{m2} = 3, \sigma_{m1} = 0,05, \sigma_{m2} = 0,01$ .

The obtained system of equations is realized in environment of a program package MATLAB-ADAM [3].

**OMS control by means the computer.** For realization of experiments on OMS control the following devices were developed:

1. The elementary controller, that fix a signal from the sensor of a zero state and transfer control efforts on an electromagnet;
2. As the second variant of interface the microcontroller on the basis of the processor i80C51GB can be used.

The elementary controller includes in its structure the following basic subblocks:

1. The converter of a sensor signal to signals, which can be fixed by TTL microcircuits;
2. The record strobe pulses and signals of an interruption inquiry generator (strobe pulses provide a record to a fixing register);
3. Fixing register, which provide temporary storage of the information from the sensor and control signals from the computer;
4. The amplifier of a control signal.

The specified controller is connected to a parallel port of a IBM PC-compatible computer and is operated with the help of the specialized program, realizing required control algorithm. Thus the distribution of control signals can be generated as by the interruptions service program, and by the program, completely accepting management by a course of computing process. The second variant of the control program building was used only, since not all microcircuits of LPT port support function of interruptions reception. Besides it is in this case possible to receive an additional gain in chronomentering accuracy and additional time for processing of sensor signals and for acceptance of the control decisions.

The controller on the basis of the microprocessor i80C51GB has essentially large opportunities. The microcircuit includes the analog-digital converter, block of timers, can give out pulse-width signals according to given from the outside or received in result of work of the program which incorporated in the microprocessor. The microcontroller is connected to PC

through a serial interface. The more detailed description of this controller can be found in [6]. The experiments with this controller in the present moment are planned.

**Control algorithm.** The development of control algorithm was made on the basis of speed-gradient algorithm[1,2], where the control aim was stabilization of a given total energy level of a mechanical system [6,7]. Control aim can be written as follows:

$$H \rightarrow H_{ref}, \text{ when } t \rightarrow \infty, \quad (4)$$

where  $H_{ref}$  is the given energy level.

As was shown in these works, such algorithm allows to achieve various types of systems behaviour at regulation of whole one parameter. For considered system the development of control algorithm complicated by absence of the complete information about its state (period of external link oscillations is measured only). However, as was shown by additional investigation, development of control algorithm, realizing the control aim, close to considered in the specified work, is possible in this case also. So, it is possible to show, that for a simple nonlinear pendulum the period of fluctuations is not constant, and is determined by a formula:

$$T = \sqrt{2} \int_{A_1}^{A_2} \frac{dx}{\sqrt{h - \Pi(x)}}, \quad (5)$$

Where  $A_1$  and  $A_2$  - deviation of a pendulum in moment, when its speed is equal to zero;

$h$  - total energy of system;

$\Pi(x)$  - potential function, i.e. a function, proportional to potential energy of the system [8].

The control of a simple nonlinear pendulum oscillation period is at the same time lead to the control of a pendulum total energy. The considered system is a double pendulum, however the influence of a second link movement on a movement of a first link is small and can be considered as revolting disturbance. This assumption used at development of a control algorithm. Other peculiarity of considered system is limitation of the control action application zone - it is limited by sector, in which a large magnet of the first link is induced registered by sensitive element EMF. Suppose that we fix input signal and apply an control action at discrete moments of time  $t_k$ , where  $k=1,2,\dots$ . Also suppose that maximal magnitude of output control voltage is equal to  $U_{max}$ . In this case the elementary control algorithm is formulated as follows:

$$U_{out}(t_k) = \begin{cases} 0, & \text{when } U_{inp}(t_k) = 0 \\ U_{max}, & \text{when } U_{inp}(t_k) \neq 0 \end{cases} \quad (6)$$

This algorithm can be rewritten also for a case of feedback control. In this case the problem of period stabilization of the first link oscillations is solved. It is offered thus to determine number of pulses in a package (i.e. in group of consecutive pulses, appropriate to the Eq. (5)) to provide the given value of oscillations period, and thus the given total energy of the system. The algorithm can be presented in the form

$$\begin{aligned} n &= n_{base} + \Delta n, \\ n_{min} &\leq n \leq n_{max}, \\ \Delta n &= \{(T_{base} - T_{ref}) / \Delta T_{base}\}, \end{aligned} \quad (7)$$

where  $\{*\}$  - denotes the nearest greater integer.

In this case we can rewrite Eq. (6) as

$$U_{out}(t_k) = \begin{cases} 0, & \text{when } U_{inp}(t_k) = 0; \\ U_{max}, & \text{when } U_{inp}(t_k) \neq 0 \text{ and } n_{min} \leq N_{contr} \leq n; \\ U_{max}, & \text{when } U_{inp}(t_k) \neq 0 \text{ and } n < N_{contr} \leq n_{max}; \end{cases} \quad (8)$$

$N_{contr}$  increase from  $n_{min}$  to  $n_{max}$ .

Included in a Eq. (7) and (8) values are determined on the basis of experiments.

Results of numerical and laboratory experiments with the specified control algorithms considered below.

**Results of numerical modeling and laboratory experiments.** The results of numerical experiments are presented on Fig. 2a and 2b. The following values of experimentally determined parameters were accepted:  $n_{base} = 7$ ,  $n_{max} = 10$ ,  $n_{min} = 4$ ,  $T_{base} = 1300ms$ ,  $\Delta T_{base} = 50ms$ ,  $U_{max} = 9V$ . Initial conditions were taken as follows:  $\theta_1 = \pi / 6 \text{ rad}$ ,  $\theta_2 = 0 \text{ rad}$ ,  $\omega_1 = 0 \text{ rad / s}$ ,  $\omega_2 = 0 \text{ rad / s}$ .

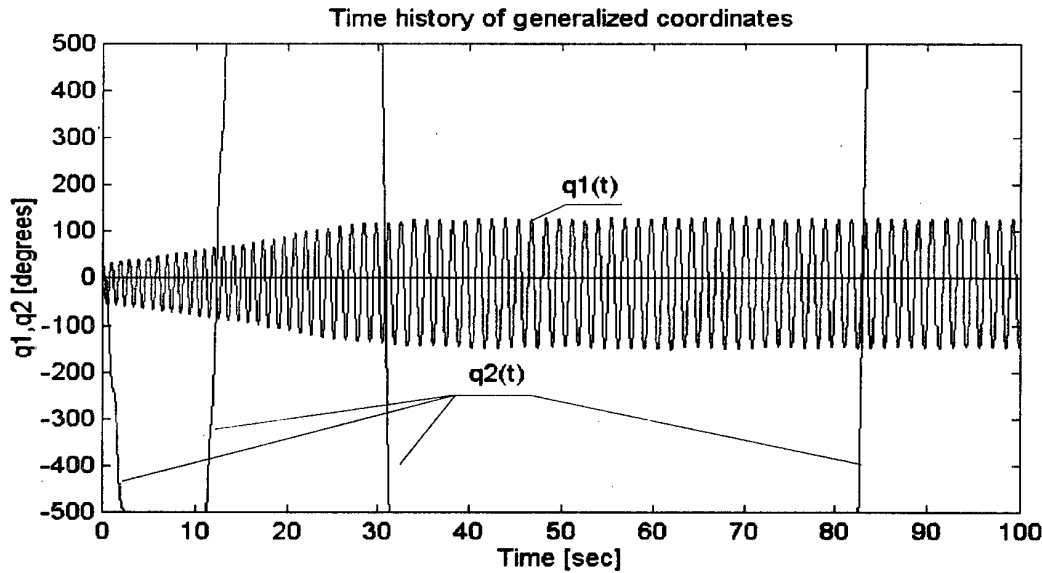


Figure 2a. Transient process (numerical experiment).

The results of laboratory research with use of the elementary controller and the same parameter values are presented on Fig. 2c.

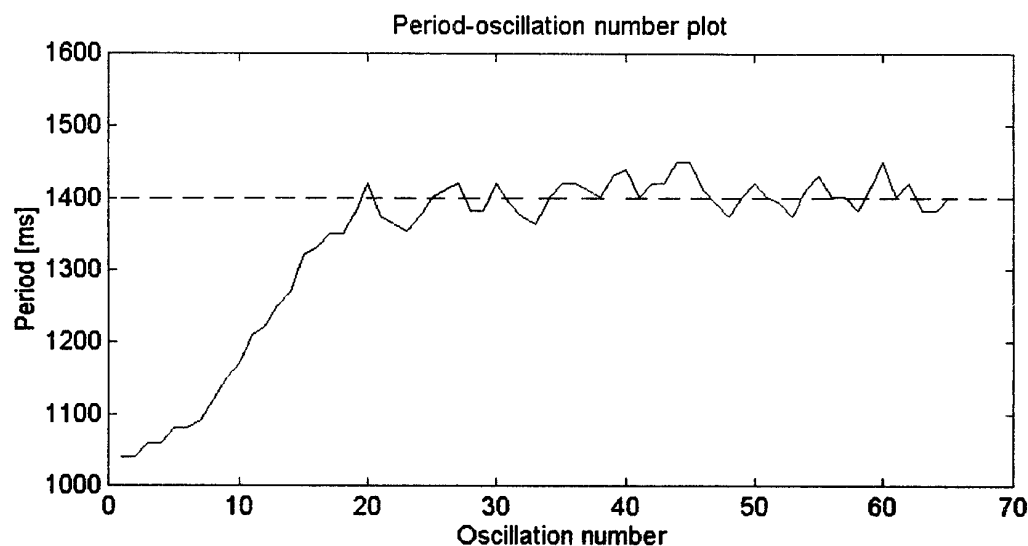


Figure 2b. Period-oscillation number plot for numerical experiment.

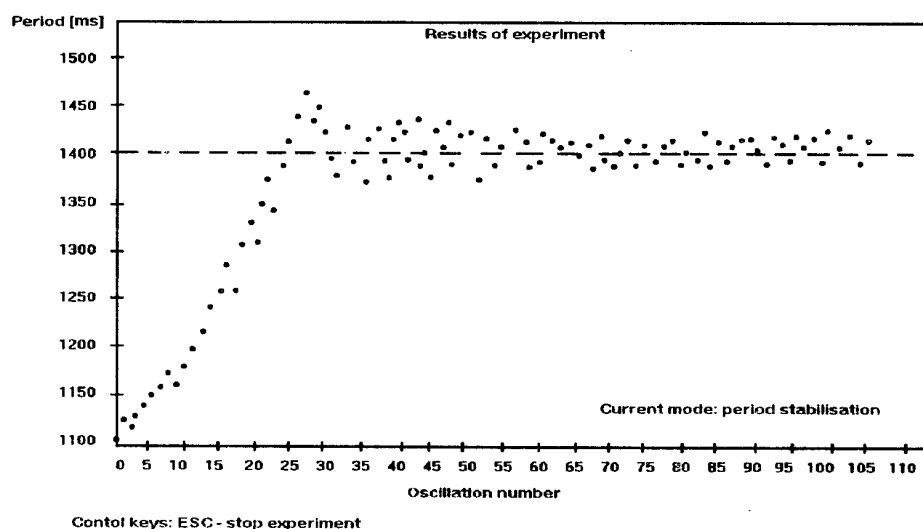


Figure 2c. Period-oscillation number plot for laboratory experiment.

As it is visible from figures, numerical and the experimental research give similar results. As it seen, curve on Fig.2b and Fig 2c are approximated by exponent equation with equal parameters and disturbances both for first and second samples restricted in the same interval. It says about high adequacy of mathematical model to an OMS. Besides it is possible to note high quality of process of regulation, that testifies to efficiencies of offered control algorithm. However it is necessary to note, that the considered system has an essential lack, namely - narrow range of the regulated variable: 1100-1500ms. Nevertheless, the obtained result is

interesting and important, as it is shown the opportunity of non-traditional methods use for control algorithm design.

### *References*

1. Fradkov A.L., Guzenko P.Yu., Hill D.J., Pogromsky A.Yu. (1995) Speed Gradient Control and Passivity of Nonlinear Oscillators, IFAC Symposium on Nonlinear Control Systems (NOLCOS'95), USA, 28-30 June 1995, pp.655-659.
2. Fradkov A.L. (1996) Swinging control of nonlinear oscillations. *International Journal of Control*, vol.6, №6, pp.1189-1202.
3. Fradkov A.L., Egorenkov D.L., Kharlamov V.Yu (1996) Basics of mathematical modeling with samples on MATLAB language. St.Petersburg, Baltic State Technical University.
4. Konoplev V.A. (1989) Aggregative Models of Mechanics of Solid Body Systems with Tree-Like Structure, *Mechanics of Solid*, vol. 24, №6, pp.45-52, USA, New-York.
5. Konoplev V.A. (1996) The aggregative mechanic of solid systems. St.Petersburg, Science.
6. Konjukhov A.P., Protopopov E.G. (1996) Oscillations Control of Coupled Pendula Systems. 5th International Student Olympiad on Automatic Control. Abstracts. St.Petersburg, Russia, October 2-4, 1996, pp.75-79.
7. Konjukhov A.P., Tomchina O.P., Nagibina O.V (1996) Energy Based Double Pendulum Control in Periodic and Chaotic Mode. The Third International Conference on Motion and Vibration Control (MOVIC'96), September 1-6, 1996, Chiba, Japan, pp.98-104.
8. Migulin V.V., Medvedev V.I., Mustel E.R., Parygin V.N. (1978) Basics of oscillation theory. Moscow, Science.

# STOCHASTIC CONTROL MINIMAX PROBLEM\*

V.N.FOMIN

*St. Petersburg State University, Russia*

e-mail: vlad@fomin.niimm.spb.su

**Abstract.** *A statement and a solution of a version of the minimax stochastic control problem are given. The control optimization is performed in the class of spectral densities of noise such that the integral of those squared is bounded on the frequency axis.*

**Keywords:** linear plant, transfer matrix, minimax stochastic control, parametrization, analytic function.

**1. Statement of the problem.** Consider a control system described by the following linear differential equations with constant real coefficients

$$a(d/dt)y(t) = b(d/dt)u(t) + v(t), \quad \alpha(d/dt)u(t) = \beta(d/dt)y(t), \quad (1)$$

where  $y(t)$  is an  $n$ -vector of the plant outputs and  $u(t)$  is an  $m$ -vector of control disturbances at instant  $t$ ;  $a(\cdot), b(\cdot), \alpha(\cdot), \beta(\cdot)$  are polynomial matrices,

$$a(p) = p^q + a_{q-1}p^{q-1} + \dots + a_0, \quad b(p) = b_{q-1}p^{q-1} + b_{q-2}p^{q-2} + \dots + b_0,$$

$$\alpha(p) = p^r + \alpha_{r-1}p^{r-1} + \dots + \alpha_0, \quad \beta(p) = \beta_{r+1}p^{r+1} + \beta_r p^r + \dots + \beta_0$$

( $q, r$  are nonnegative integers;  $a_k, b_k, \alpha_k, \beta_k$  are matrices of the appropriate dimensions),  $v = \{v(t), t \in [0, \infty)\}$  is a centered noise with a spectral densities matrix  $G_v(\cdot)$ , whose entries are square-integrable on the frequency axis. The feedback described by the second equation of (1) is assumed to be stabilizing, i.e., ensuring existence and finiteness of the limit

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \mathbf{E}(|x(t)|^2 + |u(t)|^2) dt$$

for any pair of vector functions  $x(\cdot), u(\cdot)$  satisfying system (1). Introduce the following cost functional for the control performance  $u$ :

$$J(\gamma, G_v) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \mathbf{E} \begin{pmatrix} y(t) \\ u(t) \end{pmatrix}^* N \begin{pmatrix} y(t) \\ u(t) \end{pmatrix} dt. \quad (2)$$

---

\*This research is conducted within the research grant No 95-01-00226 from RFFI.

Here  $\gamma = (\alpha, \beta)$  denotes a collection of the feedback coefficients,  $\mathbf{E}$  signifies the mathematical expectation, the Hermitian conjugation is symbolized by the asterisk, and  $N$  denotes a positive weighting matrix of the appropriate dimension.

Given the spectral density  $G_v(\cdot)$ , we arrive at the linear quadratic optimization problem [1],

$$J(\gamma, G_v) \rightarrow \inf_{\gamma \in \Gamma}, \quad (3)$$

where  $\Gamma$  denotes the set of stabilizing feedbacks. Let us know only that the spectral density  $G_v(\cdot)$  satisfies the condition

$$\int_{-\infty}^{\infty} \text{trace}\{[G_v(\omega)]^2\} d\omega \leq 1. \quad (4)$$

Denoting the set of functions  $G_v(\cdot)$  satisfying (4) by  $\mathbf{G}_1$ , we get the minimax problem

$$\sup_{G_v \in \mathbf{G}_1} J(\gamma, G_v) \rightarrow \inf_{\gamma \in \Gamma} \quad (5)$$

referred to as the stochastic control minimax problem.

**2. Reformulation of the minimax problem in frequency terms.** Due to stationarity of the control system, the cost functional (2) may be easily rearranged to give

$$J(\gamma, G_v) = \int_{-\infty}^{\infty} \text{trace}\{[W_1(\omega)]^* N W_1(\omega) G_v(\omega)\} d\omega, \quad (6)$$

where  $\text{trace}\{\cdot\}$  signifies the sum of eigenvalues of  $\{\cdot\}$  and  $W_1(\cdot)$  is a component of the transfer function

$$W(\omega) = \begin{bmatrix} a(i\omega) & -b(i\omega) \\ \beta(i\omega) & -\alpha(i\omega) \end{bmatrix}^{-1} \quad (7)$$

of the closed-loop control system (1),  $W(\cdot) = [W^{(1)}(\cdot) \ W^{(2)}(\cdot)]$  (i.e.,  $W^{(1)}(\cdot)$  is the transfer function from  $v$  to  $\text{col}(y, u)$ ).

For matrix functions  $G(\cdot)$  satisfying the condition

$$\int_{-\infty}^{\infty} \text{trace}\{[G(\omega)]^* G(\omega)\} d\omega < \infty,$$

introduce the inner product

$$\langle G', G'' \rangle = \int_{-\infty}^{\infty} \text{trace}\{[G'(\omega)]^* G''(\omega)\} d\omega. \quad (8)$$

The functional (6), in (8), can be rewritten as

$$J(\gamma, G_v) = \langle N, G_v \rangle, \quad (9)$$

where

$$N(\omega) = [W_1(\omega)]^* N W_1(\omega) \quad (10)$$

and  $G_v(\cdot)$  is the spectral density of noise  $v$  in (1).

$$\textbf{Lemma.} \sup_{G_v \in G_1} J(\gamma, G_v) = \langle N, N \rangle^{1/2}. \quad (11)$$

The proof of Lemma is almost evident from the fact that based on the Hilbert space geometry,  $G_v(\cdot)$  maximizing the linear function  $\langle N, G_v \rangle$  can be expressed as  $G_v = cN$ , where the constant  $c$  is to be subject to the condition  $c \langle N, N \rangle = 1$ . Thus the optimization problem (5) can be reduced to

$$J(\gamma) = \langle N, N \rangle \rightarrow \inf_{\gamma \in \Gamma}. \quad (12)$$

**3. Parametrization of the set of transfer functions.** Define  $W_0(\cdot)$  to be the transfer function related to a certain stabilizing feedback by

$$W_0(\omega) = \begin{bmatrix} a(i\omega) & -b(i\omega) \\ \beta_0(i\omega) & -\alpha_0(i\omega) \end{bmatrix}^{-1} = [W_0^{(1)}, W_0^{(2)}] \quad (13)$$

(polynomials  $\alpha_0(\cdot)$ ,  $\beta_0(\cdot)$  of this feedback are assumed to be known). Following [1], the component  $W^{(1)}(\cdot)$  of an arbitrary transfer function can be represented as

$$W^{(1)}(\omega) = W_0^{(1)} + W_0^{(2)}\psi(\omega), \quad (14)$$

where  $\psi(\cdot)$  is an arbitrary matrix function of the appropriate dimension. Inserting (13) in the right-hand side of (11) and making use of the notation (10), we obtain

$$\begin{aligned} J(\gamma) = \langle N, N \rangle &= \tilde{J}(\psi) = \langle \psi^* R \psi + 2\operatorname{Re} \{\psi^* r\} + \rho, \psi^* R \psi + 2\operatorname{Re} \{\psi^* r\} + \rho \rangle = \\ &= \int_{-\infty}^{\infty} \operatorname{trace} \{ [\psi(\omega)]^* [W_0^{(2)}(\omega)]^* N W_0^{(2)}(\omega) \psi(\omega) + 2\operatorname{Re} \{ [\psi(\omega)]^* r(\omega) \} + \rho(\omega) \}^2 d\omega, \end{aligned} \quad (15)$$

where an operator  $R$ , a matrix function  $R(\cdot)$ , a vector function  $r(\cdot)$ , and a scalar function  $\rho(\cdot)$  are defined by

$$\begin{aligned} (R\psi)(\omega) &= R(\omega)\psi(\omega), \quad R(\omega) = [W_0^{(2)}(\omega)]^* N W_0^{(2)}(\omega), \\ r(\omega) &= [W_0^{(2)}(\omega)]^* N W_0^{(1)}(\omega), \quad \rho(\omega) = [W_0^{(1)}(\omega)]^* N W_0^{(1)}(\omega). \end{aligned} \quad (16)$$

**4. Main assertion.** Suppose that for some  $\varepsilon > 0$  and any  $\omega \in \mathbf{R}$ , a rational function  $R(\cdot)$  (see (18)) satisfies the inequality  $R(\omega) \geq \varepsilon I$  ( $I$  is the identity matrix of the appropriate dimension). It is well known that such a function can be expressed as

$$R(\omega) = U(\omega)[U(-\omega)]^T, \quad \omega \in \mathbf{R}, \quad (17)$$

where  $U(\cdot)$  is a function admitting the analytic continuation to the lower half-plane ( $U^T$  is the transpose of  $U$ ). Relation (17), called the spectral factorization of the function  $R(\cdot)$ , defines the factor  $U(\cdot)$  up to the multiplication from the right by a diagonal unitary matrix. For an arbitrary rational matrix function  $L(\cdot)$  being square-integrable over the frequency axis, the following representation is valid:

$$L(\omega) = L_-(\omega) + L_+(\omega), \quad (18)$$

where  $L_-(\cdot), L_+(\cdot)$  are functions being analytic in the lower and upper half-plane, respectively. Relationship (18), referred to as the separation of  $L(\cdot)$ , uniquely defines its components  $L_-(\cdot), L_+(\cdot)$ .

**Theorem.** Let the matrix function  $R(\cdot)$  admit the representation (17) and the function

$$L(\omega) = \left[ \left( U^{-1}(\omega) \right)^* r(\omega) \right] \left[ \rho(\omega) + \rho_1(\omega) + \rho_2(\omega) \right] \quad (19)$$

with

$$\rho_1(\omega) = \left( \left[ \left( U^{-1}(\omega) \right)^* r(\omega) \right] \right)^* \left[ \left( U^{-1}(\omega) \right)^* r(\omega) \right], \quad (20)$$

$$\rho_2(\omega) = \left( \left[ \left( U^{-1}(\omega) \right)^* r(\omega) \right] \right)^* \left[ \left( U^{-1}(\omega) \right)^* r(\omega) \right]$$

be rational and admitting the analytic continuation to the upper half-plane where it is assumed to be bounded. Then the minimax problem (5) is solvable and its solution coincides with the solution of the linear quadratic problem (3) with  $G_v(\omega) \equiv I$ . The procedure of obtaining the solution is as follows. Define  $\psi_{opt}(\cdot)$  to be the rational function

$$\psi_{opt}(\omega) = -U^{-1}(\omega) \left[ \left( U^{-1}(\omega) \right)^* r(\omega) \right]. \quad (21)$$

Let  $\psi_1(\cdot), \psi_2(\cdot)$  be the polynomials defining the function  $\psi_{opt}(\cdot)$ ,

$$\psi_{opt}(\omega) = \psi_1^{-1}(\omega) \psi_2(\omega). \quad (22)$$

Then the polynomials  $\alpha_{opt}(\cdot), \beta_{opt}(\cdot)$  of optimal feedback are expressible as

$$\begin{aligned} \alpha_{opt}(i\omega) &= \psi_1(\omega) \alpha_0(i\omega) + \psi_2(\omega) b(i\omega), \\ \beta_{opt}(i\omega) &= \psi_1(\omega) \beta_0(i\omega) + \psi_2(\omega) a(i\omega). \end{aligned} \quad (23)$$

In addition,

$$\inf_{\gamma \in \Gamma} \sup_{G_v \in \mathcal{G}_1} J(\gamma, G_v) = \langle \tilde{r}^* \tilde{r} + r^* R r, \tilde{r}^* \tilde{r} + r^* R r \rangle, \quad (24)$$

where

$$\tilde{r}(\omega) = \left[ \left( U^{-1}(\omega) \right)^* r(\omega) \right]_+ \quad (25)$$

and  $U$  is the factor from (17).

## 5. Proof of Theorem. Setting

$$\tilde{\psi}(\omega) = U(\omega) \psi(\omega) + \left[ \left( U^{-1}(\omega) \psi(\omega) \right)^* \right]_-, \quad \tilde{\rho} = \rho - [\tilde{r}]^* \tilde{r}, \quad (26)$$

rewrite (15) as

$$\langle N, N \rangle = \langle (\tilde{\psi} + \tilde{r})^* (\tilde{\psi} + \tilde{r}) + \tilde{\rho}, (\tilde{\psi} + \tilde{r})^* (\tilde{\psi} + \tilde{r}) + \tilde{\rho} \rangle. \quad (27)$$

From (26) it follows that the matrix function  $\tilde{\psi}(\cdot)$  is analytic in the lower half-plane, so does  $\tilde{r}(\cdot)$  in the upper one, *i.e.*,  $[\tilde{\psi}(\cdot)]_+ = \tilde{\psi}(\cdot)$ ,  $[\tilde{r}(\cdot)]_- = \tilde{r}(\cdot)$ . This implies that the expansions

$$\tilde{\psi}(\omega) = \int_0^{\infty} \exp\{-i\omega t\} h_1(t) dt, \quad \tilde{r}(\omega) = \int_{-\infty}^0 \exp\{-i\omega t\} h_2(t) dt \quad (28)$$

are valid with some square-integrable functions  $h_1(\cdot), h_2(\cdot)$ . Let  $\tilde{\psi}_{opt}(\cdot)$  be a matrix function furnishing the minimum of (27). Then for any analytic (in the lower half-plane) matrix function  $\psi(\cdot)$  and sufficiently small  $\varepsilon > 0$ , formula (27) can be transformed to

$$\begin{aligned} \langle N, N \rangle &= \langle (\tilde{\psi}_0 + \varepsilon \tilde{\psi})^* (\tilde{\psi}_0 + \varepsilon \tilde{\psi}) + \tilde{\rho}, (\tilde{\psi}_0 + \varepsilon \tilde{\psi})^* (\tilde{\psi}_0 + \varepsilon \tilde{\psi}) + \tilde{\rho} \rangle = \\ &= \langle \tilde{\psi}_0^* \tilde{\psi}_0 + \tilde{\rho}, \tilde{\psi}_0^* \tilde{\psi}_0 + \tilde{\rho} \rangle + 4\varepsilon \operatorname{Re} \langle \tilde{\psi}_0^* (\tilde{\psi}_0^* \tilde{\psi}_0 + \tilde{\rho}), \tilde{\psi} \rangle + o(\varepsilon^2), \quad \tilde{\psi}_0 = \tilde{\psi}_{opt} + \tilde{r}. \end{aligned} \quad (29)$$

Due to arbitrariness of  $\varepsilon$ , the equality  $\langle \tilde{\psi}_0^* (\tilde{\psi}_0^* \tilde{\psi}_0 + \tilde{\rho}), \tilde{\psi} \rangle = 0$  must be fulfilled, and due to arbitrariness of  $\tilde{\psi}$  being analytic in the lower half-plane, from the last equality it follows that the rational function

$$\eta(\omega) = \tilde{\psi}_0^*(\omega) (\tilde{\psi}_0^*(\omega) \tilde{\psi}_0(\omega) + \tilde{\rho}), \quad \omega \in \mathbb{R}, \quad (30)$$

may not have singularities in the upper half-plane. According to the conditions of Lemma, the rational function  $\tilde{\psi}_0^*(\omega) \tilde{\psi}_0(\omega) + \rho(\omega)$  is proper and therefore from (30) it is evident that  $\tilde{\psi}_0(\omega) = \tilde{\psi}_{opt}(\omega) + \tilde{r}(\omega)$  has no singularities in the upper half-plane. The function  $\tilde{r}(\cdot)$  is analytic in the upper half-plane, and so does  $\tilde{\psi}_0(\cdot)$  in the lower one (see (28)). So the function  $\tilde{\psi}_0(\cdot)$  may be analytic in the upper half-plane provided that  $\tilde{\psi}_{opt}(\cdot)$  is the zero function. Considering (20), we get (21). For  $\tilde{\psi}_0(\omega) = \tilde{r}(\omega)$ , under the conditions of Theorem, the function (30) is analytic in the upper half-plane. For  $\tilde{\psi}_0(\omega) = \tilde{r}(\omega)$ , formula (24) results from (27). The solution of the linear-quadratic problem (3) is readily apparent from (21) when  $G_v(\omega) \equiv I$ , *i.e.*, when the control plant is acted on by white noise. The rest assertions of Theorem can be drawn from the link established between minimax and linear-quadratic problems [1].

**Conclusion.** Although the version proposed of the minimax problem is somewhat unusual because of mean square limitations imposed on a spectral density, its chief value is the clarity of the geometrical interpretation in terms of the appropriate Hilbert space.

## References

1. Cheremensky, A., V. Fomin (1996). Operator Approach to Linear Control Systems. Kluwer Academic Publishers: Dordrecht/Boston/London.

# STRUCTURE AND METHODS OF DYNAMICAL DECISION-MAKING AND STOCHASTIC OPTIMAL CONTROL

Valentin V. Baranov

Stability and Nonlinear Research Center of Russian Academy of Sciences  
5 D.Uljanov st., entr. 1, app. 312, Moscow, 117333, Russia

Valery I. Salyga

INTERNAUKA Technology and Business Development Centre  
51 Lusinovskaya, Moscow, 113833, Russia

**Abstract.** The problem of dynamical decision-making is considered. The system approach to the problem is developed. The notions of decision-making system and its structure are introduced in accordance with this approach. Three main parts can be singled out within the structure: base, envelope and deciser. The notion of base completeness is introduced. Sufficient conditions of completeness are obtained. Different variants of complete bases depending on uncertainty conditions are considered. The corresponding methods of optimal decision-making are obtained.

**Keywords.** Stochastic optimal control, dynamical decision-making system, structure, base, uncertainty, methods of optimal decision-making.

The problem of dynamical decision-making is considered, the sources of the problem are the successive analysis of A.Vald [1] and the model of markovian decision-making processes of R.Bellman [2]. As far as it is known, the latter is determined by a set of objects  $\{E, Y, Q(E | E \times Y), w(E \times Y)\}$ , where  $E$  — set of states;  $Y$  — set of controlling alternatives;  $Q(E | E \times Y)$  — transitive function from  $E \times Y$  to  $E$ , which determines probability of one step transitions on set of states under the influence of controls from  $Y$ ;  $w(E \times Y)$  — function of utility representing preferences on set  $Y$  in accordance with the condition:  $y' \succ y \Leftrightarrow w(x, y') > w(x, y)$ .

In conditions of these model random of states creates the problem of dynamic decision-making with the uncertainty of the type of risk outcome, which is well studied under different names: "dynamic programming", "controlled markovian processes", "stochastic optimal control" [3], etc.

Setting of tasks here is formalising by the following main constructions.

The measurable simple reflection  $\pi$ , which can be measured:  $E \rightarrow Y$  is called decisive function. The set of such decisive functions  $\pi = (\pi_1, \pi_2, \dots, \pi_n)$  is called strategy of decision-making at the finite horizon of length  $n < \infty$ . At infinite horizon strategy is determined by succession  $\{\pi_1, \pi_2, \dots\}$ . The strategy  $\pi^\infty = \{\pi, \pi, \dots\}$  is called stationary.

The quality of strategy is usually described by some additive criterion.

If horizon of decision-making is finite then with any quality of strategy criterion task solving does not make difficulties and is got by algorithm of inverse induction of dynamic programming type.

The difficulties appear during transition to infinite horizon. They are created by natural demand for existence of quality of strategy criterion at infinite horizon, which has a sense and receives

finite values. Two criteria satisfy this demand: of expected summary utility with discount and criterion of average utility. For stationary strategy they have appearance accordingly

$$\varphi(\pi^\infty)(x) = \lim_{n \rightarrow \infty} M_x^\pi \sum_{t=0}^{n-1} \alpha^t w(x_t, \pi(x_t)), x \in E,$$

where  $0 < \alpha < 1$  — coefficient of discount.

$$\varphi(\pi^\infty)(x) = \lim_{n \rightarrow \infty} \frac{1}{n} M_x^\pi \sum_{t=0}^{n-1} w(x_t, \pi(x_t)), x \in E,$$

where mathematical expectation is taking of product of measures created by transitive function  $Q(\pi)$ .

The task is to find optimal strategy  $\pi$ , maximising appropriate criterion, i.e.

$$\pi_*^\infty : (\pi_*^\infty)(x) \geq \varphi(\pi^\infty)(x) \quad \forall \pi^\infty, x \in E.$$

The methods of such tasks solving are known. For the task with discount the algorithm of "dynamic programming" [3] type and algorithm of strict improvements by Howard [4] are used. For the task with criterion of average utility only Howard algorithm works. It has an essential weakness of necessity deciding at each iteration of systems of algebraic equations of dimension equal to number of elements in set of states. That is why at certain increasing of dimension the task turns out practically impossible to be solved.

On the way of surmounting these difficulties we have developed optimising scheme of successive approximations, which creates packet of optimising methods of different computing efficiency. It is realising by the following main constructions.

Let set of states  $E$  be finite and  $R^E$  — vector space of dimension  $|E|$ .

Parametrical family of nonlinear operators  $L^{(m)}(\pi): R^E \rightarrow R^E$  is introduced, which in case of criterion of average utility acts according to the rule determined by superposition of operator such as

$$L^{(m)}(\pi) = U^*[F^*(\pi)]^m v, \quad m \in \{0, 1, \dots\},$$

where  $U^* = TU$ ;

$$T = v - v(z)I, \quad v(z) — z\text{-component of vector } v \in R^E, \quad 1 — \text{vector unity from } R^E;$$

$$F^*(\pi) = TF(\pi);$$

$$F(\pi)v = w(\pi) + Q(\pi)v;$$

$$Q(\pi)v = \sum_{s \in E} v(s) Q(s|x, \pi(x)), \quad x \in E;$$

If criterion with discount is considered then  $T$  is identical operator, and  $F(\pi)v = w(\pi) + \alpha Q(\pi)v$ , where  $\alpha < 1$ .

Operator  $L^{(m)}(\pi)$  creates optimising scheme of successive approximations in which speed of convergence increases according to the increasing of the value of parameter  $m \in \{0, 1, \dots\}$  and value  $0 < m^* < \infty$  exists, when speed of convergence is maximal. From this scheme a family of algorithms is following, in which speed of convergence and volume of computing operations depend on the value of parameter  $m$ . In particular cases if  $m = 0$  the algorithm of the dynamic programming type takes place, and if  $m = \infty$  — Howard algorithm. In total it allows to construct a packet of optimising algorithms of different computing efficiency and to set a task of building of the most efficient algorithm. The variant of such algorithm are built in the works [5, 6, 7, 8, 9].

Unfortunately the Bellman model of the markovian decision processes is just little adapted to solve practical tasks. Indeed in practice the necessity to choose strategic, tactical and control alternatives depending in total under different variants of uncertainty of decision-making conditions and their outcomes often appears. Their examples are the tasks of economical development, the tasks of providing of ecological security in conditions of technogenic influences and many others.

Such tasks are different and difficult, so they require unique methodology, which allows to study the problem not in particular suppositions of concrete tasks, but to build a general theory of decision-making for a broad class of conditions marked by appropriate postulates.

Minimal set of postulates necessary for building sufficiently general theory of decision-making is determined by the following suppositions.

P 1. The collective  $K = \{l, \dots, k\}$  of the interested sides and the object of their interests exist.

P 2. The interests of the sides do not coincide and are not similar in the meaning that they have hierarchy structure with at least three levels of hierarchy: strategic, tactical and direct. Achieving of the strategic interests requires some time and is realising through achieving of tactical and direct interests.

P 3. The set  $\theta$  of strategic alternatives, which determine possible ways of achieving of the strategic interests exists and is set.

P 4. The set  $G$  of tactical alternatives, which determine possible ways of achieving of the tactical interests exists. It can be a priory set and fixed or it can successively forming according to some rule.

P 5. The set  $Y$  of control alternatives, which determine possible ways of achieving of the direct interests exists. It can be a priory set and fixed or it can successively forming according to some rule.

P 6. The set of situations  $X$  exists, which elements describe operative conditions of decision-making, which the permissibility of control alternatives and their preferability depend on.

P 7. The set  $Z$  of outcomes of alternatives choice exist and is set.

P 8. Each interested side has its individual system of preferences on the set of controlling alternatives, which in general case depends on situations, outcomes, strategic and tactical alternatives and also on controlling alternatives of other interested sides as on parameters.

P 9. Choice of alternatives is directed in the meaning that each interested side seeks to choose alternative the most preferable for it.

Conception of decision-making system, which is setting by the following definition, is initial in developing of required theory.

*Decision-making system is setting by the three objects  $\{K, A, S\}$ , in which  $K$  is a collective of interested sides;  $A$  is a set of purpose alternatives;  $S$  is a structure determined by a set of formal objects, which the problem of decision-making is structured and solved by.*

In dependence of the role of objects in the structure they are divided into three main groups: base SB, envelope SO and deciser SR.

Base SB consists of formal objects, which are a priory information medium that answer the conditions of decision-making, which are set.

Envelope SO consists of those formal objects, with the help of which formal set of task is getting.

Deciser SR consists of those formal objects and constructions, with the help of which final solution of problem is getting.

The base plays determinative role in structure. That is why in its setting it is principally important to come from demand of its completeness in accordance with the following definition.

*If in conditions of set base the problem permits nontrivial setting of task, which solution exists, then base is complete.*

Setting of complete base answering to formulated postulates requires their concrete definition.

In particular, postulate P6 supposes the existence of set of situations  $X$ , which describe

operative decision-making conditions, but way of their setting is not indicated. It is determined by the following suppositions.

X 1. Situations are setting by pithy or logical statements, which determine the conditions of control alternatives choice.

X 2. Set  $X$  of situations is finite.

X 3. Situations are random.

X 4. Situations are not obliged to be available for direct observation and detection.

Same way the mode of outcomes setting foreseen by postulate P7 requires more precise definition. It is determined by the following suppositions:

Z 1. Outcomes of choice of control alternatives are random.

Z 2. Probability of outcomes depend not only on choice of control alternative  $y \in Y$  but also on situations  $x \in X$ , strategic and tactical alternatives as on parameters.

Z 3. Outcomes are available for direct observation and simply detected.

If the collective  $K$  consists of a single interested side, then in conditions of formulated suppositions complete base is set by a set of objects

$$SBDR = \{\Theta, G, X, Y, [Y_x \subset Y, x \in X], Z, P_{(\theta, g)}(Z | X \times Y),$$

$$Q_{(\theta, g)}(X | X \times Z \times Y), w_{(\theta, g)}(Y \times Z \times X), (\theta, g) \in \Theta \times G\},$$

where  $\Theta$  — set of strategic alternatives;  $G$  — set of tactical alternatives;  $X$  — set of situations;  $Y$  — set of control alternatives;  $Y_x \subset Y$  — limitations on permissibility of control alternatives in dependence of situations  $x \in X$ ;  $Z$  — set of outcomes;  $P_{(\theta, g)}(Z | X \times Y)$  — distribution of outcomes at  $Z$  under the condition  $X$  called distribution of connection;  $Q_{(\theta, g)}(X | X \times Z \times Y)$  — transitive function from  $X \times Z \times Y$  in  $X$ , which determines probabilities of transitions to  $X \times X$  in conditions  $Z \times Y \times \Theta \times G$ ;  $w_{(\theta, g)}(Y \times Z \times X)$  — function of utility, which represents preferences at  $Y$  under conditions  $Z \times X \times \Theta \times G$ .

Such base is called regular.

If the collective  $K$  consists of more than one side, then the base is determined by a set of individual bases of each side, in which is necessary to foresee their mutual dependence on individual decisions of the sides in total and on some common system decisions.

Proof of completeness of regular base is not obvious and requires analysis of possibilities of formal task setting, its solvability and building of deciser.

In process of solving the appeared methodological and mathematical questions a richness of possibilities of tasks formalization and possibilities of constructive methods building and solving appears. In particular, it is appeared that the other variants of bases with limitation conditions of decision-making follow from regular base as a particular cases.

For example, if it is supposed that sets of situations and outcomes coincide, i.e.  $X \equiv Z$ , then situation is naturally to understand as a state, which is at the same time an outcome. Then regular base comes to the base

$$SBDRM = \{\Theta, G, X, Y, [Y_x \subset Y, x \in X], Q_{(\theta, g)}(X | X \times Y), w_{(\theta, g)}(Y \times X), (\theta, g) \in \Theta \times G\},$$

where  $X$  — sets of states.

Such base is called regular markovian. It was first introduced and studied in the work [8].

In its turn, if we suppose in this base the existence of strategic and tactical imperatives, then it comes to the base

$$SBDM = \{X, Y, [Y_x \subset Y, x \in X], Q(X | X \times Y), w(X \times Y)\},$$

which is a famous Bellman model of markovian processes of decision-making [2]. That is why it is called primary markovian base.

Markovian and primary bases play an important role in proof of completeness of any base owing to the following result.

*Theorem. Let the base by some transformations comes to markovian or primary markovian base or it induces some convergence succession of primary markovian bases. Then initial base is complete.*

The formulated theorem not only determines sufficient conditions of base completeness but also indicate the ways of complete base forming if the initial base is not complete.

Indeed markovian bases determine the problem of decision-making under conditions of the type of risk outcome created by random of states. This is the most weak variant of uncertainty, in which base completeness is yet remains. But in practice state often is not available for direct observation and instead of state it is possible to observe only outcomes. If under these conditions the base is filling up by a priori distribution  $\alpha(X)$  at set of states  $X$ , then we have the base  $SBDRX = \{SBDR, \alpha(X)\}$ , which is already complete. Solving of the problem its conditions is achieved by matching of successive building of induced bases of bayes risk with the help of the process of structural filtration with building of optimal strategy of decision-making, which here is not markovian owing to dependence of the process of structural filtration on the previous decisions. Here come the possibilities of different setting of decision-making tasks under the uncertainty regarding situations and building of appropriate methods of their solving. In particular, we have developed method of broadening horizons, method of turned induction and also methods of decision-making on the principle of diagnostics and on the principle of recognition. One of such tasks is considered in [10].

More deep uncertainty appears if some structural objects of initial base a priori are not set. For example, in the structure of regular base the difficulties in setting of the distribution of connection  $P(Z | X \times Y)$  or the transitive function  $Q(X | X \times Y)$ , often appear and they turn out to be not set. In these conditions filling of base is possible by setting of set of hypotheses regarding unknown object. Here it is naturally to use hypotheses as component of control alternative or as independent strategic alternative with appropriate extension of set of alternatives.

Let, for example, distribution of connection is not set. If here may be set the set of hypotheses  $\Gamma(Z | X \times Y)$  regarding such distribution, then as alternative decision it is natural to consider pairs  $(y, \gamma): y \in Y_x, \gamma \in \Gamma(Z | x, y)$ .

By such replacement set of alternatives is broadening. Then the task gets game content Its solution is getting by matching non-markovian methods of structural filtration, identification and successive building of optimal strategy.

Analogously if the transitive function  $Q(X | X \times Y)$  is not set, then by setting the set of hypotheses  $\Omega(X | X \times Y)$  and appropriate broadening of set of alternatives the required completeness of base is getting. Task solving here requires non-markovian methods of identification and adaptive decision-making.

Some results of studying of the problems of decision-making under different variants of uncertainty are considered in the works [10 - 14].

**Conclusions.** Received results determine system methodology of decision-making, which principal propositions are developed in [15, 16]. It allows to study the problems of decision-making not in particular suppositions of concrete tasks but as a general problem for a sufficiently wide class of controlled systems marked by appropriate postulates and bases. It opens the possibility to consider and solve the problems which could not be solved earlier. In particular on the base of received results the problem of control of capacity for work and security of growing old was considered and solved [17]. At last received methodology and constructive methods are the theoretical basements for developing models and methods of

directed stable development of transforming system. Their general conception and some applications are considered in [18, 19].

### References

1. Wald A. (1947). Sequential Analysis. Wiley, N.-Y.
2. Bellman R. (1957). Markovian Decision Process. J. Math. Mech., 6, 679 — 684.
3. Bertsekas D., Shrive S. (1978). Stochastic Optimal Control. The Discrete Time Case. Academic Press, N.-Y.
4. Howard R. (1960). Dynamic Programming and Markov Processes. MIT Press.
5. Баранов В.В. (1991). Вычислительные методы оптимального стохастического управления. Принцип оптимальности и оптимизационная схема последовательных приближений. Журн. вычисл. матем. и мат. физики., том 31, № 5, с. 663 - 680.
6. Баранов В.В. (1992). Вычислительные методы последовательных исключений и оптимизации в модели стохастического оптимального управления. Журн. вычислит. матем. и мат. физики., том 32, № 2, с. 247 - 260.
7. Баранов В.В. (1992). Модель и методы равномерно оптимального стохастического управления. Автоматика и телемеханика, № 5, с. 42 - 52.
8. Баранов В.В. (1991). Параметрическая задача стохастического оптимального управления. Журн. вычислит. матем. и мат. физики, том 31, № 12.
9. Баранов В.В., Салыга В.И. (1997). Методы оптимальных динамических решений по критерию с переоценкой. Автоматика и телемеханика, № 8.
10. Баранов В.В. (1994). Оптимальные методы диагностики управляемых стохастических систем. Изв. РАН. Техническая кибернетика, № 1, с. 120 - 131.
11. Баранов В.В. (1992). Последовательные методы идентификации и адаптивного управления в стохастических системах. Кибернетика и системный анализ, № 6, с. 100 - 120.
12. Баранов В.В. (1993). Игровые методы диагностики управляемых стохастических систем. Автоматика и телемеханика, № 10, с. 108 - 121.
13. Баранов В.В. (1994). Оптимальное динамическое распознавание и принятие решений. Журн. вычислит. матем. и мат. физики, том 34, № 7, с. 987 - 1003.
14. Баранов В.В. (1994). Динамическая диагностика и принятие решений в условиях неопределенности. Кибернетика и системный анализ, № 3, с. 87 - 104.
15. Баранов В.В., Матросов В.М. (1997). Структуры систем динамического принятия решений. Часть 1. Изв. РАН. Теория и системы управления, № 1, с. 1 - 11.
16. Баранов В.В., Матросов В.М. (1997). Структуры систем динамического принятия решений. Часть 2. Изв. РАН. Теория и системы управления, № 2, с. 5-15.
17. Баранов В.В. (1995). Модели и методы управления работоспособностью и безопасностью транспортных систем. Транспорт. Наука, техника, управление, ВИНТИ, РАН, № 1, с. 18 - 31.
18. Баранов В.В. (1997). Структуры, модели и методы направляемого развития транспортных систем. Часть 1. Структуризация. Транспорт. Наука, техника, управление, ВИНТИ, РАН, № 3.
19. Баранов В.В. (1997). Структуры, модели и методы направляемого развития транспортных систем. Часть 2. Модель экономического развития. Транспорт. Наука, техника, управление, ВИНТИ, РАН, № 5.

# ROBUSTNESS MEASURES IN DISCRETE-TIME SYSTEMS WITH MULTIPLE: PARAMETERS; NONLINEARITIES, AND DELAYS

Mansour Eslami

Dept. of Electrical Engineering and Computer Science (M/C 154)  
The University of Illinois at Chicago  
Chicago, Illinois 60607-7053

## Abstract

Certain aspects of *constructing the largest class* of asymptotically convergent (stable) systems which surrounds a convergent system with nonlinearity and delays are presented. Here we consider both the *magnitude* and *computational efficiency* of generating these results which are based on the corresponding results from linear systems. In particular, we present three numerical examples to show the applications of these results, namely, in: (i) linear systems with linear multiple parameter variations; (ii) nonlinear systems with linear multiple parameter variations and multiple nonlinearities; and (iii) linear systems with multiple nonlinear parameter variations. These building-blocks examples can be assembled to solve a complicated nonlinear system with multiple nonlinear parameter variations and multiple delays.

**Keywords** – Robust and absolute stability, Parameter variations, Discrete-time, Nonlinear, Delay.

## 1. INTRODUCTION

In contemporary literature, robustness measures refer to system maneuverability regarding its stability (or in the case of a discrete-time system its *convergent* [40]) behavior. However, to describe this property precisely, we need to determine the set, preferably *the largest* of its kind, for parameter variations such that for each point in that set the underlying perturbed system remains convergent. Thus, dependently, we study the problem, bearing in mind that only those algorithms which can exhibit computational advantages merit further attention. One application area in this category of problems that has received considerable attentions is determination of the absolute stability of a class of nonlinear systems. Historical development of this topic, also known as a sector problem, is described in [2], [15], [36]. Both continuous-time and discrete-time cases are analyzed in the Russian literature extensively, however, due to the scope of this article, we consider *mainly* the pertinent literature on nonlinear *discrete-time* (sampled-data) systems.

Using a set of hybrid notation among our references, consider a discrete-time system as follows [37].

$$x(l+1, \theta) = A_0 x(l, \theta) + \sum_{j=1}^{r_1} u_j(l) A_j x(l, \theta), \quad l = 0, 1, \dots, \quad (1)$$

where in this article  $x(l) \in \mathbb{R}^n(l)$  is system state at time instant  $l$ ,  $\theta \in \mathbb{R}^r$  is a generic name for system parameters,  $A_0$  and  $A_j$ 's are  $n \times n$  constant (parameter dependent) matrices, and  $u_j(l)$ 's are *bounded* functions (in the quadratic sense). Our problem is to find conditions under which  $x(l) \equiv 0$  becomes asymptotically convergent in the large and uniformly with respect to  $u_j$ 's. We may assume that in the *same* quadratic sense these functions belong to a closed convex polyhedron or a hyperbox.

Consider the asymptotic stability of the following nonlinear discrete-time system,

$$x(l+1) = A_0 x(l) + \sum_{j=1}^{r_2} b_j \phi_j(\sigma_j(l), l), \quad \sigma_j(l) = c_j^T x(l), \quad \phi_j(0, l) \equiv 0, \quad \text{for all } j, \quad (2a)$$

$$0 \leq \phi_j(\sigma_j, l) \sigma_j \leq k_j \sigma_j^2, \quad 0 \leq k_j < \infty, \quad \text{for all } j \text{ and all } \sigma_j, \quad (2b)$$

where  $\phi_j$ 's are nonlinear functions of  $\sigma_j$ 's, and  $b_j$ 's and  $c_j$ 's are constant vectors. The absolute stability of system (2) is interpreted as uniform asymptotic stability in the large of  $x(l) \equiv 0$  for this system. There is a wealth of literature to discuss various aspects of transforming stability analysis of a nonlinear system of type (2) to that of a system of type (1) with bounded  $u_j$ 's. In this regard, system (2) is a special case of (1) with  $A_j = b_j c_j^T$ .

Now, consider asymptotic stability of the following system with multiple delays.

$$x(l+1) = A_0 x(l) + \sum_{j=1}^m A_j x(l-j). \quad (3)$$

This class of systems is not a subset of (1) and in most applications the original system model contains both nonlinearities of types (1) or (2) and delays of the type (3).

Discussions regarding the stability equivalence among systems (1), (2), (3), and perhaps any combination of these systems are outside the scope of this article (cf., [11] and its references, also [29] to [34]). Instead we concentrate on the constructional issues related to the regions of uniform stability in parameter space. In other words, when this stability determination is possible (i.e., when we may consider for the purpose of stability analysis *alone* the similar property of its corresponding linear system), we use the recent results from linear systems to construct the corresponding region for parameter variations. Specifically, we look into cases where all coefficient matrices are changing as well. This consideration advances results reported in the literature. The method that is proposed in the literature uses mainly a Lyapunov function  $v(x) = x^T P x$ , and sets up a "min max" optimization in terms of the difference between perturbed and unperturbed Lyapunov function in order to choose the "best" Lyapunov function resulting in a polyhedron (hyperbox) for parameter variations, however, this solution remains local. The underlying theme of different approaches in this area is to expand this local solution. Similar constructional methods are also reported in [7]. Our method relies on recent results from linear systems which enable us to construct these hyperboxes in a more straightforward manner than that suggested in the literature.

In Section 2 we review the pertinent results from linear systems. In Section 3 we give our proposed procedure for constructing the above hyperboxes for a nonlinear system with multiple nonlinearities. We present the similar analysis for a delay system in Section 4, followed by numerical examples,<sup>1</sup> in particular an example from linear systems with nonlinear parameter variations in Section 5. Conclusions are deferred to Section 6.

---

<sup>1</sup> From "Numerical examples for the convergent robustness measures in nonlinear discrete-time systems," in *Proc. IEEE Conf. on Decision and Contr.*, pp. 3840-3841, 1994, by this author.

## 2. REVIEW OF ANALYSIS IN LINEAR SYSTEMS

Clearly, when linearization is valid, we can relate the convergence robustness analysis of (1) or (2) to that of a corresponding set of homogeneous linear difference equations  $x(l+1, \theta) = S(\theta)x(l, \theta)$ , where  $x(l) \in \mathbb{R}^n(l)$  is system state,  $\theta \in \mathbb{R}^r$  is a generic name for system parameters and  $S(\theta) \in \mathbb{R}^{n \times n}(\theta)$  is system coefficient matrix. For more information on this type of equations refer to [26]. Here, we review the convergent behavior of  $S(\theta)$  and its corresponding computational aspects, when  $\theta$  changes. Clearly, changes in  $\theta$  (or  $S(\theta)$ ) may have been the results of possible inaccuracies in system parameters or in system model resulted from approximating or linearizing its nonlinear model by  $\Delta S(\Delta\theta)x(l)$  ( $\Delta\theta$  is the variation of  $\theta$  with respect to its nominal condition  $\theta^o$ ). In all these cases the main problem is: how to determine the convergent behavior of  $S + \Delta S$  as  $\theta \rightarrow \theta + \Delta\theta$ , and when  $\Delta S$  has *multiple-varying parameters*. Here, we are simultaneously monitoring points such as  $\Delta\theta \triangleq [\Delta\theta_1, \dots, \Delta\theta_r]^T$  in the "parameter variations space" (PVS), whose origin is at  $\theta^o$ , and the corresponding convergent matrices  $S(\cdot)$  computed at  $(\theta^o + \Delta\theta)$ .

The general stability or convergent robustness analysis of a matrix with *multiple-varying parameters* is subject of vast research in the engineering literature [14], [18]. The method that is reviewed here, [12] to [14], has two distinct differences with most results published in this area. Namely, we define *the largest* or *most extended* set of points in the PVS, whose corresponding perturbed matrices remain convergent, to be the global answer to our convergent robustness analysis, and this set is called the *E-Box*, then we approximate this box with a set of hyperboxes that are stacked appropriately *under* the *E-Box* and call this approximated set the "convergent robustness measure" (CRM). We put our emphases on the *direct* construction of this CRM (or the *E-Box*) for a convergent matrix that we have no prior information about its *region* of convergent. In this regard, we extend or construct the region of convergent by an iterative procedure that *probes* this region and yields *the largest* set for robust convergent matrices. This constructional procedure is the prerequisite for the convergent robustness analysis of the classes of difference equations discussed herein and many other perturbational problems. The following theorems form the core of our approach.

**Theorem 1 [19]:** All eigenvalues of matrix  $S \in \mathbb{R}^{n \times n}$  lie in the unit circle (convergent matrix [40]) if and only if for an arbitrary  $0 < Q = Q^T \in \mathbb{R}^{n \times n}$ , the matrix equation  $S^T P S + Q = P$ , yields a unique solution  $0 < P = P^T \in \mathbb{R}^{n \times n}$ .

If  $S \rightarrow S + \Delta S$ , then in order to compute  $\Delta S$  such that  $S + \Delta S$  remains a convergent matrix we use the following sufficient condition. If this condition is used repeatedly, then we can construct the corresponding CRM (also Theorem 3).

**Theorem 2 [9], [10]:** For a parameter perturbation  $\Delta\theta$ ,  $S \rightarrow S + \Delta S$ , then  $S + \Delta S$  remains a convergent matrix if

$$\Delta S^T (P + 2P S Q^{-1} S^T P) \Delta S < \frac{1}{2}Q. \quad (4)$$

Here  $0 < Q = Q^T \in \mathbb{R}^{n \times n}$ ,  $0 < P = P^T \in \mathbb{R}^{n \times n}$  satisfies  $S^T P S + Q = P$ . Also by  $M < N$  for two matrices  $0 \leq M = M^T \in \mathbb{R}^{n \times n}$  and  $0 < N = N^T \in \mathbb{R}^{n \times n}$  we mean  $N - M > 0$ .

Inequality (4) which is *parameterized* in terms of  $Q$  serves as a "set generator" [14], in order to construct a class of admissible  $\Delta S$ 's. Despite various attempts, no "optimal"  $Q$  is known which yields a global  $\Delta S$ . Clearly this "local-upper bound" is only a sufficient condition and is a function of  $Q = Q^T > 0$ . It seems one way to rectify this parameter dependency (relative to the global solution) when utilizing (4) is to use an iterative method as described for single-parameter variations in [9] to [11] and for multiple-parameter variations in [12] to [14] (Theorem 3, below). Indeed, as a consequence of this approach we let  $Q = I$  (an identity matrix) which simplifies the remaining computations. However, efforts to parameterize (4) in different forms are also reported in the literature. For instance, recently a new parameterization for this upper bound is presented in [35], and if we use that procedure in Theorem 2 by letting  $Q = I$  and assuming that  $P + 2PSS^TP \leq \frac{1}{2}(1 + \alpha^{-1})P$ , for some  $0 < \alpha < \infty$ , then  $\Delta S^T(P + 2PSQ^{-1}S^TP)\Delta S < \frac{1}{2}Q$  becomes  $(1 + \alpha^{-1})\Delta S^TP\Delta S < I$ . Here this  $\alpha$  is chosen such that  $P = P^T > 0$  becomes the unique solution of a scaled-Lyapunov equation  $(1 + \alpha)S^TPS - P = -I$ .

**Lemma 1 [14]:** Consider a perturbation policy of the type  $\Delta S = \epsilon \Delta S'$  for a convergent matrix  $S$ , where  $\Delta S'$  is a numerically known matrix and  $\epsilon$  changes according to  $\epsilon^2 < 1/[(1 + \alpha^{-1})\|\Delta S'^TP\Delta S'\|]$ ,  $P = P^T > 0$  is the solution of  $(1 + \alpha)S^TPS + I = P$  with  $0 < \alpha < \infty$ . Then the least-upper bound (l.u.b.) of  $\epsilon^2$  is maximized at  $\alpha = 1/\|S\| - 1$ .

We can show that the above  $\alpha$  does not provide the *global*  $\Delta S$  either, and it seems that the only viable method to generate the global (non-convex)  $\Delta S$  is by an iterative one. We also note that to "solve" for the global  $\Delta S$ , we need to know the structure of  $\Delta S$ . In [10], a number of possible candidates for  $\Delta S$ , and an algorithm to "solve" for certain  $\Delta S$ 's, is proposed. We can generally divide this structure into two groups: (i) single-parameter variations; and (ii) multiple-parameter variations. In (i) we let  $\Delta S = \epsilon \Delta S'$ , where  $\Delta S'$  is numerically known and is a *candidate* for  $\Delta S$ , then we search for the *best*  $\epsilon$ , in order that  $\epsilon \Delta S'$  becomes an *admissible*  $\Delta S$ . Because in effect we have only one parameter  $\epsilon$  here, therefore this is also called the *largest directional*  $\Delta S$ . The situations corresponding to (ii), however, require additional attentions, and in this case we break the structure of  $\Delta S$  into the following two categories [10]. (Note that both cases refer to linear systems.)

*F: Multiple parameters, linear perturbation affecting two or more entries.*

*G: Multiple parameters, multiple nonlinear perturbations affecting many entries.*

In each of the above two cases for  $\Delta S$ , the unknowns are a set of scalar parameter variations  $\Delta \theta_j$ 's. Substituting any one of these candidates for  $\Delta S$  (or any other comparable  $\Delta S$ ) in (4) yields an involved algebraic problem for determination of  $\Delta \theta_j$ 's, a situation that must be avoided. Indeed as the consequence of the next theorem, which in effect advances a *single by single* (or *directional*) search for a convergent perturbed matrix (or a *point by point* search in the PVS) to that of a *set by set* search for such matrices (or a *set by set* search for points in the PVS), we avoid such an involved algebraic problem.

**Theorem 3 (E-Box) [12] to [14]:** Consider (4) with  $Q = I$  repeated below.

$$\Delta S^T(\Delta \theta)(P + 2PSS^TP)\Delta S(\Delta \theta) \triangleq \Delta S^T(\Delta \theta)X\Delta S(\Delta \theta) < \frac{1}{2}I. \quad (5)$$

Suppose from this inequality we have extracted, in principle,  $2^r$ -directional "solutions":  $\Delta S_1, \Delta S_2, \dots, \Delta S_{2^r}$ ; each corresponding to one point  $[\Delta \theta_1, \dots, \Delta \theta_r]$  in the PVS, which form

a hyperbox in that space. If for each and every point in this hyperbox we have

$$\Delta S = \sum_{i=1}^{2r} w_i \Delta S_i, \text{ with } \sum_{i=1}^{2r} w_i = 1, \quad 0 \leq w_i < 1, \text{ for all } i. \quad (6)$$

Then every point of this hyperbox corresponds to an asymptotically convergent  $S + \Delta S$ .

*Remark 1:* Clearly (6) is true for all linear perturbation matrices including polynomials. The situations for the nonlinear perturbation matrices require additional attentions. In this case, using the Taylor series expansion, we can approximate a nonlinear perturbation matrix with a set of perturbation matrices each is computed for a *small*- parameter variation, in order to comply with the convexity requirement of (6). The implication is that we must increase our computations as is the case with any complicated nonlinear problem. Example 3 shows one such application.

To elucidate this technique consider Example 1 that is revisited in Example 2.

*Example 1 (Linear systems with linear multiple parameter variations):* Consider an asymptotically convergent matrix  $S$  which is perturbed according to  $\theta_1 \geq 0$  and  $\theta_2 \geq 0$  with zero initial values as follows.

$$S (\equiv S_1) \triangleq \begin{bmatrix} 0.06\theta_2 & 0.6 - 0.21\theta_2 \\ -1.5 + 0.168\theta_1 - 0.054\theta_2 & 0.2 + 0.224\theta_1 + 0.189\theta_2 \end{bmatrix} \equiv \begin{bmatrix} 0.0 & 0.6 \\ -1.5 & 0.2 \end{bmatrix}. \quad (7)$$

Our objective is to study the ways of constructing *the largest* set of  $r (= 2)$  independently varying parameters  $\Delta\theta_1$  and  $\Delta\theta_2$  in  $\Delta S$  such that  $S + \Delta S$  remains convergent, where

$$\Delta S \triangleq \begin{bmatrix} 0.06\Delta\theta_2 & -0.21\Delta\theta_2 \\ 0.168\Delta\theta_1 - 0.054\Delta\theta_2 & 0.224\Delta\theta_1 + 0.189\Delta\theta_2 \end{bmatrix}. \quad (8)$$

*Solution:* Let  $J = 1, 2, \dots, N$  be the iteration number. The perturbation matrices must satisfy (5). Now, we take the following steps to complete the solution to this example.

*STEP 0 (To update S):* We let  $S_J = S_{J-1} \pm \Delta S_{J-1}$ , with  $S_1 \equiv S_0 \equiv S$  being the initial asymptotically convergent matrix and  $\Delta S_0 \equiv 0$ . (The  $(\pm)$  means in both directions.)

*STEP 1 (To solve the Lyapunov equation):* For the current  $S_J$  and  $Q = I$  we solve for  $P_J$  according to  $S_J^T P_J S_J + I = P_J$ . At  $J = 1$  we have  $P_1 = \begin{bmatrix} 17.286 & -1.143 \\ -1.143 & 7.238 \end{bmatrix}$ .

*STEP 2 (Directional perturbations):* We perturb the system iteratively along all the  $r$  independent axes and in a few "45° lines". We compute the corresponding largest directional perturbations along each axis and according to the procedure in [10], Fig. 1. For  $\Delta S_{\Delta\theta_1} = \Delta\theta_1 \begin{bmatrix} 0.000 & 0.000 \\ 0.168 & 0.224 \end{bmatrix}$ ,  $S + \Delta S_{\Delta\theta_1}$  is convergent if  $0 < \Delta\theta_1 < 5.234$  (point  $B_n$ ). For  $\Delta S_{\Delta\theta_2} = \Delta\theta_2 \begin{bmatrix} 0.06 & -0.21 \\ -0.054 & 0.189 \end{bmatrix}$ ,  $S + \Delta S_{\Delta\theta_2}$  is convergent if  $0 < \Delta\theta_2 < 3.272$  (point  $A_n$ ). For  $\Delta S_{45^\circ} = \epsilon_{45^\circ} \begin{bmatrix} 0.06 & -0.21 \\ 0.114 & 0.413 \end{bmatrix}$ ,  $S + \Delta S_{45^\circ}$  is convergent if  $0 < \epsilon_{45^\circ} < 2.325$  (point  $C_n^{++}$ ).

*STEP 3 (To interpret the sufficient conditions):* We need to study  $\Delta S^T X \Delta S < \frac{1}{2}I$ , where  $X \triangleq P + 2PSS^T P$ , in order to establish a set of inequalities whose intersection yields the set of sufficient conditions for all parameter variations. In most practical cases,

however, we use  $\text{Tr}(\Delta S^T X \Delta S) < 1/2$  which is computed symbolically once and is amenable for computer computations. Here (5) becomes

$$\text{Tr}(\Delta S^T X \Delta S) \triangleq \text{Tr} \begin{bmatrix} \Delta s_{11} & \Delta s_{21} \\ \Delta s_{12} & \Delta s_{22} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} \\ x_{12} & x_{22} \end{bmatrix} \begin{bmatrix} \Delta s_{11} & \Delta s_{12} \\ \Delta s_{21} & \Delta s_{22} \end{bmatrix} < 1/2. \quad (9)$$

The sufficient condition for the asymptotic convergent of (9) is

$$(\Delta s_{11}^2 + \Delta s_{12}^2)x_{11} + 2(\Delta s_{11}\Delta s_{21} + \Delta s_{12}\Delta s_{22})x_{12} + (\Delta s_{21}^2 + \Delta s_{22}^2)x_{22} < 1/2. \quad (10)$$

*STEP 4 (To compute  $X \triangleq P + 2 P S S^T P$ ):* With the current  $S_J$  and  $P_J$  we compute  $X$ . At  $J = 1$ , we have

$$X \triangleq \begin{bmatrix} x_{11} & x_{12} \\ x_{12} & x_{22} \end{bmatrix} = \begin{bmatrix} 228.93 & -22.92 \\ -22.92 & 244.15 \end{bmatrix}. \quad (11)$$

*STEP 5 (To search for corner points):* The set of admissible parameter variations must satisfy the sufficient condition (10). At  $J = 1$ , we have

$$19.14\Delta\theta_1^2 + 17.94\Delta\theta_1\Delta\theta_2 + 22.32\Delta\theta_2^2 < 1/2. \quad (12)$$

From (12) we must extract  $2^r = 2^2 = 4$  sets of corner points for each corresponding hyperbox. Clearly the following points which we directly compute from (12) are possible candidates for the corner points of the desired set:

$$\begin{aligned} C_1^{++} &\triangleq (\Delta\theta_1 = \Delta\theta_2 = 0.092), \quad A_1 \triangleq (\Delta\theta_1 = 0.0, \Delta\theta_2 = 0.092), \\ O_1 &\triangleq (\Delta\theta_1 = \Delta\theta_2 = 0.0), \quad B_1 \triangleq (\Delta\theta_1 = 0.092, \Delta\theta_2 = 0.0). \end{aligned} \quad (13)$$

Substituting (13) in (8) yields the corresponding  $\Delta S$ , which are "solutions" to (4) in this example. Note that we remain in the first quadrant of  $PVS$  by our previous assumption.

*STEP 6 (To construct the admissible set):* Connecting the above  $2^r = 2^2 = 4$  corner points in (13) yields a set (or a hyperbox) of admissible points in the  $PVS$ . Every point of this set yields a  $\Delta S$  such that  $S + \Delta S$  remains convergent as a result of Theorem 3, Fig. 1.

*Comment 1:* From Step 2 we know that at least one other point which we call  $C_n^{++} \triangleq (\Delta\theta_1 = \Delta\theta_2 = \epsilon_{45^\circ} = 2.325)$  results in a convergent perturbed matrix. Therefore we now search for iterative ways of extending the initial set of admissible points in the  $PVS$ , in order to reach point  $C_n^{++}$ .

*Iteration:  $J+1$*

*STEP 7 (Construction of the largest set):* We now propose to translate the coordinates of the  $PVS$  to the farthest corner of the current convex hull, in order to construct additional regions of admissible points in that space. That means we start with  $S_{J+1} = S_J \pm \Delta S_J$  (Step 0), where  $\Delta S_J = \Delta S(\Delta\theta_1, \Delta\theta_2, \dots, \text{at } O_{J+1})$  and invoke Steps 1 to 6 to  $S_{J+1}$ . In this manner we continue our construction until we find the largest set for our problem. If we continue to apply this *probing* method to Example 1, we can construct the largest set of points in the  $PVS$  whose corresponding perturbed matrices are

convergent. This is done by enlarging the set  $O_1 B_1 C_1^{++} A_1$ , as has been carried out for similar examples in [12] to [14]. The final approximate result is set  $O_1 B_n C_n^{++} A_n$  shown in Fig. 1.

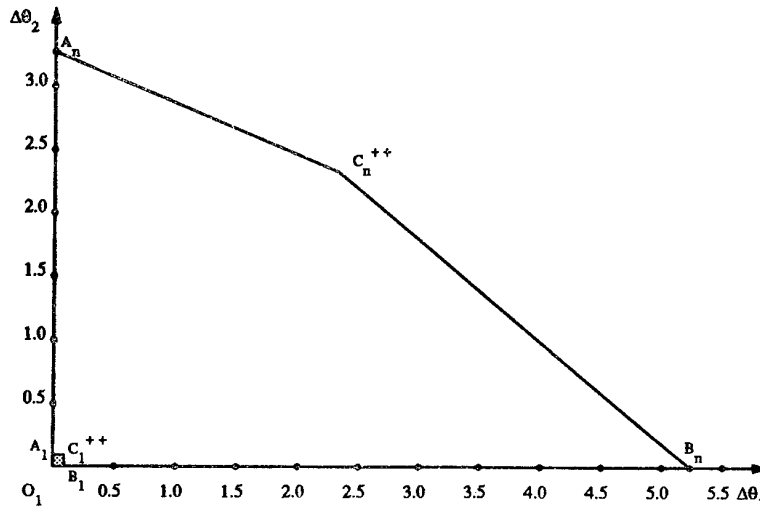


Fig. 1. Direct construction of an admissible set of parameter variations.

### 3. ANALYSIS FOR DISCRETE - TIME NONLINEAR SYSTEMS

Consider the following set of nonlinear difference equations.

$$x(l+1) = Ax(l) + b\phi(\sigma), \quad \sigma(l) = c^T x(l), \quad (14)$$

where  $x(l) \in R^n(l)$  is state vector;  $A$  is an  $n \times n$  convergent matrix (this assumption only simplifies our presentation);  $\phi(\sigma)$  is a scalar nonlinear function of  $\sigma$  representing system nonlinearity or modeling inaccuracy; and  $b$  and  $c$  are constant vectors. Replacing  $\phi(\sigma)$  by a linear function  $\mu\sigma$  ( $\mu$  is a scalar), results in

$$x(l+1) = (A + \mu bc^T)x(l) \triangleq A(\mu)x(l). \quad (15)$$

This linearized system is asymptotically convergent for all time and for  $\mu$  in an interval  $[\mu_1'', \mu_2'']$ ; while the nonlinear function  $\phi(\sigma)$  is satisfying  $\mu_1' < \phi(\sigma)/\sigma < \mu_2'$  at a given operating point considering the *entire* state of the system, provided that such a linearization is possible [11]. We now propose the following problem.

**Problem 1:** Find the largest interval  $[\mu_1, \mu_2] \equiv [\mu_1', \mu_2'] \cap [\mu_1'', \mu_2'']$ .

The continuous-time version of this classical control problem has been recently solved (cf., [11]), and similarly Problem 1 can be solved using Theorem 2 together with the iterative algorithm of [10]. Indeed for the above convergent  $A$ ,  $\mu bc^T$  becomes a perturbation matrix  $\Delta A$ , therefore  $A + \Delta A$  remains asymptotically convergent if

$$(\mu bc^T)^T X (\mu bc^T) < \frac{1}{2}Q, \quad (16)$$

where  $X \triangleq P + 2PAQ^{-1}A^TP$  and  $A^TPA + Q = P$ . Since the rank of  $[(bc^T)^TX(bc^T)]$  is one, we may use one of the following two conservative-upper bounds: namely,

$$\mu^2 < \lambda_{\min}(Q) / 2\text{Tr}[(bc^T)^T X(bc^T)], \quad (17)$$

where  $\lambda_{\min}(Q)$  is the minimum eigenvalue of  $Q$ . Or

$$\mu^2 < 1/2\lambda, \quad (18)$$

where  $\lambda$  is the unique nonzero generalized eigenvalue of  $(bc^T)^T X(bc^T)$  with respect to  $Q$  [9], [10]. Using (17) or (18) and the same iterative procedure suggested in [10], we can reach the largest  $[\mu_1'', \mu_2'']$ . On the other hand, the nonlinearity in each problem determines  $[\mu_1', \mu_2']$ . From these two intervals, the solution to Problem 1 follows immediately.

For the case wherein we have multiple nonlinearities, i.e.,  $x(l+1) = Ax(l) + B\phi(\sigma)$ , and  $\phi(\sigma)$  is a vector, then the linearized version of this problem becomes  $x(l+1) = (A + \Delta A)x(l)$ , with  $\Delta A(B, \Delta B, \sigma)$  having multiple-varying parameters and the system having multiple nonlinearities. This problem is now manageable in the set-generator context of Theorem 3. In other words, when we no longer have a single-varying parameter, we must seek a necessary and sufficient condition for determination of the largest or most extended set (instead of interval) for parameter variations shown by  $\chi$ , for the above nonlinear state equation, whose corresponding set of perturbed systems is convergent. In this regard we propose the following problem.

**Problem 2:** Find the largest set  $\chi \equiv \chi_{\text{linearization}} \cap \chi_{\text{perturbation}}$ .

This intersection, as in Problem 1, requires that we consider the contribution and/or validity of linearization, and the contribution of linear homogeneous part of the underlying system, in order to determine the overall convergent robustness measure (CRM) of the nonlinear system. The conceptual solution for Problem 2 consists of generating two sets. The first set  $\chi_{\text{linearization}}$  depends on the encountered nonlinear problem and this has to be treated in each situation accordingly. This set may also represent physical constraints on various system parameters, or an intersection of these constraints and system nonlinearities. Generation of the second set  $\chi_{\text{perturbation}}$  depends on convergent robustness properties of the corresponding class of homogeneous linear difference (discrete-time) equations, which we have provided Theorem 3 whose typical application is described in Example 1.

#### 4. ANALYSIS FOR DISCRETE – TIME DELAY SYSTEMS

To present a CRM for the delay system corresponding to (3), i.e., to study the corresponding variations in coefficient matrices  $A_0$  and  $A_j$ 's, we note that we can transform (3) into the following system.

$$y_1(l) = x(l - m) \quad (19a)$$

$$y_2(l) = x(l - m + 1) \quad (19b)$$

...

$$y_{m+1}(l) = x(l). \quad (19c)$$

Here  $y_j \in \mathbb{R}^n$ , for all  $j$ , and grouping (19) and (3) yields.

$$\begin{Bmatrix} y_1(l+1) \\ y_2(l+1) \\ \vdots \\ y_{m+1}(l+1) \end{Bmatrix} = \begin{bmatrix} 0 & I & 0 & \cdot & 0 \\ 0 & 0 & I & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & I \\ A_m & A_{m-1} & A_{m-2} & \cdot & A_0 \end{bmatrix} \begin{Bmatrix} y_1(l) \\ y_2(l) \\ \vdots \\ y_{m+1}(l) \end{Bmatrix}. \quad (20)$$

The CRM for (20), when all  $A_j$ 's are changing, can be generated using Theorems 2 and 3. Thus, the *largest* perturbation matrix corresponding to the augmented coefficient matrix in (20) can be easily generated using results presented in this article. Therefore we have a viable technique to construct the *largest* class of convergent perturbed difference equations with multiple delays.

## 5. NUMERICAL EXAMPLES

Here, we present the following examples to illustrate our approach and to spur interests in this aspect of our study. After all, this is an applied area of research and different methods cited in the literature must be compared for their usefulness when applied to the same nontrivial example.

*Example 2 (Nonlinear systems with linear multiple parameter variations and multiple nonlinearities) [37]:* Consider the following second-order sampled-data nonlinear system.

$$x_1(l+1) = 0.6x_2(l) + 0.1\phi_2(\sigma_2(l), l) \quad (21a)$$

$$x_2(l+1) = -1.5x_1(l) + 0.2x_2(l) + 0.14\phi_1(\sigma_1(l), l) - 0.09\phi_2(\sigma_2(l), l),$$

where

$$\sigma_1 = 1.2x_1 + 1.6x_2 \quad (22a)$$

$$\sigma_2 = 0.6x_1 - 2.1x_2 \quad (22b)$$

$$0 \leq \phi_j(\sigma_j(l), l)\sigma_j \leq \theta_j\sigma_j^2 \quad (j = 1, 2). \quad (22c)$$

This system is uniformly asymptotically convergent when  $\theta_j > 0$ ,  $j = 1, 2$ , are changing according to Fig. 2 [these  $\theta_j$ 's are shown by  $k_j$ 's in [37]]. No explicit information on the actual computational steps for construction of this set is provided in [37]. Compare this result (Fig. 2) with the method of this article.

*Solution:* If we use the method of Section 3 and upon linearizing (21), i.e., substituting for  $\phi_j$ 's in terms of  $\sigma_j$ 's as well as  $x_j$ 's, we get

$$\begin{Bmatrix} x_1(l+1) \\ x_2(l+1) \end{Bmatrix} = S(\theta_1, \theta_2) \begin{Bmatrix} x_1(l) \\ x_2(l) \end{Bmatrix}, \quad (23)$$

where  $S(\theta_1, \theta_2)$  is exactly the same as  $S_1$  in (7). Here we assume that from the convergent behavior point of view, (23) and (21) are the same. However, we have already established the CRM of  $S(\theta_1, \theta_2)$  in Fig. 1, which shows that for this linearized system the set of parameter variations can be maneuvered in a larger area than that proposed in [37].

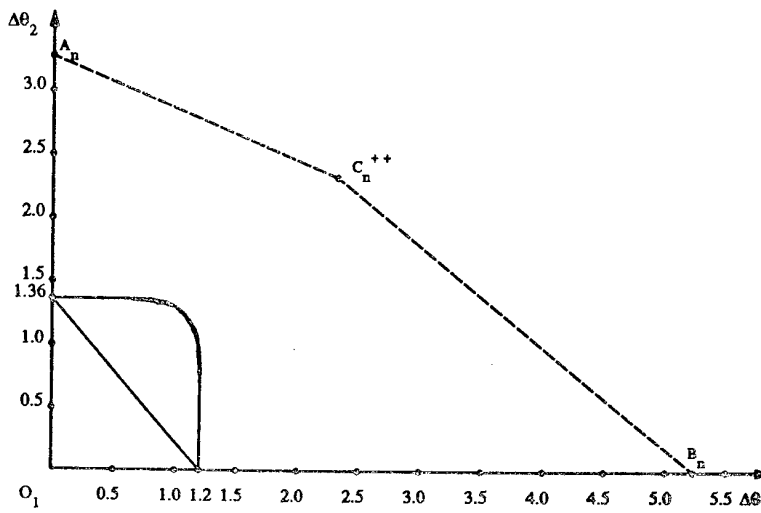


Fig. 2. The admissible set of parameter variations from Pyatnitskii and Skorodinskii method.

*Comment 2:* Our main observation from Example 2 is that, the proposed method is more systematic and broadly applicable to various problems involving parameter variations than those suggested in our references, and provides a larger area for parameter variations as well, i.e., this method meets our objective.

Often a question is raised, what if our perturbation matrix changes as a nonlinear function of parameter variations, for instance, a type  $G$  [10], while the underlying system is linear. In the following we tailor an academic example to shed some light on this case.

*Example 3 (Linear systems with multiple nonlinear parameter variations) [14]:* Consider the following system which is asymptotically convergent at its operating condition  $\theta^o \triangleq \frac{1}{4}[2, \sqrt{2}]^T$  as follows.

$$S(\equiv S_1) \triangleq \begin{pmatrix} 0 & 1 \\ \theta_2^2 & \theta_1^2 \end{pmatrix} \equiv \begin{pmatrix} 0 & 1 \\ 0.125 & 0.25 \end{pmatrix}. \quad (24)$$

Our objective is to construct the largest set of  $r (= 2)$  independently varying parameters  $\Delta\theta_1$  and  $\Delta\theta_2$  from their operating condition  $\theta^o \triangleq \frac{1}{4}[2, \sqrt{2}]^T$  such that  $S + \Delta S$  remains convergent, where the perturbation matrix  $\Delta S$  is a type  $G$  [10]. Here,

$$\Delta S|_{\theta^o} \triangleq \begin{pmatrix} 0 & 0 \\ 2\theta_2^o\Delta\theta_2 + (\Delta\theta_2)^2 & 2\theta_1^o\Delta\theta_1 + (\Delta\theta_1)^2 \end{pmatrix}. \quad (25)$$

*Solution:* Clearly at any given  $\theta^o$ , (25) does not meet the convexity requirement of Theorem 3 and we must seek other approach to construct its CRM, but first, we explore

any special property of the original system, if we may. In fact, because of the way in which nonlinearity has appeared in (24), and as suggested in [10] to [14], we may group the (2,1) entry as one parameter and the (2,2) entry as another. For instance, we let  $\theta_2^2 = \alpha$  and  $\theta_1^2 = \beta$  with the corresponding initial values given in (24). Next, (24) becomes  $\begin{bmatrix} 0 & 1 \\ \alpha & \beta \end{bmatrix}$ . Perturbing this matrix in the space of  $(\alpha, \beta)$  results in the corresponding CRM. Indeed this measure is already established in [12], also shown in Fig. 3, but to interpret that CRM in terms of  $\theta_1$  and  $\theta_2$  may not be so easy as it looks. Nevertheless, this is a thought that has to be explored. On the other hand, we can always use Theorem 2 with (25) as its corresponding  $\Delta S$ , but we face a complicated algebraic problem. Therefore in the following instead of an *ad hoc* procedure, we review a systematic method which works and its complexity depends on the encounter problem.

As Remark 1 suggests, we propose to expand (25) by its corresponding Taylor series expansion as follows.

$$\begin{aligned} \Delta S(\Delta\theta) = \Delta S(\Delta\theta) \Big|_{\Delta\theta^o} + \sum_{j=1}^r \frac{\partial \Delta S}{\partial \Delta\theta_j} \Big|_{\Delta\theta^o} \frac{\Delta\theta_j}{1!} \\ + \left[ \frac{\partial^2 \Delta S}{\partial (\Delta\theta_1)^2} \frac{(\Delta\theta_1)^2}{2!} + \frac{\partial^2 \Delta S}{\partial \Delta\theta_1 \partial \Delta\theta_2} \frac{(\Delta\theta_1)(\Delta\theta_2)}{2!} \right. \\ \left. + \dots + \frac{\partial^2 \Delta S}{\partial (\Delta\theta_r)^2} \frac{(\Delta\theta_r)^2}{2!} \right] \Big|_{\Delta\theta^o} + \dots, \end{aligned} \quad (26)$$

where by working in the PVS,  $\Delta\theta^o \equiv 0$ . Applying (26) to (25) results in

$$\Delta S = \begin{pmatrix} 0 & 0 \\ 0 & 2\theta_1^o \end{pmatrix} \Delta\theta_1 + \begin{pmatrix} 0 & 0 \\ 2\theta_2^o & 0 \end{pmatrix} \Delta\theta_2 + \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \frac{(\Delta\theta_1)^2}{2} + \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix} \frac{(\Delta\theta_2)^2}{2}. \quad (27)$$

As expected in this case, all the remaining higher-order terms are zero. In order that (27) meets the convexity requirement, we must be able to justify dropping the second-order perturbations. For instance, if we let  $\Delta\theta_1 < 0.3$  and  $\Delta\theta_2 < 0.3$ , resulting in  $(\Delta\theta_1)^2 < 0.09$ , and  $(\Delta\theta_2)^2 < 0.09$ , then we can approximate  $\Delta S$  as follows.

$$\Delta S \approx \begin{pmatrix} 0 & 0 \\ 0 & 2\theta_1^o \end{pmatrix} \Delta\theta_1 + \begin{pmatrix} 0 & 0 \\ 2\theta_2^o & 0 \end{pmatrix} \Delta\theta_2, \quad (28)$$

where the initial  $\theta^o \triangleq 1/4[2, \sqrt{2}]^T$  remains as a running variable for each new operating condition which is selected at the farthest vertex of a new generated set. Clearly, to approximate (27) with (28) in order to meet the convexity requirement of Theorem 3, one must consider the norms of the coefficient matrices as well as the magnitude of parameter variations. When these conditions are met, by using Theorem 3 we can construct the corresponding CRM. However, in each iteration, only that portion of this CRM is acceptable where  $\Delta\theta_1 < 0.3$  and  $\Delta\theta_2 < 0.3$ . A limitation that we inherently face. If the

coefficient matrices of higher-order perturbations had larger norms, then the chosen upper bound would be even smaller. Thus our computation heavily depends on the encounter problem. Now, the remaining steps to complete the construction of *CRM* for (28) are the same as in Example 1. We note that using this procedure, and as we translate the coordinates of the *PVS* to various corner points of each generated set, the symbolic computations remain unchanged. This method is easily amenable with computer computations. Furthermore we are taking advantage of convergent behavior of linear time-invariant system which depends only on the locations of its eigenvalues. In other words, as long as *S* in (24) remains convergent, irrespective of the perturbation mechanism used to changed its entries, we can perturb that *S* and find its *CRM*, using results from Section 2.

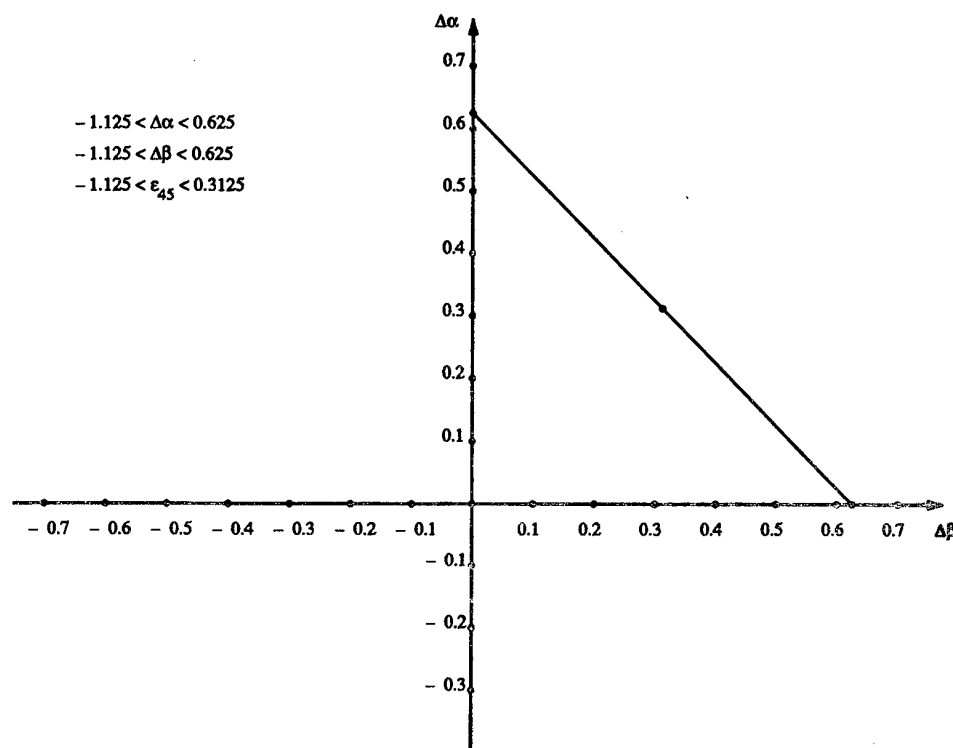


Fig. 3. Toward the direct construction of the largest admissible set of parameter variations.

## 6. CONCLUSIONS

In this article we describe the necessary and sufficient conditions for the absolute stability of a class of nonlinear difference equations and possibly with delays. This work depends on the result of Theorem 3, which enables us to construct *the largest set* of admissible parameter variations for a class of homogeneous linear difference equations having multiple large-parameter variations. In this article using three building-block examples, in effect we have shown how to *break* a complicated nonlinear system with multiple nonlinear parameter variations and possibly with multiple delays *into* a set of manageable problems, thus to construct the largest set of parameter variations for the overall robust stability or convergent analysis of the underlying system.

## 7. REFERENCES

- [1] E.Y. Aivazyan, "A frequency criterion of stability of nonlinear sampled-data systems," *Auto. Remote Contr.*, vol. 49, no. 3, pp. 279-283, 1988.
- [2] M.A. Aizerman and F.R. Gantmacher, *Absolute Stability of Regulator Systems*. San Francisco: Holden-Day, Inc. 1964. [Translated from Russian by E. Polak.] Reviewed by R.W. Brockett, *IEEE Trans. Auto. Contr.*, vol. AC-11, no. 3, p. 634, 1966, and by R.E. Kalman, *SIAM Review*, vol. 12, no. 1, pp. 161-162, 1970.
- [3] N.E. Barabanov, "The Lyapunov indicator of discrete inclusions," I, II, III, *Auto. Remote Contr.*, vol. 49, no. 2, pp. 152-157, no. 3, pp. 283-287, and no. 5, pp. 558-565, 1988.
- [4] \_\_\_\_\_, "Absolute stability of sampled-data control systems," *Auto. Remote Contr.*, vol. 49, no. 8, pp. 981-988, 1988.
- [5] J. Chegancas and C. Burgat, "Polyhedral cones associated to  $M$ -matrices and stability of time varying discrete time systems," *J. Math. Anal. Appl.*, vol. 118, pp. 88-96, 1986.
- [6] Y.N. Chekhovoy, "Stability of discrete automatic systems containing nonlinear and quasilinear blocks," *Soviet Auto. Contr.*, vol. 17, no. 2, pp. 86-89, 1984.
- [7] H.-D. Chiang and J.S. Thorp, "Stability regions of nonlinear dynamical systems: A constructive methodology," *IEEE Trans. Auto. Contr.*, vol. AC-34, no. 12, pp. 1229-1241, Dec. 1989.
- [8] K.L. Cooke and J.M. Ferreira, "Stability conditions for linear retarded functional difference equations," *J. Math. Anal. Appl.*, vol. 96, pp. 480-504, 1983.
- [9] M. Eslami, "On sensitivity of nonlinear digital systems," in *Proc. Allerton Conf.*, pp. 424-432, Oct. 1980.
- [10] \_\_\_\_\_, "Computer-aided determination of stability robustness measure of linear discrete-time systems," *Automatica*, vol. 26, n. 3, pp. 623-627, May 1990.
- [11] \_\_\_\_\_, Robust nonlinear dynamic systems - The old question with a new answer, *Systems and Control Letters*, vol. 14, pp. 445-451, 1990.
- [12] \_\_\_\_\_, "On stability robustness analysis of discrete-time dynamic systems," in *Proc. 31st IEEE Conf. on Decision and Contr.*, pp. 1569-1574, 1992.
- [13] \_\_\_\_\_, "Stability robustness analysis of discrete-time systems with multiple large parameter variations," *Automatica*, vol. 30, 1994.
- [14] \_\_\_\_\_, *Theory of Sensitivity in Dynamic Systems, an introduction*. New York: Springer Verlag, 1994.
- [15] A.I. Gaiduk, "Absolute stability of control systems with several nonlinearities," *Auto. Remote Contr.*, vol. 37, no. 6, pp. 815-821, Nov. 1976.
- [16] A.K. Gelig and N.I. L'yanova, "A circle criterion of stability of nonlinear sampled-data systems with a new type of quadratic constraints," *Auto. Remote Contr.*, vol. 49, no. 6, pp. 714-719, 1988.
- [17] A. Hmamed, "On the stability of time-delay systems: New results," *Int. J. Contr.*, vol. 43, no. 1, pp. 321-324, 1986.
- [18] E.I. Jury, "Robustness of a discrete system," *Auto. Remote Contr.*, vol. 51, no. 5, pp. 571-592, 1990.
- [19] R.E. Kalman and J.E. Bertram, "Control system analysis and design via the 'second method' of Lyapunov, II. Discrete-time systems," *Trans. ASME - J. Basic Eng.*, vol. 82, no. 2, pp. 394-400, June 1960.
- [20] A.I. Korshunov, "Analysis of stability in the large of linearizable sampled-data systems with the aid of two Lyapunov functions," *Auto. Remote Contr.*, vol. 51, no. 5, pp. 647-657, 1990.
- [21] \_\_\_\_\_, "Robustness of pulse-width systems," *Auto. Remote Contr.*, vol. 51, no. 12, pp. 1702-1709, 1990.

- [22] V.S. Kozyakin, "Algebraic unsolvability of problem of absolute stability of desynchronized systems," *Auto. Remote Contr.*, vol. 51, no. 6, pp. 754-759, 1990.
- [23] \_\_\_\_\_, "Stability of phase-frequency desynchronized systems under a perturbation of the switching instants of the components," *Auto. Remote Contr.*, vol. 51, no. 8, pp. 1034-1040, 1990.
- [24] \_\_\_\_\_, "Absolute stability of systems with asynchronous sampled-data elements," *Auto. Remote Contr.*, vol. 51, no. 10, pp. 1349-1355, 1990.
- [25] M.A. Krasnosel'skii and A.V. Pokrovskii, "Absolute stability of systems with discrete time," *Auto. Remote Contr.*, vol. 39, no. 2, pp. 189-199, 1978.
- [26] V. Lakshmikantham and D. Trigiante, *Theory of Difference Equations: Numerical Methods and Applications*. New York: Academic, 1988.
- [27] M.R. Liberzon, "New results on absolute stability of nonstationary controlled systems," (Survey) *Auto. Remote Contr.*, vol. 40, no. 8, pp. 1124-1140, Aug. 1979.
- [28] A.I. Lur'e, *Certain (Some) Nonlinear Problems in the Theory of Automatic Control [in Russian]* (Gostekhizdat, Moscow, 1951). (English Translation: H.M.S.O., London, 1957).
- [29] A.P. Molchanov, "Absolute stability of sampled-data systems with several nonstationary nonlinear elements," *Auto. Remote Contr.*, vol. 40, no. 3, pp. 355-362, March 1979.
- [30] \_\_\_\_\_, "Criterion of absolute stability for sampled-data systems with a nonstationary nonlinearity. I," *Auto. Remote Contr.*, vol. 44, no. 5, pp. 607-616, May 1983.
- [31] \_\_\_\_\_, "A criterion of absolute stability of sampled-data systems with a nonstationary nonlinearity. II," *Auto. Remote Contr.*, vol. 44, no. 6, pp. 719-729, June 1983.
- [32] A.P. Molchanov and E.S. Pyatnitskii, "Absolute instability of nonlinear nonstationary systems," Part I, *Auto. Remote Contr.*, vol. 43, no. 1, pp. 13-20, Jan. 1982, Part II, *Auto. Remote Contr.*, vol. 43, no. 2, pp. 147-157, Feb. 1982, Part III, *Auto. Remote Contr.*, vol. 43, no. 3, pp. 289-299, March 1982.
- [33] \_\_\_\_\_, "Lyapunov functions that specify necessary and sufficient conditions of absolute stability of nonlinear nonstationary control systems," Part I, *Auto. Remote Contr.*, vol. 47, no. 3, pp. 344-354, March 1986, Part II, *Auto. Remote Contr.*, vol. 47, no. 4, pp. 443-451, April 1986, Part III, *Auto. Remote Contr.*, vol. 47, no. 5, pp. 620-630, May 1986.
- [34] A.P. Molchanov and Y.S. Pyatnitskiy, "Criteria of asymptotic stability of differential and difference inclusions encountered in control theory," *Systems Contr. Lett.*, vol. 13, pp. 59-64, 1989.
- [35] X. Niu, J.A. De Abreu-Garcia, and E. Yaz, "Improved bounds for linear discrete-time systems with structured perturbations," *IEEE Trans. Auto. Contr.*, vol. AC-37, pp. 1170-1173, Aug. 1992. Correction, *ibid.*, vol. AC-38, no. 5, p. 832, 1993, Comments by M. Eslami, *IEEE Trans. Auto. Contr.*, vol. AC-39, pp. 1768-1769, Aug. 1994.
- [36] E.S. Pyatnitskiy, "New research on absolute stability of automatic control systems, (Review)," *Auto. Remote Contr.*, vol. 29, no. 6, pp. 855-881, June 1968.
- [37] E.S. Pyatnitskii and V.I. Skorodinskii, "A criterion of absolute stability of nonlinear sampled-data control systems in the form of numerical procedures," *Auto. Remote Contr.*, vol. 47, no. 9, pp. 1190-1198, 1986.
- [38] M.R.M. Rao and S. Sivasundaram, "Asymptotic stability for equations with unbounded delay," *J. Math. Anal. Appl.*, vol. 131, pp. 97-105, 1988.
- [39] R.H. Seacat, L.F. Judd, and D.R. Frannin, "A method of specifying parameter changes to insure absolute stability of nonlinear systems," in *Proc. Midwest Symp. Circuit Theory*, vol. 11, pp. 273-280, 1968.
- [40] B.A. Shane and S. Barnett, "Sensitivity of convergent matrices and polynomials," *J. Math. Anal. Appl.*, vol. 43, pp. 114-122, 1973.

# MULTIDIMENSIONAL FREQUENCY ROBUST CONTROL SYSTEMS

Taalaibek A. Akunov, Akilai Akunova and Anatoly V. Ushakov  
Saint-Petersburg Institute of Fine Mechanics and Optics  
Department of Automatic and Remote Control  
Sablinskaya St., 14, Saint-Petersburg, 197101, RUSSIA  
e-mail: myk@beam.ifmo.ru

**Abstract.** The procedure of synthesis of frequency robust control systems is offered, in which as a criterion frequency robustness a frequency condition number of a ratio an input-output of system is used, calculated on the basis of singular value decomposition of real-valued frequency transfer function matrices. The minimization of the condition number is reached by minimization of condition number of a modal matrix of system. The results are illustrated by examples

**Key words.** Multidimensional systems, real-valued frequency transfer function matrices, robustness, frequency condition number, singular value decomposition of matrices, generalized modal control.

## **Introduction. Initial algebraical concepts.**

**Concept 1.** Let research problem of multidimensional control systems is reduced to vector-matrix representation

$$\xi(\tau) = \pi(\tau)\beta(\tau); \quad \tau = t, k; \quad (1)$$

where  $\xi, \beta$  are the vectors over a field of real numbers,  $\xi \in R^p$ ,  $\beta \in R^v$ ;  $\pi$  - some criterion matrix,  $\pi \in R^{p \times v}$ , (accepts meaning of continuous time  $t$  in case of research of continuous multidimensional controllable processes and meaning of discrete time  $k$ , expressed among intervals of step-type behaviour by duration  $\Delta t$  so, that continuous time  $t$  and discrete  $k$  are connected by a ratio  $t = (\Delta t)k$  in case of research of discrete multidimensional controllable processes. Let the matrix  $\pi(\tau)$  has the singular value decomposition [1]

$$\pi(\tau) = U_\pi(\tau) \Sigma_\pi(\tau) V_\pi^T(\tau), \quad (2)$$

where  $\Sigma_\pi$  is a  $(\rho \times \nu)$  diagonal matrix, having singular values on a main diagonal of matrix  $\pi(\tau)$ ;  $U_\pi(\tau)$  - an orthogonal  $(\rho \times \rho)$  matrix, which columns form a left singular basis of the matrix  $\pi(\tau)$ ;  $V_\pi(\tau)$  - an orthogonal  $(\nu \times \nu)$  matrix, which columns form the right singular basis of the matrix  $\pi(\tau)$  so, that

$$\pi(\tau) V_{\pi i}(\tau) = \alpha_i(\tau) U_{\pi i}(\tau); \quad i = \overline{1, \nu}$$

where  $\alpha_i(\tau)$  is the  $i$ -th element of an algebraic spectrum  $\sigma_\alpha \{ \pi(\tau) \}$  of singular values of the matrix  $\pi(\tau)$ ,  $(o)_i$  is the  $i$ -th column of a matrix  $(o)$ , thus a vectorial Euclidean norm  $\| (o)_i \| = 1$ .

If in an algebraic spectrum of singular values  $\sigma_\alpha\{\pi(\tau)\}$  and geometric spectra  $U_\pi(\tau)$  and  $V_\pi(\tau)$  left and right singular bases of the matrix  $\pi(\tau)$  to select two groups  $\{U_{\pi M}(\tau), \alpha_M(\tau), V_{\pi M}(\tau)\}$  and  $\{U_{\pi m}(\tau), \alpha_m(\tau), V_{\pi m}(\tau)\}$ , where  $\alpha_M(\tau)$ ,  $\alpha_m(\tau)$  are the maximum and minimum singular values of the matrix  $\pi(\tau)$  for  $\forall \tau$  accordingly,  $\{U_{\pi M}(\tau), V_{\pi M}(\tau)\}$  and  $\{U_{\pi m}(\tau), V_{\pi m}(\tau)\}$  are the elements coordinated with them of the left and right singular bases, then the sphere  $\|\beta(\kappa)\| = \text{const}$  by virtue of (1) is mapped in an ellipsoid in  $R^p$ , the maximum and minimum half-axes of which belong to linear envelopes  $\mathcal{L}\{U_{\pi M}(\tau)\}$  and  $\mathcal{L}\{U_{\pi m}(\tau)\}$ , and the lengths of these half-axes are accordingly equal  $\alpha_M(\tau)\|\beta(\tau)\|$  and  $\alpha_m(\tau)\|\beta(\tau)\|$ , vectors in  $R^v$ , mapped in  $\mathcal{L}\{V_{\pi M}(\tau)\}$  and  $\mathcal{L}\{V_{\pi m}(\tau)\}$  belong to linear envelopes  $\mathcal{L}\{V_{\pi M}(\tau)\}$  and  $\mathcal{L}\{V_{\pi m}(\tau)\}$  accordingly. When passing from (1) to Euclidean vectorial norms, there are the correct estimated inequalities

$$\alpha_m(\tau)\|\beta(\tau)\| \leq \|\xi(\tau)\| \leq \alpha_M(\tau)\|\beta(\tau)\|, \forall \tau. \quad (3)$$

It is necessary to notice, that if the matrix  $\pi(\tau)$  can be represented in one of the forms,

$$\pi(\tau) = \bar{\pi}R(\tau), \pi(\tau) = W(\tau)\bar{\pi}, \pi(\tau) = W(\tau)\bar{\pi}R(\tau), \quad (4)$$

where  $R(\tau), W(\tau)$  are an orthogonal or unitary matrices, the estimated inequalities (3) become stationary on  $\tau$

$$\bar{\alpha}_m \leq \|\xi(\tau)\|/\|\beta(\tau)\| \leq \bar{\alpha}_M, \forall \tau. \quad (5)$$

where  $\bar{\alpha}_m, \bar{\alpha}_M$  are the extreme elements of the algebraic spectrum  $\sigma_\alpha\{\bar{\pi}\}$  of singular values of the matrix  $\bar{\pi}$

*Concept 2.* Let the matrix  $\pi(\tau)$  and the vector  $\beta(\tau)$  can get an increment  $\Delta\pi(\tau)$  and  $\Delta\beta(\tau)$  accordingly, thus by virtue of (1) they generate an increment  $\Delta\xi(\tau)$ . Then an majoring inequality becomes correct

$$\delta_\xi(\tau) \leq C\{\pi(\tau)\}(\delta_\beta(\tau) + \delta_\pi(\tau) + \delta_\pi(\tau)\delta_\beta(\tau)) \quad (6)$$

where  $\delta_{(\circ)}(\tau) \triangleq \|\Delta(\circ)(\tau)\|/\|(\circ)(\tau)\|$ ,  $(\circ)$  – it is meaningful  $\xi, \beta$  and  $\pi$ .

$$C\{\pi(\tau)\} \triangleq \|\pi(\tau)\|/\|\pi^+(\tau)\| = \alpha_M(\tau)/\alpha_m(\tau) \quad (7)$$

$$C\{\pi(\tau) = \bar{\pi}R(\tau)\} = C\{\pi(\tau) = W(\tau)\bar{\pi}\} = C\{\pi(\tau) = W(\tau)\bar{\pi}R(\tau)\} = C\{\bar{\pi}(\tau)\} = \bar{\alpha}_M/\bar{\alpha}_m \quad (8)$$

$C\{\bar{\pi}\}$  – condition number as factor of amplification of a relative error of the linear problem (1) can be used as a measure of sensitivity of the linear problem (1) to variations of its matrix and vectorial components.

**Frequency condition number.** If the task of control is reduced to an aspect (1), supposing representation of a matrix  $\pi(\tau)$  in one of the forms (4), where the matrix  $\bar{\pi}$  depends on frequency  $\omega$  so, that  $\bar{\pi} = \bar{\pi}(\omega)$ , therefore the condition number  $C\{\bar{\pi}\} = C\{\bar{\pi}(\omega)\}$  depends on frequency.

For designing realizations of matrices  $\bar{\pi}$  of relations "input—state" and "input—output" we shall use following methodological reception. According to this reception the description of a movement of multidimensional system at first is built at external finite dimensional signal of an any kind, and then the obtained vector-matrix representations are used for a case of vectorial real-valued external harmonic input signal.

In connection with marked continuous system with a vector-matrix "input—state—output" description is considered

$$\dot{x}(t) = Fx(t) + Gg(t); \quad x(0) = x_0; \quad y(t) = Cx(t) \quad (9)$$

where  $x(t)$  is a state vector,  $g(t)$  is a vector of input signal,  $F, G, C$  – matrices of the state, input and output accordingly, the input signal  $g(t)$  is generated by autonomous finite dimensional system

$$\dot{z}(t) = Ez(t); \quad z(0) = z_0; \quad g(t) = Hz(t), \quad (10)$$

where  $x \in R^n$ ;  $g, y \in R^m$ ;  $z \in R^l$ ;  $F \in R^{n \times n}$ ,  $G, C^T \in R^{n \times m}$ ,  $E \in R^{l \times l}$ ;  $H \in R^{m \times l}$ .

We shall formulate the following statements for the purposes of further researches.

**Statement 1.** For the state vector  $x(t)$  of the system (9), in which the input signal  $g(t)$  is set (10), the representation is fair

$$x(t) = \exp(Ft)x_0 + [T \exp(Et) - \exp(Ft)T]z_0, \quad (11)$$

where the matrix  $T$  satisfies to the matrix Sylvester equation

$$TE - FT = GH. \quad (12)$$

**Proof.** For the purposes of the proof of the statement we shall formulate the following lemma.

**Lemma.** Let a scalar series

$$f(\alpha) = a_0 + a_1\alpha + a_2\alpha^2 + \dots + a_p\alpha^p + \dots \quad (13)$$

generate the matrix series of a rather square matrix  $N$  regarding to the scalar variable  $\alpha$

$$f(\alpha) = a_0I + a_1N + a_2N^2 + \dots + a_pN^p + \dots \quad (14)$$

If the matrix  $N$  has bloc matrix representation by matrices

$$N = \begin{bmatrix} F & GH \\ O & E \end{bmatrix} \quad (15)$$

where the matrix components  $N$  are connected by the Silvester equation (12), the matrix function  $f(N)$  (14) from a matrix  $N$  can be written down in the form

$$f(N) = \begin{bmatrix} f(F) T f(E) - f(F) T \\ O & f(E) \end{bmatrix} \quad (16)$$

*Proof.* The proof lemma is built on direct substitution (14) matrixes  $N$  (15), in which the matrix element  $GH$  by virtue of the matrix Silvester equation (21) is replaced by the matrix composition  $TE - FT$ , thus the first term  $a_0I$  matrix series (14) is represented in the form

$$a_0 \begin{bmatrix} I_{n \times n} & TI_{l \times l} - I_{n \times n} T \\ O & I_{l \times l} \end{bmatrix}, \text{ where } I_{n \times n}, I_{l \times l} - \text{ accordingly single } (n \times n) \text{ and } (l \times l) \text{ matrices.}$$

Lemma is proved.

We form of system (17) and source of finite dimensional input signal (10) an autonomous system

$$\dot{\tilde{x}}(t) = \tilde{F}\tilde{x}(t); \tilde{x}(0) = \tilde{x}_0; \quad (17)$$

$$\text{where } \tilde{x} = \begin{bmatrix} x^T & z^T \end{bmatrix}^T; \tilde{F} = \begin{bmatrix} F & GH \\ O & E \end{bmatrix};$$

The solution of system (17) is written down

$$\tilde{x}(t) = \exp(\tilde{F}t)\tilde{x}_0; \quad (18)$$

which by virtue of proved lemma can be submitted in the form

$$\tilde{x}(t) = \begin{bmatrix} \exp(Ft) T \exp(Et) - \exp(Ft) T \\ O & \exp(Et) \end{bmatrix} \tilde{x}_0;$$

The last ratio written down on vectorial components  $x(t)$  and  $z(t)$  of vector  $\tilde{x}(t)$  results in the ratio (11). The statement is proved.

We shall be limited in further researches by a case, when  $F$  is stable matrix, and initial conditions are zero so, that  $x_0 = 0$ , and we consider a forced component of the solution (11). Then the researches of forced processes in a multidimensional system (9) on the state vector  $x(t)$  and output  $y(t)$  with external finite-dimensional input signal  $g(t)$  formed by virtue of (10) are reduced to the analysis of representation (1) in the form

$$x(t) = \pi_x(t)z_0; y(t) = \pi_y(t)z_0, \quad (19)$$

where 
$$\pi_x(t) = T \exp(Et), \quad \pi_y(t) = C\pi_x(t).$$

*Statement 2.* For the case of vectorial harmonic real-valued input signal  $g(t) = g(t, \omega)$  the matrices  $\pi_x(t) = \pi_x(t, \omega)$ ,  $\pi_y(t) = \pi_y(t, \omega)$  are depend on frequency  $\omega$  so, that they are can be presented in the form (4)

$$\pi_x(t, \omega) = \bar{\pi}_x(\omega)R(t), \quad \pi_y(t, \omega) = \bar{\pi}_y(\omega)R(t), \quad (20)$$

where the matrices  $\bar{\pi}_x(\omega)$  and  $\bar{\pi}_y(\omega)$  are determined for each value of frequency  $\omega$  with the help of the decision  $T$  of the Silvester equation (12) in the form of a ratio

$$\bar{\pi}_x(\omega) = T, \quad \bar{\pi}_y(\omega) = CT, \quad (21)$$

Thus the matrix  $R(t) = \exp(Et)$  is orthogonal

*Proof.* The proof of the statement is built on direct calculation matrix exponent  $\exp(Et)$ . In the case of vectorial real-valued harmonic input signal the matrices  $E$  and  $H$  of the source of input signal (10) can be written down in the form.

$$E = \text{diag} \left\{ E_{ii} = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}; i = \overline{1, m} \right\}, \quad H = I_{m \times m} \otimes [1 \ 0], \quad (22)$$

where  $I_{m \times m}$  – unique  $(m \times m)$  matrix,  $\omega$  – frequency of real-valued harmonic input signal, applied to  $i$ -th input of system (9), forming thus vectorial real-valued input signal,  $\otimes$  – Kronecker delta of product of matrices. It is easy to see, that for the matrix exponent  $\exp(Et)$  with a matrix  $E$  of the kind (22) it is possible to note

$$\exp(Et) = \text{diag} \left\{ \exp(E_{ii}t) = \begin{bmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & \cos \omega t \end{bmatrix}; i = \overline{1, m} \right\}. \quad (23)$$

The representation (23) discovers an orthogonality of  $\exp(Et)$ .

The statement is proved.

The matrices  $\bar{\pi}_x(\omega)$  and  $\bar{\pi}_y(\omega)$  can be used for the analysis of processes at vectorial real-valued input harmonic signal with frequency  $\omega$  on base of the algebraic concepts 1 and 2. Thus the inequality (5) for the variables  $x(t)$  and  $y(t)$  appears stationary in time and for each of them accepts the kind

$$\bar{\alpha}_{xm}(\omega) \leq \|x(t)\|/\|z_0\| \leq \bar{\alpha}_{xM}(\omega), \quad \bar{\alpha}_{ym}(\omega) \leq \|y(t)\|/\|z_0\| \leq \bar{\alpha}_{yM}(\omega), \quad \forall t. \quad (24)$$

where  $\{\bar{\alpha}_{xm}(\omega), \bar{\alpha}_{xM}(\omega)\}$ ,  $\{\bar{\alpha}_{ym}(\omega), \bar{\alpha}_{yM}(\omega)\}$  are the extreme elements of the algebraic spectrum of singular values  $\sigma_\alpha\{\bar{\pi}_x(\omega)\}$  and  $\sigma_\alpha\{\bar{\pi}_y(\omega)\}$  of matrices  $\bar{\pi}_x(\omega)$  and  $\bar{\pi}_y(\omega)$ .

For calculation of matrices  $\bar{\pi}_x(\omega)$  and  $\bar{\pi}_y(\omega)$  we shall use the following statement.

*Statement 3.* The matrices  $\bar{\pi}_x(\omega)$  and  $\bar{\pi}_y(\omega)$  can be written down in the form

$$\bar{\pi}_x(\omega) = (\omega^2 I + F^2)^{-1} \text{row} \left[ \begin{array}{c|c} -FG_i & \omega G_i \end{array} \right]; \quad i = \overline{1, m}; \quad \bar{\pi}_y(\omega) = C \bar{\pi}_x(\omega), \quad (25)$$

where  $\text{row}\{(\circ)_i\}$  – lower case matrix structure from elements  $(\circ)_i$ ;  $G_i$  –  $i$ -th column of the matrix  $G$ .

*Proof.* By virtue of (21) the proof of the statement is reduced to the explicit decision of the Sylvester equation (12) concerning the matrix  $T$ . For two adjacent columns of matrix components of the Sylvester equation the row form (12) has representation

$$T \begin{bmatrix} E_{2i-1} & E_{2i} \end{bmatrix} - F \begin{bmatrix} T_{2i-1} & T_{2i} \end{bmatrix} = G \begin{bmatrix} H_{2i-1} & H_{2i} \end{bmatrix}; \quad i = \overline{1, m}, \quad (26)$$

where by virtue of (23) it is possible to write down

$$\begin{bmatrix} E_{2i-1} & E_{2i} \end{bmatrix} = \begin{bmatrix} O_{2(i-1)} & O_{2(i-1)} \\ 0 & \omega \\ \omega & 0 \\ O_{2(m-i)} & O_{2(m-i)} \end{bmatrix} \quad (27)$$

$O_{2(\circ)}$  – the  $2(\circ)$ -dimensional zero matrix-column, where  $(\circ)$  accepts meanings  $(i-1)$ ,  $(m-i)$  accordingly,  $E_{2(i-1)}$ ,  $T_{2(i-1)}$ ,  $H_{2(i-1)}$  –  $2(i-1)$ -th columns of the matrices  $E$ ,  $T$  and  $H$ . The substitution (27) and matrix  $H$  (22) in (26) after transformations, gives for adjacent columns  $T_{2(i-1)}$  and  $T_{2i}$  of matrix  $T$  representation

$$T_{2(i-1)} = -(\omega^2 I + F^2)^{-1} F G_i \quad T_{2i} = (\omega^2 I + F^2)^{-1} \omega G_i \quad ; i = \overline{1, m} \quad (28)$$

The expression (28) proves the validity of the statement.

For the estimation of robustness of the input-state and input-output relations dependent on frequency, we shall use the definition of condition number (7), in which matrix spectral norms are used. Then input-state relation dependent on frequency with the matrix  $\bar{\pi}_x(\omega)$  is characterized by condition number  $C_x(\omega)$ , and input-output relation dependent on frequency with the matrix  $\bar{\pi}_y(\omega)$  is characterized by condition number  $C_y(\omega)$ , calculated by virtue of ratio

$$C_x(\omega) \triangleq C\{\bar{\pi}_x(\omega)\} = \bar{\alpha}_{xM}(\omega) / \bar{\alpha}_{xm}(\omega), \quad C_y(\omega) \triangleq C\{\bar{\pi}_y(\omega)\} = \bar{\alpha}_{yM}(\omega) / \bar{\alpha}_{ym}(\omega) \quad (29)$$

In pithy, the matrices  $\bar{\pi}_x(\omega)$  and  $\bar{\pi}_y(\omega)$  are real-valued transfer function matrices of input-state relation and input-output relation. Accordingly,  $\bar{\alpha}_{xM}(\omega)$ ,  $\bar{\alpha}_{xm}(\omega)$  are majorants and minorants of the amplitude frequency characteristics of input-state relations, and  $\bar{\alpha}_{yM}(\omega)$ ,  $\bar{\alpha}_{ym}(\omega)$  are majorants and minorants of the amplitude frequency characteristics of input-output relations. Thus, the frequency condition numbers  $C_x(\omega)$  and  $C_y(\omega)$  by virtue of (29) are characterized by a measure of distinction of frequency majorants and minorants accordingly on the state and output. Hereinafter the researches are limited by frequency condition number  $C_y(\omega)$  as measure of sensitivity of input-output relation.

**Synthesis of frequency robust systems.** For synthesis of multidimensional frequency robust control systems we shall take advantage of possibilities of modal control in the form, oriented rather input signal. For what we shall formulate the following statement.

**Statement 4.** Let system (9) is obtained by aggregation of control plant, having vector-matrix description

$$\dot{x}(t) = Ax(t) + Bu(t); \quad x(0) = x_0; \quad y(t) = Cx(t) \quad (30)$$

where  $u(t)$  - vector of control,  $A$ ,  $B$  - matrices of the state and control accordingly,  $u \in R^r$ ,  $A \in R^{m \times n}$ ,  $B \in R^{r \times n}$ , and law of modal control, oriented with regard to input signal  $g(t)$ , given in the form

$$u(t) = K_g g(t) - Kx(t) \quad (31)$$

where  $K_g$  is the matrix of direct connection on the vector of the input  $g(t)$ ,  $K$  the matrix of a feed-back on the state vector  $x(t)$   $K_g \in R^{r \times m}$ ,  $K \in R^{r \times n}$  so, that matrices  $F$  and  $G$  of the system (9) are represented in the form

$$F = A - BK, G = BK_g, \quad (32)$$

thus the matrix  $K$  can be calculated by virtue of a matrix relation

$$K = LM^{-1}, \quad (33)$$

where the matrix  $M$  is a solution of the matrix Sylvester equation

$$M\Gamma - AM = -BL, \quad (34)$$

here  $\Gamma$  is the  $(n \times n)$ -dimensional matrix-carrier of a desirable structure of eigenvalues of the system matrix  $F$ ,  $L$  is an arbitrary  $(r \times n)$ -dimensional matrix forming with  $\Gamma$  observable pair; and the matrix  $K_g$  calculated as a result of the decomposition of the matrix  $K$ .

*Proof.* Let's note (33) in the equivalent form

$$L = KM. \quad (35)$$

The substitution (35) in (34) in view of representation of the matrix  $F$  (32) reduces in a matrix relation

$$M\Gamma = FM, \quad (36)$$

being the matrix form of a condition of a similarity of matrices  $\Gamma$  and  $F$ , that supplies to the matrix  $F$  the structure of eigenvalues of the matrix  $\Gamma$ . For a proof of the statement, that the matrix  $K_g$  is calculated as a result of decomposition of the matrix  $K$  we shall note control (31) in two equivalent forms

$$u(t) = K_g g(t) - K_y y(t) - K_x x(t) \quad (37)$$

$$u(t) = K_e e(t) - K_x x(t) \quad (38)$$

where  $e(t) = g(t) - y(t)$  is an error vector of the system on the output,  $K_e$  is the matrix of direct connection on the error,  $K_y$  is the feed-back matrix on an output. Equivalence of representations (37) and (38) of control law (31) reduce to necessity of realization of matrix equality  $K_g = K_y = K_e$ , that because of equivalence (31) and (37), and also with use (30) allows to note a matrix relation

$$\begin{bmatrix} K_g & K_x \end{bmatrix} \begin{bmatrix} C \\ I \end{bmatrix} = K. \quad (39)$$

The statement is proved.

*Note.* The matrix relation (39) is undetermined. Available freedom of decomposition of the matrix  $K$  on the elements  $K_g$  and  $K_x$  can be used for providing of additional properties of

input-output relations. So, if the providing of a condition of single closure of the system on output is required, it is necessary to supplement (39) by a matrix relation

$$-CF^{-1}BK_g = I. \quad (40)$$

The use of possibilities in statement 3 and the procedure of synthesis of oriented modal control for the purposes of designing of multidimensional frequency robust control systems develops in the following statement.

*Statement 5.* The synthesis problem of frequency robust systems in sense of a minimum of frequency condition number  $C_y(\omega)$  of input-output relation, dynamic properties of which is set by an observable pair  $(F, H)$  of modal model, can be formulated and solved as a problem of generalized modal control.

*Proof.* Using an equivalent representation of real-valued transfer function matrix  $\bar{\pi}_y(\omega)$  (26) for input-output relation

$$\bar{\pi}_y(\omega) = CT = C(\omega^2 I + F^2)^{-1} [-F \ \omega I] \text{row} \left\{ \begin{bmatrix} G_i & O \\ O & G_i \end{bmatrix}; i = 1, m \right\} \quad (41)$$

and property of condition number of a matrix  $(\circ)$

$$C\{(\circ)\} = C^{1/2}\{(\circ)(\circ)^T\} \quad (42)$$

that from (41) with the account (42) it is possible to write down

$$C_y(\omega) = C_y\{\bar{\pi}_y(\omega)\} = C^{1/2}\{\bar{\pi}_y(\omega)\bar{\pi}_y(\omega)^T\}. \quad (43)$$

If in (43) to substitute  $\bar{\pi}_y(\omega)$  from (41), and also to take into account property of matrix functions from matrices to keep the ratio of similarity, the expression for  $\bar{\pi}_y(\omega)\bar{\pi}_y(\omega)^T$  can be written down in the form

$$\begin{aligned} \bar{\pi}_y(\omega)\bar{\pi}_y(\omega)^T &= CM(\omega^2 I + \Gamma^2)^{-1} [-\Gamma \ \omega I] M^{-1} \begin{bmatrix} M^{-1}GG^T(M^{-1})^T & O \\ O & M^{-1}GG^T(M^{-1})^T \end{bmatrix} = \\ &= [-\Gamma \ \omega I]^T ((\omega^2 I + \Gamma^2)^{-1})^T M^T C^T \end{aligned} \quad (44)$$

Multiplication of matrix components in (44) with the taking into account (42), and also use of an estimated inequality for condition number of product of matrices in the form  $C\{(\circ)(*)\} \leq C\{(\circ)\}C\{(*)\}$  and property  $C\{(\circ)^{-1}\} = C\{(\circ)\}$  allows to write down

$$C_y(\omega) \leq C^2\{M\} \cdot C\{C\} \cdot C\{G\} \cdot C^{3/2}\{\omega^2 I + \Gamma^2\}. \quad (45)$$

From the ratio (45) it is visible, that minimized functional of robustness of the kind (29) for the input-out relations includes as a factor  $C\{M\}$  condition number of the modal matrix  $M$ . Thus, if the matrix  $F$  is selected as diagonal, the matrix of similarity transformation

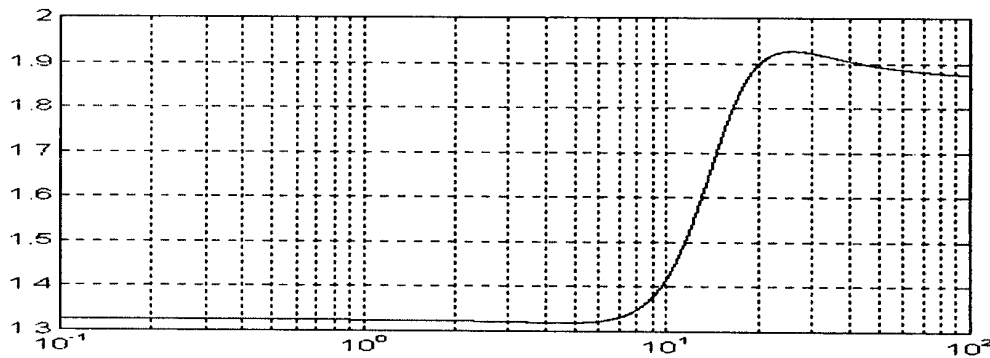
becomes modal, which columns represent of eigenvectors of the state matrix  $F$  of the synthesized multidimensional control system (9). Obviously,  $C\{M\}$  reaches a minimum value with the given structure of eigenvalues, when the geometric spectrum of the eigenvectors of the matrix  $F$  is as much as possible approximate to an orthonormalized system.

It is thus shown, that the problem of providing with frequency robustness of multidimensional systems can be reduced to a generalized problem of modal control, consisting in synthesis of the law of control, supplying to the system state matrix  $F$  desirable structure of eigenvalues and desirable geometrical spectrum of eigenvectors.

**Example.** The continuous two-input multivariable system of the fourth order with the matrices  $F, G, C$

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -236.1965 & -15.4464 & -14.7608 & -14.4921 \\ 0 & 0 & 0 & 1 \\ -2.09560 & 1.6614 & -168.0439 & -14.5536 \end{bmatrix}; G = \begin{bmatrix} 1 & 0 \\ 238200 & 7380 \\ 0 & 0 \\ 2098.6 & 64022 \end{bmatrix}; C = \begin{bmatrix} 0.001 & 0 & 0 & 0 \\ 0 & 0 & 0.002 & 0 \end{bmatrix}$$

is considered as an example. On fig. the curve of frequency condition number  $C_y(\omega)$  of input-output relation is shown.



**Conclusion.** In the paper the synthesis problem of frequency robust systems is solved in a class modal robust systems. However, it is necessary to notice, that  $C_y(\omega)$  is an equivalent element of space  $H^\infty$ , that allows to use opportunities  $H^\infty$ -ideology for the purposes of synthesis of frequency robust systems.

### References

1. Voevodin, V.V. (1977) Computing bases of linear algebra, Nauka, Moscow.
2. Ushakov, A.V. (1992) Modal estimations of processes quality in linear multivariable systems at external finite dimensional input signal // Automatic and telemechanics, N11,
3. Gantmaxer, F.R. (1988) The theory of matrices, Nauka, Moscow.,

# THE ROBUSTNESS ANALYSIS FOR DYNAMIC SYSTEMS WITH INTERVAL PARAMETRIC UNCERTAINTY: FREQUENCY DOMAIN TECHNIQUE DEVELOPMENT

Farit M.Akhmedjanov, Yury M.Gusev, Victor G.Krymsky  
Ufa State Aviation Engineering University  
12 K.Marx Street, Ufa, Bashkortostan, 450000, Russia  
E-mail: pel@admin.ugatu.ac.ru

**Abstract.** The paper is devoted to the problem of dynamic systems (DS) with interval parametric uncertainty investigation. The methodology for DS robust stability analysis using frequency domain technique is developed. The authors' approach is based on the constructing of DS frequency characteristics close upper and lower bounds in relation to open-loop system interval model.

The main conclusions are oriented towards effective application to robust control systems design and analysis.

**Keywords.** dynamic system, interval uncertainty, robust stability, frequency domain.

**Introduction.** The continuous or discrete dynamic systems (DS) creation with regard to parametric uncertainties in a plant model is a problem of paramount importance in different control applications [1]. For example, closed-loop DS for aircraft engines are usually developed in the cases when the consequences of some flight conditions and engines functioning regimes combinations are unknown or imperfectly known. This fact causes the necessity to consider some coefficients in the engine description as uncertain parameters. Several approaches have been proposed to characterize the uncertainty and subsequently deal with various aspects of the noted problem. The main groups of the mentioned approaches can be classified as:

- (i) probabilistic uncertainties influence assessment,
- (ii) fuzzy sets utilization,
- (iii) interval models development and treatment.

The present paper concerns the third group of approaches investigation. This kind of DS uncertainties descriptions allows to estimate system main performances using only the bounds for uncertain parameters values intervals. A considerable amount of literature started from [2] has been devoted to such direction promotion. Meanwhile the great majority of the obtained conclusions correspond to analysis of the whole closed-loop system model. In particular, famous Kharitonov's theorems [2] on DS robust stability assume that all the coefficients of the whole system interval characteristic polynomial have been calculated. In the real world, however, the initial information on DS behaviour can be composed of the plant model (including interval parametric uncertainty) and of the controller model (containing parameters with fixed values). So the problem how to find the closed-loop system resulting interval characteristics using the open-loop DS components description appears. The attempts to fulfil direct calculation of closed-loop DS dynamic equations coefficients for the case of plant model interval uncertainty cause a lot of specific difficulties:

- (i) the sequence of designer's operations becomes awkward,
- (ii) the convergence of some computing algorithms cannot be guaranteed,
- (iii) the length of uncertain parameters intervals essentially increases during step-by-step data conversion process (this circumstance causes erroneous estimation of system performances).

The noted difficulties can be overcome in many practical situations if frequency domain technique is used. Interval analogues of Nyquist methodology are able to create a foundation for closed-loop DS specific features investigation based only on the open-loop system components description (without any additional data treatment). Some important results regarding this fruitful way are obtained in the publications [3,4]. It is necessary to take into account that the mentioned results are oriented towards gain-phase loci fields constructing in the complex plane. The computing procedures for such fields boundaries determination become too complicated.

Here the authors propose to transform the analysis technique for a possibility to operate with usual real functions: gain-frequency and phase-frequency characteristics. This approach can be considered as more suitable for current applications.

**Interval models and frequency domain characteristics.** Let open-loop DS system with single input and single output can be described by the transfer function  $W(s)$ , where  $s$  is Laplace transform variable.

In the case of DS interval model

$$W(s) = \frac{\sum_{i=0}^m a_i s^i}{\sum_{i=0}^n b_i s^i}, \quad (1)$$

where the coefficients  $a_i, i \in \overline{0, m}$ ;  $b_i, i \in \overline{0, n}$  of numerator and denominator polynomials belong to corresponding intervals  $[a_i, \bar{a}_i], [b_i, \bar{b}_i]$  with fixed lower  $a_i, b_i$  and upper  $\bar{a}_i, \bar{b}_i$  bounds. Only the mentioned bounds are assumed to be known to designer.

The frequency domain characteristics for open-loop DS can be obtained by the substitution of  $s=j\omega$  into  $W(s)$  (1). Meanwhile

$$W(j\omega) = \text{Re}(\omega) + j \text{Im}(\omega) = H(\omega) \exp(-j\Theta(\omega)). \quad (2)$$

Here  $\text{Re}(\omega), \text{Im}(\omega)$  are the real and imaginary parts of  $W(j\omega)$ ;

$$H(\omega) = \sqrt{\text{Re}^2(\omega) + \text{Im}^2(\omega)} \quad (3)$$

is a gain-frequency characteristic;

$$\Theta(\omega) = \text{Arctg}\{\text{Im}(\omega)/\text{Re}(\omega)\} \quad (4)$$

is the phase-frequency characteristic.

With regard to the fact that  $W(j\omega)$  contains the so called "interval parameters"  $a_i, i \in \overline{0, m}$ ,  $b_i, i \in \overline{0, n}$ , all types of frequency characteristics for each fixed value  $\omega=\omega^0$  become also the interval numbers with corresponding lower and upper bounds. For example,  $\text{Re}(\omega^0) \in [\underline{\text{Re}}(\omega^0), \bar{\text{Re}}(\omega^0)], \underline{\text{Re}}(\omega^0) \leq \bar{\text{Re}}(\omega^0)$ .

A useful idea which allows to calculate close upper and lower bounds for many types of characteristics is based on the results of interval polynomials depending on  $s=j\omega$  properties analysis. Let

$$P(s) = p_0 + p_1 s + p_2 s^2 + \dots \quad (5)$$

be the polynomial with interval coefficients  $p_i \in [\underline{p}_i, \bar{p}_i], i = 0, 1, 2, \dots$ , where  $\underline{p}_i, \bar{p}_i$  are the real positive numbers ( $\underline{p}_i \leq \bar{p}_i$ ).

After substitution  $s=j\omega$  we can obtain

$$P(j\omega) = \text{Re}(P) + j\text{Im}(P). \quad (6)$$

Here  $\text{Re}(P)$  and  $\text{Im}(P)$  are the real part and the imaginary part of  $P(j\omega)$ :

$$\text{Re}(P) = p_0 - p_2\omega^2 + p_4\omega^4 - p_6\omega^6 + \dots, \quad (7)$$

$$\text{Im}(P) = \omega(p_1 - p_3\omega^2 + p_5\omega^4 - p_7\omega^6 + \dots). \quad (8)$$

The consideration of expressions (7), (8) gives us a possibility to find upper ( $\overline{\text{Re}}(P)$ ,  $\overline{\text{Im}}(P)$ ) and lower ( $\underline{\text{Re}}(P)$ ,  $\underline{\text{Im}}(P)$ ) bounds for  $\text{Re}(P)$ ,  $\text{Im}(P)$  [5]:

$$\underline{\text{Re}}(P) = \underline{p}_0 - \underline{p}_2\omega^2 + \underline{p}_4\omega^4 - \underline{p}_6\omega^6 + \dots; \quad (9)$$

$$\overline{\text{Re}}(P) = \overline{p}_0 - \overline{p}_2\omega^2 + \overline{p}_4\omega^4 - \overline{p}_6\omega^6 + \dots; \quad (10)$$

$$\underline{\text{Im}}(P) = \omega(\underline{p}_1 - \underline{p}_3\omega^2 + \underline{p}_5\omega^4 - \underline{p}_7\omega^6 + \dots); \quad (11)$$

$$\overline{\text{Im}}(P) = \omega(\overline{p}_1 - \overline{p}_3\omega^2 + \overline{p}_5\omega^4 - \overline{p}_7\omega^6 + \dots). \quad (12)$$

Using (9)-(12) one has an opportunity to consider frequency characteristics of interval system as specific "tubes"; the inside field of such arbitrary "tube" corresponds to all probable combinations of uncertain parameters. For example, consider the gain- frequency characteristic. The upper and lower bounds of this function values are represented by equations:

$$\overline{H}(\omega) = \sqrt{\frac{(\overline{\text{Mod}}[\text{Re}_a(\omega)])^2 + (\overline{\text{Mod}}[\text{Im}_a(\omega)])^2}{(\underline{\text{Mod}}[\text{Re}_b(\omega)])^2 + (\underline{\text{Mod}}[\text{Im}_b(\omega)])^2}}, \quad (13)$$

$$\underline{H}(\omega) = \sqrt{\frac{(\underline{\text{Mod}}[\text{Re}_a(\omega)])^2 + (\underline{\text{Mod}}[\text{Im}_a(\omega)])^2}{(\overline{\text{Mod}}[\text{Re}_b(\omega)])^2 + (\overline{\text{Mod}}[\text{Im}_b(\omega)])^2}}, \quad (14)$$

where  $\text{Re}_a(\omega)$ ,  $\text{Im}_a(\omega)$  are real and imaginary parts of  $W(s)$  numerator after substitution  $s=j\omega$ ;  $\text{Re}_b(\omega)$ ,  $\text{Im}_b(\omega)$  are analogous functions calculated for  $W(s)$  denominator;  $\underline{\text{Mod}}[\bullet]$ ,  $\overline{\text{Mod}}[\bullet]$  are lower and upper bounds of the corresponding function module. For example, if  $\text{Mod}[\mu(\omega)]$  denotes the module of arbitrary real function  $\mu(\omega)$  with upper  $\overline{\mu}(\omega)$  and lower  $\underline{\mu}(\omega)$  bounds, then

$$\overline{\text{Mod}}[\mu(\omega)] = \max\{\text{Mod}[\underline{\mu}(\omega)], \text{Mod}[\overline{\mu}(\omega)]\};$$

$$\underline{\text{Mod}}[\mu(\omega)] = \min\{\text{Mod}[\underline{\mu}(\omega)], \text{Mod}[\overline{\mu}(\omega)]\} \quad \text{if } \overline{\mu}(\omega) \times \underline{\mu}(\omega) > 0;$$

$$\underline{\text{Mod}}[\mu(\omega)] = 0 \quad \text{if } \overline{\mu}(\omega) \times \underline{\mu}(\omega) \leq 0.$$

The resulting "tube" is shown in Fig. 1.

This "tube" can be also represented via the logarithmic scales for axes.

The phase frequency function must be constructed as the difference between the corresponding functions for  $W(s)$  numerator and denominator polynomials after substitution  $s=j\omega$ . So it is necessary to investigate the phase behaviour for arbitrary interval polynomial (5). According to (6) analysis results the upper  $\overline{\Theta}(P)$  and lower  $\underline{\Theta}(P)$  bounds for the phase  $\Theta(P)$  of the noted polynomial can be calculated when the signs of  $\text{Re}(P)$  and  $\text{Im}(P)$  have been taken into account. Here 9 special cases are to be considered.

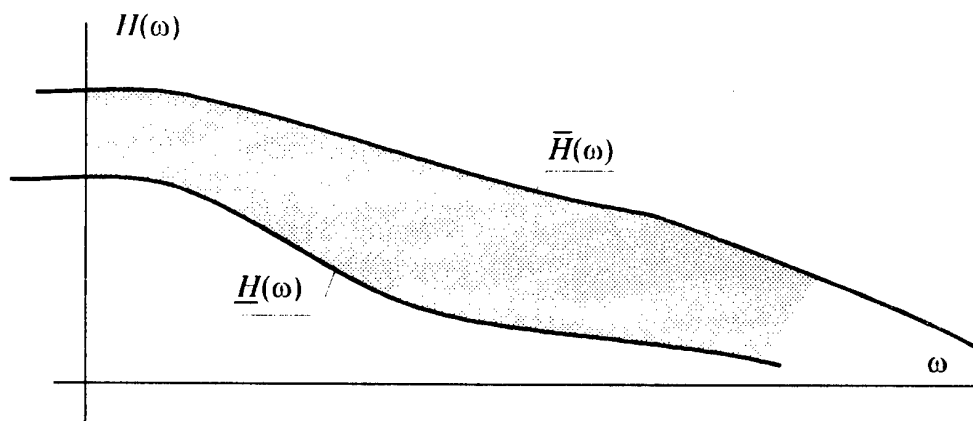
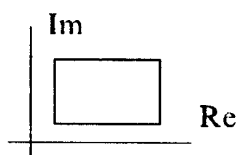


Fig.1 The "tube" of system gain-frequency characteristics

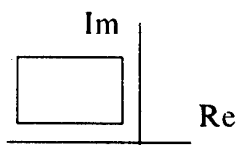
Situation 1. Upper and lower bounds of  $\text{Re}(P)$ ,  $\text{Im}(P)$  for concrete value of  $\omega$  are positive numbers.



$$\underline{\Theta}(P) = \text{Arctg}(\underline{\text{Im}}(P)/\underline{\text{Re}}(P));$$

$$\overline{\Theta}(P) = \text{Arctg}(\overline{\text{Im}}(P)/\overline{\text{Re}}(P)).$$

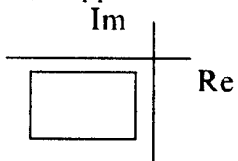
Situation 2. Upper and lower bounds of  $\text{Re}(P)$  are negative but upper and lower bounds of  $\text{Im}(P)$  are positive numbers.



$$\underline{\Theta}(P) = \pi + \text{Arctg}(\overline{\text{Im}}(P)/\overline{\text{Re}}(P));$$

$$\overline{\Theta}(P) = \pi + \text{Arctg}(\underline{\text{Im}}(P)/\underline{\text{Re}}(P)).$$

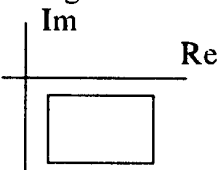
Situation 3. Upper and lower bounds of  $\text{Re}(P)$ ,  $\text{Im}(P)$  are negative numbers.



$$\underline{\Theta}(P) = \pi + \text{Arctg}(\overline{\text{Im}}(P)/\overline{\text{Re}}(P));$$

$$\overline{\Theta}(P) = \pi + \text{Arctg}(\underline{\text{Im}}(P)/\underline{\text{Re}}(P)).$$

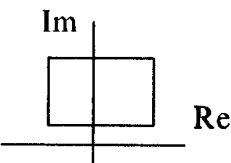
Situation 4. Upper and lower bounds of  $\text{Re}(P)$  are positive but upper and lower bounds of  $\text{Im}(P)$  are negative numbers.



$$\underline{\Theta}(P) = 2\pi + \text{Arctg}(\underline{\text{Im}}(P)/\underline{\text{Re}}(P));$$

$$\overline{\Theta}(P) = 2\pi + \text{Arctg}(\overline{\text{Im}}(P)/\overline{\text{Re}}(P)).$$

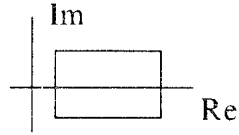
Situation 5. Upper and lower bounds of  $\text{Im}(P)$  are positive numbers; upper bound of  $\text{Re}(P)$  is positive number but lower bound of  $\text{Re}(P)$  is a negative number.



$$\underline{\Theta}(P) = \text{Arctg}(\underline{\text{Im}}(P)/\underline{\text{Re}}(P));$$

$$\overline{\Theta}(P) = \pi + \text{Arctg}(\overline{\text{Im}}(P)/\overline{\text{Re}}(P)).$$

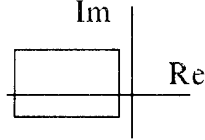
Situation 6. Upper and lower bounds of  $\text{Re}(P)$  are positive numbers; upper bound of  $\text{Im}(P)$  is positive number but lower bound of  $\text{Im}(P)$  is a negative number.



$$\underline{\Theta}(P) = 2\pi + \text{Arctg}(\underline{\text{Im}}(P)/\underline{\text{Re}}(P));$$

$$\overline{\Theta}(P) = 2\pi + \text{Arctg}(\overline{\text{Im}}(P)/\underline{\text{Re}}(P)).$$

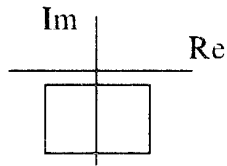
Situation 7. Upper and lower bounds of  $\text{Re}(P)$  are negative numbers; upper bound of  $\text{Im}(P)$  is positive number but lower bound of  $\text{Im}(P)$  is a negative number.



$$\underline{\Theta}(P) = \pi + \text{Arctg}(\overline{\text{Im}}(P)/\overline{\text{Re}}(P));$$

$$\overline{\Theta}(P) = \pi + \text{Arctg}(\underline{\text{Im}}(P)/\overline{\text{Re}}(P)).$$

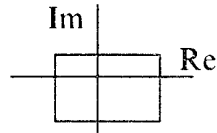
Situation 8. Upper and lower bounds of  $\text{Im}(P)$  are negative numbers; upper bound of  $\text{Re}(P)$  is positive number but lower bound of  $\text{Re}(P)$  is a negative number.



$$\underline{\Theta}(P) = \pi + \text{Arctg}(\overline{\text{Im}}(P)/\underline{\text{Re}}(P));$$

$$\overline{\Theta}(P) = 2\pi + \text{Arctg}(\overline{\text{Im}}(P)/\overline{\text{Re}}(P)).$$

Situation 9. Upper bounds of  $\text{Re}(P)$  and  $\text{Im}(P)$  are positive numbers but lower bounds of  $\text{Re}(P)$  and  $\text{Im}(P)$  are negative numbers.



$$\underline{\Theta}(P) = 0;$$

$$\overline{\Theta}(P) = 2\pi.$$

Here  $\underline{\text{Re}}(P)$ ,  $\overline{\text{Re}}(P)$ ,  $\underline{\text{Im}}(P)$ ,  $\overline{\text{Im}}(P)$  correspond to the expressions (9) - (12).

The described investigation gives a possibility to calculate the "tube" of phases for any arbitrary polynomial and given interval of frequency  $\omega$  values. Now using the denotation

$$W(s) = P_1(s)/P_2(s), \quad (15)$$

where  $P_1(s)$ ,  $P_2(s)$  are the polynomials of the noted transfer function numerator and denominator, one can obtain

$$\underline{\Theta}(\omega) = \underline{\Theta}(P_1) - \overline{\Theta}(P_2); \quad (16)$$

$$\overline{\Theta}(\omega) = \overline{\Theta}(P_1) - \underline{\Theta}(P_2). \quad (17)$$

In the equations (16), (17)  $\underline{\Theta}(\omega)$  and  $\overline{\Theta}(\omega)$  denote lower and upper bounds for  $\Theta(\omega)$  respectively.

The "tube" of interval open-loop system phase-frequency characteristics is illustrated by Fig.2. Now we can analyze the performances of closed-loop system. In order to consider this DS with uncertainty robust stability problem the special version of Nyquist approach has been developed.

**Nyquist methodology version for the systems with interval parametric uncertainty.** Further contents corresponds to the case when polynomials  $P_1(s)$ ,  $P_2(s)$  in (15) have not roots in the right half of complex plane. Note that the test for this requirement to be satisfied can be realized by means of Kharitonov's result [2] easily.

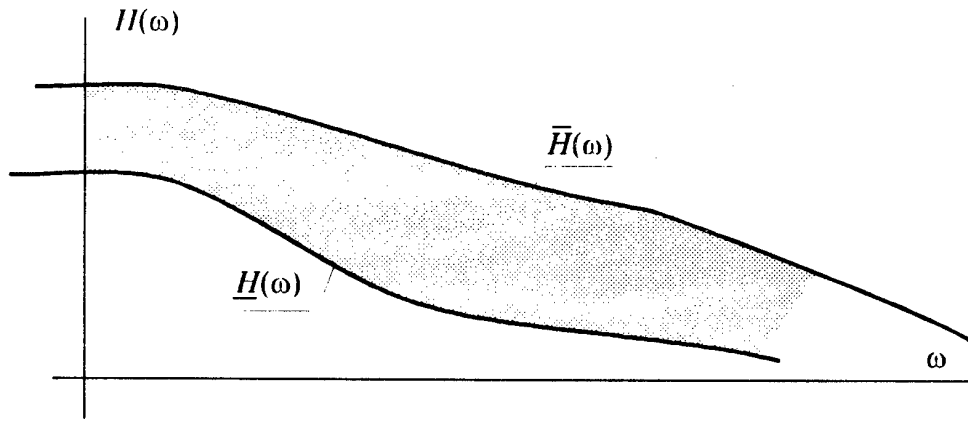
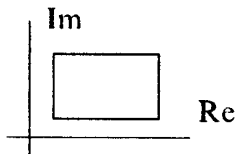


Fig. 1 The "tube" of system gain-frequency characteristics

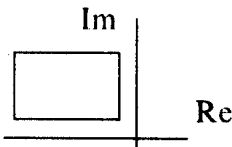
Situation 1. Upper and lower bounds of  $\text{Re}(P)$ ,  $\text{Im}(P)$  for concrete value of  $\omega$  are positive numbers.



$$\underline{\Theta}(P) = \text{Arctg}(\underline{\text{Im}}(P)/\underline{\text{Re}}(P));$$

$$\overline{\Theta}(P) = \text{Arctg}(\overline{\text{Im}}(P)/\overline{\text{Re}}(P)).$$

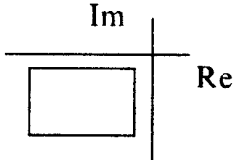
Situation 2. Upper and lower bounds of  $\text{Re}(P)$  are negative but upper and lower bounds of  $\text{Im}(P)$  are positive numbers.



$$\underline{\Theta}(P) = \pi + \text{Arctg}(\overline{\text{Im}}(P)/\overline{\text{Re}}(P));$$

$$\overline{\Theta}(P) = \pi + \text{Arctg}(\underline{\text{Im}}(P)/\underline{\text{Re}}(P)).$$

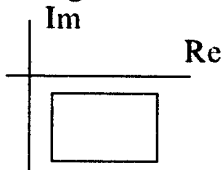
Situation 3. Upper and lower bounds of  $\text{Re}(P)$ ,  $\text{Im}(P)$  are negative numbers.



$$\underline{\Theta}(P) = \pi + \text{Arctg}(\overline{\text{Im}}(P)/\overline{\text{Re}}(P));$$

$$\overline{\Theta}(P) = \pi + \text{Arctg}(\underline{\text{Im}}(P)/\underline{\text{Re}}(P)).$$

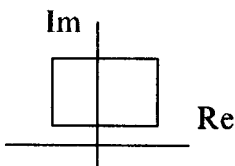
Situation 4. Upper and lower bounds of  $\text{Re}(P)$  are positive but upper and lower bounds of  $\text{Im}(P)$  are negative numbers.



$$\underline{\Theta}(P) = 2\pi + \text{Arctg}(\underline{\text{Im}}(P)/\underline{\text{Re}}(P));$$

$$\overline{\Theta}(P) = 2\pi + \text{Arctg}(\overline{\text{Im}}(P)/\overline{\text{Re}}(P)).$$

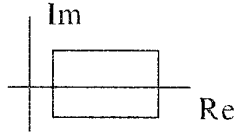
Situation 5. Upper and lower bounds of  $\text{Im}(P)$  are positive numbers; upper bound of  $\text{Re}(P)$  is positive number but lower bound of  $\text{Re}(P)$  is a negative number.



$$\underline{\Theta}(P) = \text{Arctg}(\underline{\text{Im}}(P)/\underline{\text{Re}}(P));$$

$$\overline{\Theta}(P) = \pi + \text{Arctg}(\overline{\text{Im}}(P)/\overline{\text{Re}}(P)).$$

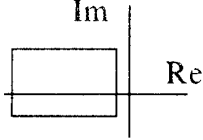
Situation 6. Upper and lower bounds of  $\text{Re}(P)$  are positive numbers; upper bound of  $\text{Im}(P)$  is positive number but lower bound of  $\text{Im}(P)$  is a negative number.



$$\underline{\Theta}(P) = 2\pi + \text{Arctg}(\underline{\text{Im}}(P)/\underline{\text{Re}}(P));$$

$$\overline{\Theta}(P) = 2\pi + \text{Arctg}(\overline{\text{Im}}(P)/\underline{\text{Re}}(P)).$$

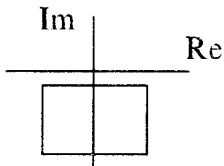
Situation 7. Upper and lower bounds of  $\text{Re}(P)$  are negative numbers; upper bound of  $\text{Im}(P)$  is positive number but lower bound of  $\text{Im}(P)$  is a negative number.



$$\underline{\Theta}(P) = \pi + \text{Arctg}(\overline{\text{Im}}(P)/\overline{\text{Re}}(P));$$

$$\overline{\Theta}(P) = \pi + \text{Arctg}(\underline{\text{Im}}(P)/\overline{\text{Re}}(P)).$$

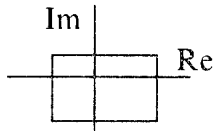
Situation 8. Upper and lower bounds of  $\text{Im}(P)$  are negative numbers; upper bound of  $\text{Re}(P)$  is positive number but lower bound of  $\text{Re}(P)$  is a negative number.



$$\underline{\Theta}(P) = \pi + \text{Arctg}(\overline{\text{Im}}(P)/\underline{\text{Re}}(P));$$

$$\overline{\Theta}(P) = 2\pi + \text{Arctg}(\underline{\text{Im}}(P)/\underline{\text{Re}}(P)).$$

Situation 9. Upper bounds of  $\text{Re}(P)$  and  $\text{Im}(P)$  are positive numbers but lower bounds of  $\text{Re}(P)$  and  $\text{Im}(P)$  are negative numbers.



$$\underline{\Theta}(P) = 0;$$

$$\overline{\Theta}(P) = 2\pi.$$

Here  $\underline{\text{Re}}(P)$ ,  $\overline{\text{Re}}(P)$ ,  $\underline{\text{Im}}(P)$ ,  $\overline{\text{Im}}(P)$  correspond to the expressions (9) - (12).

The described investigation gives a possibility to calculate the "tube" of phases for any arbitrary polynomial and given interval of frequency  $\omega$  values. Now using the denotation

$$W(s) = P_1(s)/P_2(s), \quad (15)$$

where  $P_1(s)$ ,  $P_2(s)$  are the polynomials of the noted transfer function numerator and denominator, one can obtain

$$\underline{\Theta}(\omega) = \underline{\Theta}(P_1) - \overline{\Theta}(P_2); \quad (16)$$

$$\overline{\Theta}(\omega) = \overline{\Theta}(P_1) - \underline{\Theta}(P_2). \quad (17)$$

In the equations (16), (17)  $\underline{\Theta}(\omega)$  and  $\overline{\Theta}(\omega)$  denote lower and upper bounds for  $\Theta(\omega)$  respectively.

The "tube" of interval open-loop system phase-frequency characteristics is illustrated by Fig.2. Now we can analyze the performances of closed-loop system. In order to consider this DS with uncertainty robust stability problem the special version of Nyquist approach has been developed.

**Nyquist methodology version for the systems with interval parametric uncertainty.** Further contents corresponds to the case when polynomials  $P_1(s)$ ,  $P_2(s)$  in (15) have not roots in the right half of complex plane. Note that the test for this requirement to be satisfied can be realized by means of Kharitonov's result [2] easily.

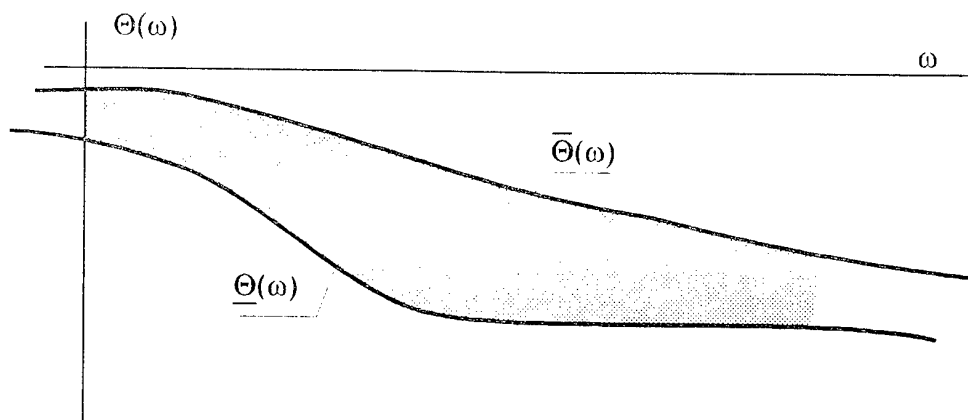


Fig.2 The "tube" of system phase-frequency characteristics

Compare the "tubes" placement for the open-loop DS characteristics  $H(\omega)$  and  $\Theta(\omega)$  in relation to similar logarithmic scales along frequency axes (Fig.3).

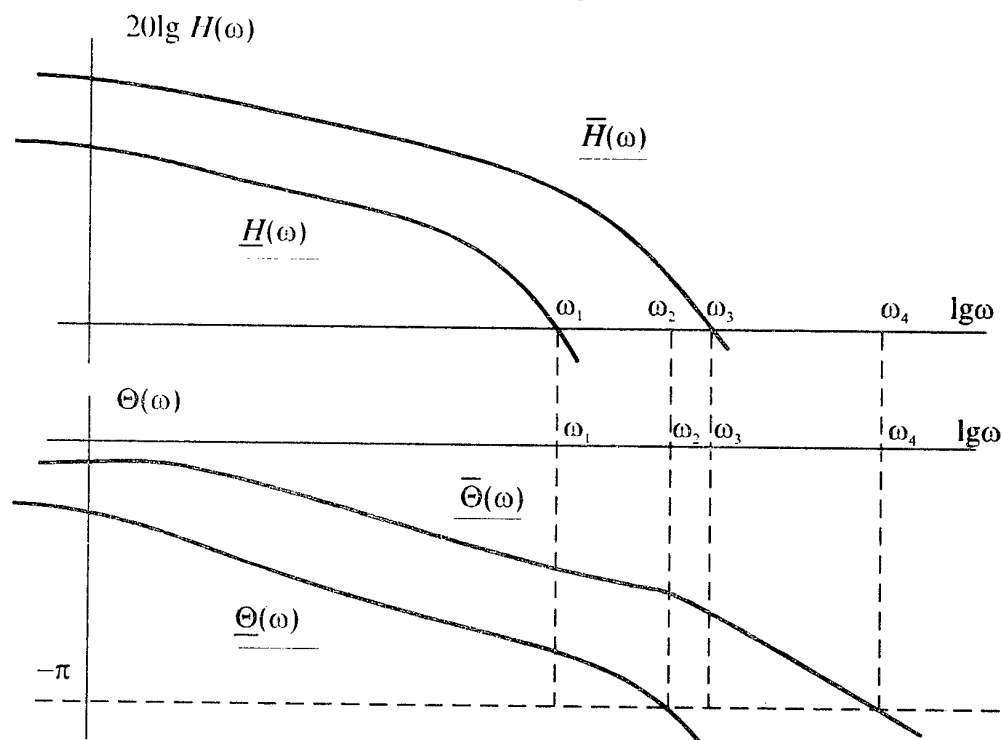


Fig.3 The "tubes" of open-loop DS frequency characteristics

According to well-known Nyquist stability criterion for the case

$$\omega_3 < \omega_2 < \omega_4$$

the closed-loop DS will possess stability in relation to all possible combinations of uncertain parameters values. In other words, the system is robustly stable [1,2].

If

$$\omega_2 < \omega_4 < \omega_1$$

then the closed-loop system will be always unstable.

If

$$\omega_1 < \omega_2 < \omega_4 < \omega_3$$

there are parameters values for which DS is unstable.  
At last if

$$\omega_1 < \omega_2 < \omega_3 < \omega_4$$

or

$$\omega_2 < \omega_1 < \omega_4 < \omega_3$$

it is difficult to form final conclusion because  $H(\omega)$  and  $\Theta(\omega)$  are influenced by the same parameters and we don't know the real correspondence between concrete gain and phase frequency characteristics values.

The additional investigation procedure can be developed specially for the mentioned situation. Let's introduce the functions

$$A(\omega) = \text{Re}_a^2(\omega) + \text{Im}_a^2(\omega); \quad (18)$$

$$B(\omega) = \text{Re}_b^2(\omega) + \text{Im}_b^2(\omega). \quad (19)$$

These functions totalities close lower  $\underline{A}(\omega), \underline{B}(\omega)$  and upper  $\overline{A}(\omega), \overline{B}(\omega)$  bounds can be obtained as

$$\underline{A}(\omega) = (\underline{\text{Mod}}[\text{Re}_a(\omega)])^2 + (\underline{\text{Mod}}[\text{Im}_a(\omega)])^2; \quad (20)$$

$$\overline{A}(\omega) = (\overline{\text{Mod}}[\text{Re}_a(\omega)])^2 + (\overline{\text{Mod}}[\text{Im}_a(\omega)])^2; \quad (21)$$

$$\underline{B}(\omega) = (\underline{\text{Mod}}[\text{Re}_b(\omega)])^2 + (\underline{\text{Mod}}[\text{Im}_b(\omega)])^2; \quad (22)$$

$$\overline{B}(\omega) = (\overline{\text{Mod}}[\text{Re}_b(\omega)])^2 + (\overline{\text{Mod}}[\text{Im}_b(\omega)])^2. \quad (23)$$

Consider also the additional variable  $x$  which values belong to the interval  $[\underline{x}, \overline{x}]$  with the bounds

$$\underline{x} = \max\{\underline{A}(\omega), \underline{B}(\omega)\}; \quad (24)$$

$$\overline{x} = \min\{\overline{A}(\omega), \overline{B}(\omega)\}. \quad (25)$$

The condition

$$A(\omega) = B(\omega) = x \quad (26)$$

satisfaction causes

$$H(\omega) = 1. \quad (27)$$

So the provision of (26) means that  $\omega = \omega_c$ , where  $\omega_c$  is cut-off frequency for open-loop DS characteristics.

Consider two planes: the first plane with axes  $\text{Re}_a(\omega), \text{Im}_a(\omega)$  and the second plane with axes  $\text{Re}_b(\omega), \text{Im}_b(\omega)$ . The rectangles of the noted variables possible values in the corresponding planes are shown in Fig.4.

The equations  $A(\omega) = x$  and  $B(\omega) = x$  become the sources for the circles with radius  $r = \sqrt{x}$  to appear.

It is clear that arbitrary circle counterclockwise going around (inside the noted rectangles) provides the conditions for  $\Theta_1(\omega) = \text{Arctg}(\text{Im}_a(\omega)/\text{Re}_a(\omega))$  and  $\Theta_2(\omega) = \text{Arctg}(\text{Im}_b(\omega)/\text{Re}_b(\omega))$  monotonous increase.

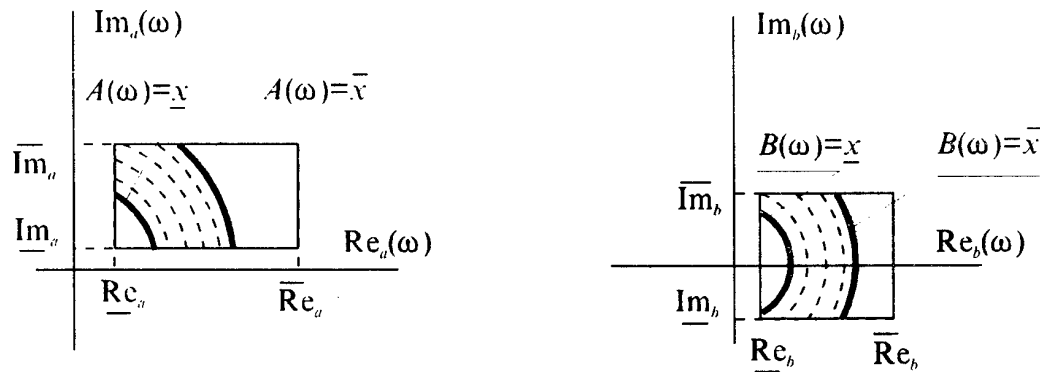


Fig.4 Additional investigation technique: rectangles and circles in the planes of chosen variables

This fact allows to realize the following operations sequence in order to find  $\Theta(\omega)$ . For some  $x$  fixed value the points of the circles with  $r = \sqrt{x}$  and of the rectangles boundaries intersection must be obtained for both planes. By phase values comparison in relation to the mentioned intersection points one can obtain  $\inf\{\Theta_1(\omega, x)\}$  and  $\sup\{\Theta_2(\omega, x)\}$ . So

$$\Theta(\omega, x) = \inf\{\Theta_1(\omega, x)\} - \sup\{\Theta_2(\omega, x)\}, \quad (28)$$

where  $\Theta(\omega, x)$  is the value of  $\Theta(\omega)$  with regard to fixed value of  $x$  in (26). If

$$\Theta(\omega, x) < -\pi \quad (29)$$

then the system is unstable.

Scanning the whole interval  $[\underline{x}, \bar{x}]$  and using these operations one can verify the inequality (29) satisfaction. In the case when it is not satisfied for all  $x \in [\underline{x}, \bar{x}]$  the system is robustly stable.

Finally the procedure of the additional frequency characteristics "tubes" investigation is based on the following stages:

- 1) the frequencies interval  $[\underline{\omega}, \bar{\omega}]$  (Fig.3) must be scanned with the step of  $\Delta\omega$ ;
- 2) for each value of  $\omega$  the bounds  $\underline{x}$  and  $\bar{x}$  of the interval  $[\underline{x}, \bar{x}]$  in (24), (25) must be found;
- 3) for the noted value of  $\omega$  and each  $x$  from  $[\underline{x}, \bar{x}]$  obtained using the step  $\Delta x$  -
  - (i) circles with radius  $r = \sqrt{x}$  at the both planes must be constructed;
  - (ii) the intersection points with corresponding rectangles boundaries must be computed;
  - (iii) the values of  $\Theta(\omega, x)$  must be obtained and the inequalities of (29) type must be verified.

It is necessary to note that steps  $\Delta\omega$  and  $\Delta x$  values can be chosen with regard to results accuracy and computing complexity requirements.

Now we have an opportunity to consider robust stability problem in relation to DS standard configuration with the serial interconnection between plant and controller (Fig.5, where  $Z(s)$  is the transfer function of the plant with uncertain parameters,  $R(s)$  is the analogous function of the controller with fixed parameters).

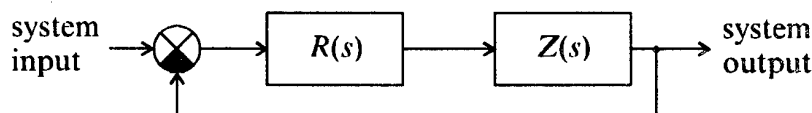


Fig.5 Closed-loop dynamic system standard configuration

In the frequency domain

$$Z(j\omega) = H_z(\omega) \exp(j\Theta_z(\omega)), \quad (30)$$

$$R(j\omega) = H_R(\omega) \exp(j\Theta_R(\omega)). \quad (31)$$

Here  $H_z(\omega)$ ,  $H_R(\omega)$  are the gain-frequency characteristics of the plant and controller respectively;  $\Theta_z(\omega)$ ,  $\Theta_R(\omega)$  are the correspondent phase-frequency characteristics. For the mentioned problem state  $H_z \in [\underline{H}_z, \overline{H}_z]$ ,  $\Theta_z \in [\underline{\Theta}_z, \overline{\Theta}_z]$ , where  $\underline{H}_z, \overline{H}_z, \underline{\Theta}_z, \overline{\Theta}_z$  are the fixed bounds of the intervals to be considered.

So for the close lower bound of open-loop phase-frequency characteristic  $\Theta(\omega)$  it is easy to obtain

$$\underline{\Theta}(\omega) = \underline{\Theta}_R(\omega) + \underline{\Theta}_z(\omega). \quad (32)$$

Analysis of  $\underline{\Theta}(\omega)$  behaviour can sometimes require the additional investigation (as it is shown in Fig.3).

To overcome the difficulty it is inherent to use the above stated methodology with regard to some specific features. In particular, the circles in the planes of  $(\text{Re}_a(\omega), \text{Im}_a(\omega))$  and  $(\text{Re}_b(\omega), \text{Im}_b(\omega))$  must have different radii now: the circles in the first plane must correspond to  $r_1 = \sqrt{x}/H_R(\omega)$  and the circles in the second plane must correspond to  $r_2 = \sqrt{x}$ .

In other aspects the system (Fig.5) analysis algorithm and above stated procedure are similar.

**The perspectives of proposed approach utilization.** The proposed approach can be considered as the foundation not only for the robust stability problems solving. The main results can be also applied to DS with uncertainties control quality estimation. This fact gives a possibility to create interval system design algorithms oriented towards frequency domain technique utilization.

Today the authors have developed the procedures which allow to obtain robust stability margins for gain and phase frequency characteristics. So the important system performances indices intervals (in particular, for settling time or damping ratios) can be constructed.

Design algorithms consider the requirements regarding system with uncertainty dynamic behaviour from view point of robust stability margins limitations. That is why controller design problem is proposed to be solved by step-by-step change of the open-loop DS frequency characteristics "tubes" placement.

Finally the developed approach gives some new possibilities for systems with uncertainty simulation technique. The noted DS are investigated as the compositions of subsystems whose inputs and outputs are represented by frequency characteristics "tubes". The designer can estimate the bounds of different system time responses totality.

According to the obtained conclusions authors decided to introduce the developed frequency domain algorithms into universal programming package "Robustness-1". This package aims to provide the support of decisions regarding DS with interval uncertainties design and analysis [6].

The accumulated experience in "Robustness-1" exploitation confirms the efficiency of the proposed approaches.

## References

1. Gusev Yu.M., Efanov V.N., Krymsky V.G., Rutkowsky V.Yu. Analysis and Design of Linear Interval Dynamic Systems (The State of the Problem)// Izv. AN SSSR. Techn. kibernetika. No.1, No.2. 1991.

2. Kharitonov V.L. Asymptotic Stability of an Equilibrium Position of a Family of Systems of Linear Differential Equations// Differ. Uravnen. V.14. No.11. 1978.
3. Polyak B.T., Tsytkin Ya.Z. Nyquist Robust Criterion// Automatica and Telemechanics. No.7. 1992.
4. Le Khung Lan. Robust Stability Modified Frequency Criterion for Closed-Loop Systems// Automatica and Telemechanics. No.8. 1993.
5. Barmish B.R. Generalization of Kharitonov's Four Polynomial Concept for Robust Stability Problems with Linearly Dependent Coefficient Perturbations// IEEE Transaction on Autom. Contr. V.34. No.2. 1989.
6. Akhmedjanov F.M., Krymsky V.G. Risk-Based Design of Complex Control Systems: Uncertainties Interval Models Utilization// American Society of Mechanical Engineers Proceedings. V.PVP-320. New York: United Engineering Centre. 1995.

# PARAMETRICAL SYNTHESIS OF THE CONTROL SYSTEM IN THE PROCESS OF THE IMITATIVE TESTS

Oleg Degtyarjov, Valentin Evstifeyev  
Moscow State Technical University  
named by Bauman  
5, Second Baumanskaya Street  
Moscow, 107005, Russia

**Abstract.** The stage of synthesis is considered following the initial stage of the control system analytical synthesis. The structure and initial value of regulator parameters of the control system are determined after the first stages. The goal of next stage is precisising of these parameters by means of using the complete mathematical model of the control system during imitative modelling (simulation computer). Algorithms of the optimal parametrical synthesis of the control systems during simulation testing or developmental tests are offered.

**Keywords.** Parametrical synthesis, simulation computer, nonlinear programming, optimization.

The control system structure is defined and the regulator parameters are determined at the initial stages of the synthesis. For this purpose their simplified mathematical models are used. So the determined values of the regulator parameters not always fulfil the technical demands imposed on the control system under all possible conditions of work. It is typical, for example, for the aircraft control systems, taking account of vibrations of elastic elements and liquid waves in tanks. The sensitivity for the parameter change of these systems is especially large and a-priory data are not exact. To decrease the control system developmental tests it is necessary to carry out the following stage of the system parametrical synthesis to the full extent as possible. For this purpose the methods of imitative and semi-natural modelling are used as well as the rational test organization.

This report deals with a number of procedures and algorithms of the imitative test organization to solve the optimization tasks of the parametrical synthesis of the control system. In its turn in a real system must be fulfilled all restrictions concerning the parameters of its functioning under all possible conditions of work. The influence of the all disturbance factors should be taken account of as well.

The first algorithm group solves the parametrical synthesis problems of the control system in the imitative modelling process.

The technical problem of the parametrical synthesis of the control system [1,2] may be formulated as optimization problem of nonlinear programming

$$\min_{(k)} \{ f(k) : q_i(k) + F_{\text{set}_i} \leq 0, \quad i = 1, 2, \dots, I \}, \quad (1)$$

where  $f$  - the scalar criterion of optimality, that is the function of the vector of synthesized parameters  $k$ , it has dimensions  $n_k$ ;  $q_i = p_i - B_i$ ;  $p_i$  - the characteristic number  $i$  of the control system, its upper range is set up by the extremely allowable values  $B_i$ ;  $F_{\text{set}_i} \geq 0$  - the "margin", that is set in the synthesis of the system to fulfil restriction  $i$ . The value of the "margin" is

chosen when the influence of the various factors on the corresponding parameter of the system functioning is investigated. These factors are not taken account of in the control system model and are considered, for example, in [3]. The choice of the size  $F_{\text{set}_i}$  depends on a concrete system and is not considered in this report.

The formulation of the optimization problem (1) allows to transfer vector of the synthesized parameters  $k$  inside the allowable value area with the required "margin". This "margin" fulfils all restrictions - inequalities when the factors act upon, that are not taken account of in the system model. In its turn in the "rigid" area of the allowable parameter values the size of criterion  $f(k)$  appears the least.

The procedure of the problem decision (1) when carried out specially organized imitative tests is based on linearization method [1,4] and is iterative. First decision  $d$  of the auxiliary problem of the square-law programming is solved

$$\begin{aligned} \min_{(d)} \{ f^l(k^{(r)}, d) + 0,5 \|d\|^2 : (q_i^l(k^{(r)}, d) + q_i(k^{(r)}) + \\ + F_{\text{set}_i} \leq (1 - \delta) F_M(k) \}, \quad i = 1, 2, \dots, I \end{aligned} \quad (2)$$

for the first value of parameter  $\delta$  from the series  $\delta = 2^{-w}$ ;  $w = 0, 1, 2, \dots$

In (2)  $f^l$  and  $q_i^l$  - vectors - gradients of functions  $f$  and  $q_i$  in point  $k$ ; the designation  $(x, y)$  - corresponds to scalar product of vectors  $x$  and  $y$ ;

$$F_M(k) = \max(0, q_i + F_{\text{set}_i}), \quad i = 1, 2, \dots, I. \quad (i)$$

Then step  $\alpha_w d$  of the movement to the optimum is determined when the first fulfilment condition is checked

$$f(k^{(r)} + \alpha_w d) + N F_M(k^{(r)} + \alpha_w d) \leq f(k^{(r)}) + N F_M(k^{(r)}) - \varepsilon \alpha_w \|d\|^2,$$

where  $\alpha_w = 2^{-w}$ ,  $w = 0, 1, 2, \dots$ ;  $\varepsilon$  - the given small size (the algorithm parameter);  $N$  - the parameter that is corrected during the decision process  $N \geq \Sigma u^{(i)}$ ;  $u$  - the vector of the problem decision dimension  $I$ , dual (2).

The advantages of the given algorithm consist in the opportunity to get decision at any initial values of the synthesized parameters  $k_0$  that fulfil the minimum default of the requirements imposed on the system.

The second group of the algorithms solves the problem of the control system parametrical synthesis during the imitative modelling process in view of their parametrical uncertainty. In this case the four problem statements are of interest.

**Problem A.** The determination of the allowable decision.

To determine any parameters  $k$  of the control system that fulfil all  $I$  restrictions written in the technical requirements:

$$q_i(k, v) \leq 0 \text{ for any } v \in V_i,$$

where  $k$  - the vector of the synthesized parameters dimension  $n_k$ ;  $v$  - the vector of the parametrical disturbances dimension  $n_v$ ;  $V_i$  - area of possible parametrical disturbances values. The area corresponds to the parameter (restriction)  $i$  and is determined by the terminated by the inequality system :

$$s_{ij}(v) \leq 0, \quad j = 1, 2, \dots, n_{s_i}.$$

Problem B. The problem of the guaranteeing synthesis.

To determine the vector of the parameter values  $k$  that fulfil

$$\min_{(k)} \{ f(k) : q_i(k, v) \leq 0, \quad i = 1, 2, \dots, n_q \text{ for any } v \in V_i \}.$$

Problem C. The synthesis with maximum "margins" to fulfil the technical demands imposed on the control system.

To find the parameter values  $k$ , to fulfil

$$\min_{(k)} \{ \max_{\substack{(i=1, 2, \dots, n_q) \\ (v \in V_i)}} (q_i(k, v)) \}.$$

Problem D. The special case of problem C.

The problem decision allows to determine such values of the synthesized parameters that keep the maximum probability of the restrictions fulfilment for the most unfavourable combination of the parametrical disturbances (if it is impossible to fulfil all restrictions for any parametrical disturbances combination). In its turn the optimization problem C decision is affords

$$\min_{(k)} \{ \max_{(i, v)} (q_i(k, v)) \} > 0.$$

This problem is of practical interest in these case when it is impossible to fulfil the technical demands with probability  $P = 1$ , but it is enough to afford, at least, the maximum probability of their fulfilment. The problem approximately solves this question.

The same problems are solved using other algorithms [5,6]. They differ by the regulations of the inclusion of the new constraints and by the computation efficiency as a result. The given algorithms of all problem decision realize the iterative process of the synthesis solving the following auxiliary problems at every step.

1. The decision of the auxiliary optimization problem

$$\min_{(k)} \{ f(k) : q_i(k, v_{ij}) + F_{\text{set}}^{(r)} \leq 0, \text{ for all } i = 1, 2, \dots, n_q \} \quad (3)$$

and all  $v_{ij} \in A_{v_i}^{(r)}$ ,

where:  $A_{v_i}^{(r)} = (v_{i1}, v_{i2}, \dots, v_{iJ_i})$ ,  $i = 1, 2, \dots, n_q$  - the discrete sets of the disturbed parameter combinations  $v_i$ , that are taken account of when the first auxiliary optimization problem is solved - the search of the synthesized parameters  $k$  at synthesized step  $r$ ;  $J_i$  - the quantity of the disturbance parameter combinations corresponding to step  $r$ , that demand (and control) the restriction fulfilment  $i$ ;  $F_{\text{set}}^{(r)}$  - the current value of the demanded "margin" when the restrictions are fulfilled. The correct choice of parameter  $F_{\text{set}}^{(r)}$  affords a fast movement to the optimal decision and satisfactory characteristics of the calculation procedure.

The new values of the synthesized parameters  $k^{(r)}$  are determined as a result of the problem decision. Also the "margin" achieved at step  $r$  is determined when the restrictions are fulfilled

$$F_{\text{mar}}^{(r)} = -\max_{(i, v_{ij} \in A_{v_i}^{(r)})} \{ q_i(k^{(r)}, v_{ij}) \}.$$

2. The consistent decision of the auxiliary optimization problems for all parameters  $q_i$ , that depend on the disturbed parameters

$$\max_{(v)} \{ q_i(k^{(r)}, v) : s_{ij}(v) \leq 0, j = 1, 2, \dots, n_{s_i} \}$$

with determination  $v_i^{(r)}$  and  $q_i^{(r)}$ .

3. The check of the fulfilment of the new restrictions inclusion conditions into the auxiliary optimization problem (3):

$$q_i^{(r)} > -F_{\text{mar}}^{(r)} + G^{(r)},$$

where:  $G^{(r)}$  - the parameter of the synthesis algorithm, that determines the condition of the new restrictions fulfilment. The correct choice of value  $G^{(r)}$  affords the satisfactory characteristics of the calculation procedure.

4. The correction of the current values of the algorithm synthesis parameters  $F_{\text{set}}^{(r)}$  and  $G^{(r)}$ .

$$\text{Thus } F_{\text{set}}^{(r+1)} = F_{\text{mar}}^{(r)}; G^{(r+1)} = \alpha_g G^{(r)}, \text{ coefficient } \alpha_g < 1,$$

when solved problems A, C, and D;

$$F_{\text{set}}^{(r+1)} = \alpha_f F_{\text{mar}}^{(r)}; \quad G^{(r+1)} = \alpha_g G^{(r)}, \quad \text{coefficients } \alpha_g > 1; \alpha_f < 1, \\ \text{when solved problem B.}$$

The given problem decision algorithm of the parametrical synthesis of the control systems differ from known by the restriction inclusion regulations in number of active and by the formulation of the auxiliary optimization problems. This affords a fast movement of the current decision to the allowable area and approaching to the optimum decision. The algorithms are the same for four problems. The distinctions are in the choice of the algorithm parameters.

The software made for the decision of the given problems allows to realize automatic and interactive processes of the control system parametrical synthesis during the imitative tests. The efficiency of both algorithm groups has been checked in solving of a large number of the various problems of the parametrical synthesis of the aircraft control system. In particular, the following problems have been solved.

The problems of the parametrical synthesis of the stabilization acceleration of the aircraft on the process of imitative modelling (interactive model). The number of the synthesized parameters was 2...4. The satisfaction of the technical requirements imposed on the stabilization system has been reached for 2...3 steps. The results of the optimization problem decision bear a recommendational or an obligatory feature.

The problem of the parametrical synthesis of the plane stabilization system (roll channel). As a result of the problem decision the fulfilment of the technical requirements imposed on the stabilization system has been afforded in the whole range of the flight conditions and in view of the spread in date of the plane aerodynamic coefficients. The decision agrees for any four given problems of the second group algorithms. When solved the number of additionally included restrictions did not exceed 7...8. This is quite allowable for computing procedures.

**Conclusions.** Suggested procedures and algoritms of optimal parametrical synthesis can be realized during simulation computer of the control systems when the various parametrical disturbances act upon. All algorithms have been program realized and their computer efficiency is confirmed in the parametrical synthesis for a number of the aircraft control systems.

## References

1. Degtyarjov, O., Korolejov, V. and others (1985) Выбор параметров регулятора заданной структуры в процессе доводочных испытаний. В сб. Исследование операций и аналитическое проектирование в технике, Казань, КАИ, pp. 58-65.
2. Vermichev, G (1983) Основы автоматизации проектирования. М., Радио и связь.
3. Abramov, O., Bernacskiy, F., Zdor, V. (1987) Параметрическая коррекция систем управления. М., Энергоиздат.
4. Pshenichnyi, B. (1983) Методы линеаризации. М., Наука..
5. Gonzaga, C. and Polac, E. (1979) On Constraint Dropping Schemes and Optimality Functions for a Class of Outer-Approximation Algorithms. SIAM Journal on Control and Optimization, vol.17.
6. Polac, E. (1982) An Implementable Algorithm for the Optimal Centering Tolerancing and Tuning Problem. Journal of Optimization Theory and Applications, vol.37, N1.

# IDENTIFICATION OF CRITICAL PHENOMENA IN ENERGETIC SYSTEMS

Almaskhan Gugushvili, Nika Gomareli,

Gocha Dalakishvili

Georgian Technical University,

Department of Automatic Systems,

Kostava 77, Tbilisi, Georgia

**Abstract.** Chaotic phenomena in dynamic systems represents an urgent task. It is particularly vivid in controlling of technical systems. The work deals with Hopf bifurcation and chaos in power systems. A three-machine system which is most close to real systems. Conditions are revealed in which bifurcation and chaos take place in a power systems. For a three-generator system torus is built in the phase space which corresponds Arnold diffusion. A characteristic equation is received which describes a three-machine system and the analysis of the characteristic equation makes it possible to analyse Hopf bifurcation and random motions in power systems.

**Keywords.** Bifurcation, chaos, model, three-generator dynamic system, Arnold diffusion, torus.

**Introduction.** Study of bifurcation and chaos in dynamic systems represents one of the important problems in non-linear theory of oscillations. Researches in this field are conducted with the help of numerical and natural experiments.

At present quite a few researches have been conducted in dynamic systems with changing controlling parameter.

In this work bifurcation and chaos origination in power systems is studied by means of solving and modelling corresponding equations with the help of computer.

**Environmental Impact.** Random motion is considered in the form of phase trajectories on a plane for which angle of deflection from established regime and corresponding slip are chosen as axes. These trajectories are obtained as cuttings of a three-dimensional space which has the value of integration step as its third axis. The latter performs the part of an impressed periodic force and controlling parameter.[1-3]

Transition processes in a complex generator system considerably differ from motions in a one-generator system. That is why it is expedient to generalise and check the results on a three-generator system. The choice of the given system is not random. It is known that in practising power system analysis the present system occupies a particular place. This is caused by the fact that the system with three generators reflect many basic characteristics of real complex systems. That is why consideration brought for the systems with a random number of generators may be illustrated on this example. This makes it possible to make a better visual presentation and construction of drawings which can not be constructed through dimension increase.[7]

The system of differential equations describing motions in a system with three generators looks as follows: [4]

$$\begin{aligned}
 I_1 \frac{dS_1}{dt} &= -E_1 E_2 Y_{12} \cos(\delta_{*1} - \delta_{*2}) \sin(\Delta\delta_1 - \Delta\delta_2) - E_1 U Y_{1u} \cos\delta_{*1} \sin\Delta\delta_1 + E_1 E_2 Y_{12} \sin(\delta_{*1} - \delta_{*2}) \\
 I_2 \frac{dS_2}{dt} &= -E_1 E_2 Y_{12} \cos(\delta_{*1} - \delta_{*2}) \sin(\Delta\delta_1 - \Delta\delta_2) E_2 U Y_{2u} \cos\delta_{*2} \sin\Delta\delta_2 - \\
 &\quad - E_1 E_2 Y_{12} \sin(\delta_{*1} - \delta_{*2}) [1 - \cos(\Delta\delta_1 - \Delta\delta_2)] + E_2 U Y_2 \sin(1 - \cos\Delta\delta_2) - T_2 \frac{d\Delta\delta_2}{dt}; \\
 \frac{d\delta_1}{dt} &= S_1, \quad \frac{d\delta_2}{dt} = S_2
 \end{aligned} \tag{1}$$

where  $I$  - is a constant inertial of a generator;  $\delta_{*}$  - the size of the angle in the established regime;  $\Delta\delta_{*}$  - the size of angle digression from the established values;  $E$  - electromotive force of generators;  $Y$  - transfer conductance of branches;  $S$  - slips.

Results of the researches while changing controlling parameters for the present system are given in fig. 1 and fig. 2.

When  $\delta=0.2$  rad.  $\frac{EUY}{I} = 5$   $T=0.45$  sec.  $\delta_{*}=0.9$  rad, chaos is not present (fig. 1) and

when

$\delta=0.2$  rad,  $\frac{EUY}{I} = 5$   $T=0.9$  sec,  $\delta_{*}=0.9$  rad, random motions are clearly observable (fig. 2).

Chaos rises at even small excitation of the system's orbits as a result of a special local instability.

In the phase space and in the area of the system's parameter values there are always the areas in which the system's dynamics is stochastic. These areas may be very small, but to

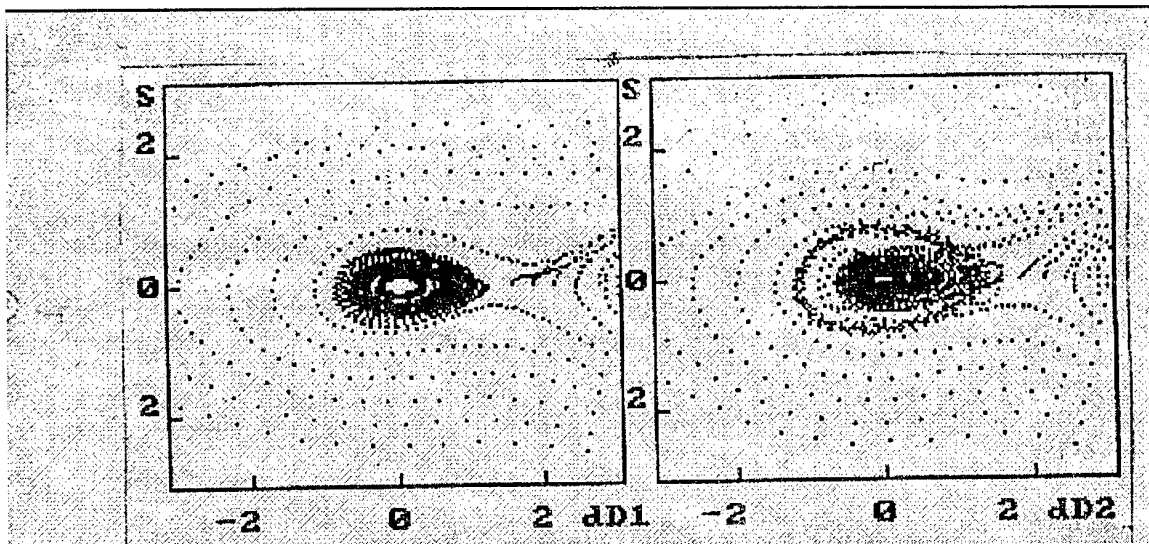
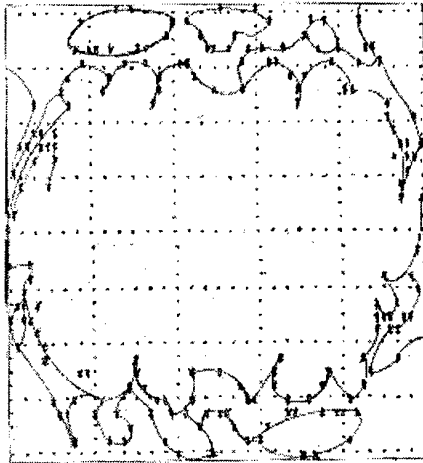


Figure 1,2

get rid of them is impossible for any finite value and any fixed construction of the dynamic system. In such condition a clear example is Arnold's diffusion - the universal unlimited transportation of particles in stochastic network channels for systems with more than two degrees of freedom. While transition to chaotic systems small areas emerge. In systems with two and more dimensions stochastic layer and stochastic network represent such germs. [6,8]

A clear example of a stochastic layer is a phase portrait of one-generator system which should be considered as a torus cutting [7]. This is the case when excitation  $K=0.09$ . Fig. 3 consists of the following elements: the largest is the separatrix cell with saddle. The separatrix is disintegrated and in its place a stochastic layer has appeared. Inside this main stochastic layer there are invariant curves sitting in each other which describe a point (0,0). Outside the main stochastic layer there are separatrix cell "beads" possessing smaller stochastic layer. They correspond to non-linear resonances of different order and describe a torus dropping perpendicularly to the first axis. Between the resonances there are invariant curves describe the torus.



**Figure 3**

Bifurcation connected with finite cycles represents Hopf bifurcation. It may be considered for the systems with one, two and three-dimensional generators.

Let us consider Hopf bifurcation for a system with three generators which completely characterises the processes taking place in multidimensional systems.

Theorem of Hopf bifurcation concerns obtaining of closed trajectories from the fixed equilibrium point or from fixed equilibrium point bifurcation in finite cycle takes place and the already obtained finite cycles describe stability of the system. [7,8,9] For three-generator systems we use equation (1).

Deriving from the group of parameters bifurcation points in a conservative system are obtained.

This makes it possible to obtain a characteristic equation of the function  $G(j\omega)J$ .

$$\lambda(j\omega) = -[\alpha \pm \sqrt{z + \beta + y}] \frac{(w^2 + 1) + J\omega y}{w^4 + (2 + y)w^2 + 1} \quad (2)$$

where

$$G(s) = \begin{bmatrix} \frac{1}{s^2 + ys - 1} & 0 \\ 0 & \frac{1}{s^2 + ys - 1} \end{bmatrix} \quad (3)$$

J-jacobian.

It is clear that the obtained pole defined by Smith-Macmillan formula (3) is the function of travel on the open node. [10]

$$P(S) = (S^2 + yS - 1) \quad (4)$$

If the values of parameters are put when  $y=0.05$  it will be sufficient for calculations and simulation on a computer as bifurcation parameter  $P_1$  - power changes at its critical value. When the value of damping  $y=0.05$  the critical value of  $P_1$  - is shifted from  $-1.155$  approximately to  $-1.146$ . When  $P < -1.146$  the chart will describe  $(-1, j0)$  point two times and reach an equilibrium position. But at first the chart tends to this point. The system converges to equilibrium with the  $1.2$  rad/sec. Fig. 4 corresponds to the given conditions. If  $P$  passes a certain value the trajectory will pass the point  $(-1, j0)$  but does not coincide with it, i.e. describes the unstable system (fig. 4). this is the case when the roots cross the false axis. Hopf bifurcation takes place in the system.

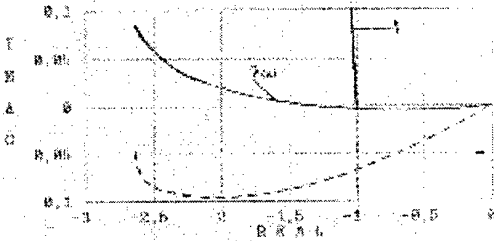


Figure 4

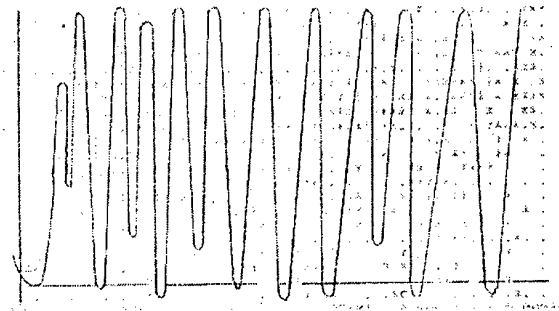


Figure 5

When  $P_1 = 1.144$  the system tends towards equilibrium. If  $P_1 < -1.146$  then the systems reach equilibrium and finally, when  $P_1 = -1.146$  Hopf bifurcation takes place in the system.

The curve of the transition process of this experiment is given on fig. 5. Fig. 6 shows its view on phase plane.

Fig. 7 gives the final result of the experiment: 3-dimensional torus.

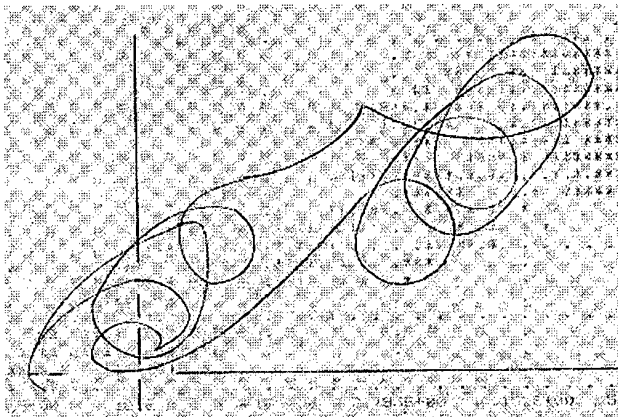


Figure 6

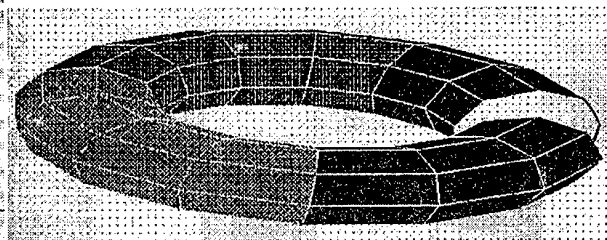
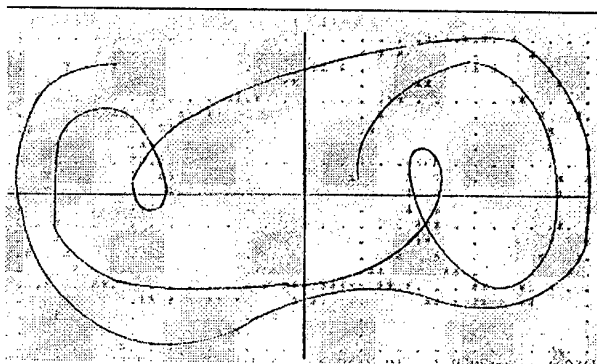
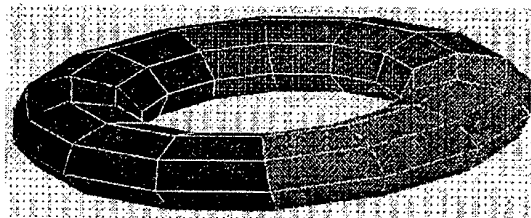


Figure 7

To make the comparison clearly evident, the phase portrait and torus of the two-generator system look correspondingly as shown on fig. 8 and fig. 9.



**Figure 8**



**Figure 9**

**Conclusions.** The differential equation describing a power system and corresponding a three-generator system has been obtained. The effluence of parameter changes of the equation on the state of the system has been studied. Values of parameters have been ascertained when Hopf bifurcation and random motions appear. With the help of computer bifurcation portraits and Arnold's diffusion have built in a phase space.

### **References**

1. Gugushvili, A.S., Muzashvili, Z.A., (1988) The Bulletin of the Academy of Sciences of the Georgian SSR;
2. Kopel, N., Washburn, R.B. (1982) IR Mener IEEE. IEEE Transactions and Systems. Vol. Cas. 29 N11;
3. Holmes, P.I., (1980) Siam Appl Mohr., vol. 38., 65-80;
4. Vennikov, A.V., (1982) The Control Systems of Transient Response, M.;
5. Barinov, V. A., Savalov, C.A., (1990) Regimes of Power Systems: Analysis Methods and Control. M.;
6. Likxtenberg, A., Liberman, M. , (1992), The Regular and Stochastic Dynamic, M.;
7. Gugushvili, A.Sh., Dalakishvili, G.N., Gomareli, N.M., (1996) Chaos in Power Systems. Hopf bifurcation in power systems. Transaction of the international conference "Information Management System" - Tbilisi. 39-47;
8. Zaslavski, G.M., Sagdeev R.Z., Usikov D.A., Chernikov, A.A. (1991) Low chaos and cvaziregular structures.M.;
9. Parker, T.S., Chua, L.O., (1992) Practical Numerical Algorithms for Chaotic Systems. Springer-Verlag. New-York;
10. Kwatny, H.G., Piper, G.E. (1996) Frequency Domain Analysis of Hopf Bifurcation in Electric Power Networks. IEEE transactions on circuits and systems, vol 37, N10 October

# CONFLICT-FREE IDENTIFICATION IN ADAPTIVE SYSTEMS

**Popow O.S.**

Institute of Computer Science and Information Systems, Technical University of

Szczecin, Poland

71-210 Szczecin, 49, Żołnierska Street,

phone: (4891) 487 65 77, fax: (4891) 487 64 39,

e-mail: orest\_popow@beta.ii.tuniv.szczecin.pl

**Goosev S.A.**

State Academy of Aerospace Instrumentation of St.Petersburg, Russia

190000 St.Petersburg, 67, Bolshaia Morskaja,

phone: (812) 110 62 87, (812) 210 70 65, fax: (812) 315 77 78

e-mail: postmaster@acts.cit.spb.su

**Abstract.** The construction of control systems for dynamic objects is frequently connected to necessity of synthesis of regulators on the basis of the limited items of information on object and conditions of its functioning. This dictates a necessity of making the abilities of self-set-up, self-training and self-organizing in systems. In the given article a solution of the outlined tasks is offered by organization of strategic identification's contour, operating in a mode of a system's normal functioning.

**Keywords.** Adaptive control, parametric identification, strategic identification, training systems.

**Introduction.** One of the intensive developed approaches in adaptive control expects inclusion parametric identifier in a real functioning dynamic system. Operation of control's contour and identification's contour in such system is subordinate to different purposes. It is known that in dual control systems, to as are to be referred the adaptive systems with identifier, exist contradictions between processes of training and control [1]. The creation of real functioning systems is associate, practically, with ensuring an acceptable interaction of conflict processes.

Practical realizations of schemes of parametric identification are often based on the hypothesis of quasistationarity, in accordance with which parameters of controlled process or object are to be consider constant within a period of adaptive system identifications. This restriction is highly essential and can't to be extend to highinertial controlled processes, for which a sample, minimum required for identifications, is accompanying by significant changing the conditions of operation.

Alongside with the nonstationarity an essential problem is a continuous lack of an information about dynamics of object, and also probable inflow of a false or incomplete information with implying from here problems of numerical problem solving of identification, mainly, bad condition and degeneration of the algebraic equations.

Outlined problems, as follows:

- the conflicting interaction between processes of control and identifications;
- essential nonstationarity of controlled objects;
- small selfdescriptiveness of measurements

reflect on reliability of adaptive system and prevent wide spreading based on adaptation approach in practice control by complex processes. The concept of such problem's solution is offered in the given article.

## **1. Organization of two-contour identification**

As is well known, current identification of object's parameters in the mode its normal operation requires presence a priori information about the dynamics of object, for getting which parametric identification is anticipated by the strategic identification. Accepted to consider that strategic identification includes to itself solution of such problems, as an evaluation of structure and parameters models, degrees of stationarity and possibilities of presentation of concrete object by the stationary model, degrees of nonlinearity and possibilities of using single-line models; selection of informative input variable; estimation of adequacy models and real object [2].

For ensuring a reliable work of parametric identifier reasonable, at least partly, to use principles of strategic identification in the mode of normal adaptive system operation. Thereby, adaptive unsteady object control is suggested to organize on the base of division of information stream of parametric identification's process upon two contours: contour of operative identification, which supply estimations for the circuit of control, and contour of strategic identification, which supply estimations for the organize of parametric identification's recursion.

Strategic identification, traditionally intended for shaping a priori given about of object, is started in the mode of object's normal operation.

Hereunder is organized a periodic sop a priori given for contour of operative parametric identification. Operative be name parametric estimations, used in contour of control upon syntheses parameters of regulator.

In process of parametric identification a posteriori estimations, recognized in determined meaning best, assign status a priori estimations for renewal iteration of identification.

Strategic be name parametric estimations, used as a priori for renewal iteration

parametric estimation.

## **2. Organization of algorithms, uncritical to the of degeneration equations to identifications. Recursion of parametric estimation**

Changing of structure and parameters of dynamic model, occur in the process of object's operation, are to be caused by inaccuracy of procedures of parametric estimation and changing the conditions of object's operation.

In different states of working object value of modes, characterizing its dynamics, ranges from the zero to greatly possible value. For formed modes distinctive reduction of amount of significant modes with result thence degeneration of parametric identification's problems.

Let's turning to the mathematical device, allowing decide problems of parametric identification, including situations of degeneration and bad conditions. Equation of parametric identification will present in the manner of  $\mathbf{P}\Theta = \mathbf{Y}$ , where  $\mathbf{P}$  - a regression matrix (matrix an input-output measurements);  $\mathbf{Y}$  - an output measurement vector;  $\Theta$  - a parametric estimation vector. Procedure of parametric identification is build on base of formula of normal pseudosolution, in accordance with which is calculate parametric estimation [3]

$$\Theta = \mathbf{P}^+\mathbf{Y} + (\mathbf{I} - \mathbf{P}^+\mathbf{P}) \mathbf{C},$$

where  $\mathbf{P}^+$  - pseudoinverse matrix;  $\mathbf{C}$  - any vector. Using an operation of pseudoinversion supplies algorithmic protection from malfunctions of procedure in the event of the degeneration, but if the problem of identification is nondegenerated, its normal pseudosolution complies with the exact solution.

For LSA-identifiers distinctive two types of information handling, according to one of which information is beforehand accumulate (is form sample from the given amount an input-output measurements), but then dares a problem of parametric estimation by some approach numerical method. Such type of processing a regression is accepted to name packet [4]. Other type LSA-identifiers expects a simultaneous renovation of regression sample and solution of problem of parametric estimation with provision for solutions, received on preceding steps. Build thereby procedure to is a recursion, but identifier - recurrence (RLSA-identifier). Recurrence identifiers possess beside advantages: in the procedure of estimation may be expelled evident referencing the regression matrixes, adjusting an identifier are to be correct in the process of its work

with the smaller risk of loss of stability of procedure, as far as direction and depth of adjustments easy co-ordinates with results of parametric estimation.

For a reason of organizations of recursion parametric estimation at quality of vector  $\mathbf{C}$  shall use a priori (strategic) estimations (with, constantly updating them in process of identifications [5]

$$\Theta = \mathbf{P}^* \mathbf{Y} + (\mathbf{I} - \mathbf{P}^* \mathbf{P}) \Theta_C,$$

Thereby, supplied relationship of parametric estimations, received on every step of recursion. The primary task of contour of strategic identification is conclude in the accumulation, analysis and selection of strategic estimations for the reason optimization of adaptation's processes in conditions of the system's normal operation.

### **3. Reduction of models and principle of their ranking**

Solution of singularity and bad conditioned problems of identification is associate with greater inaccuracy, but often turns out to be simply impossible. So at the process of object's operation it is reasonable flexibly to change the dimensionality of equation of identification, preventing of hereunder its degeneration.

It is undifficult to notice, that such strategy is fraught gradual by reduction of model over the measure of turning a system in the stationary mode. The information, accumulated in the process of identification, turns out to be gradually lost. At the moment of necessity of activation of assigning action for translation of object to a new condition the system appear of a deprived information about dynamics of object.

There are recommendations to switch off a procedure of identification for protection models from the reduction. However this reasonable make only in conditions of guarantee stationarity of system at least once in certain modes of operation. Obviously, that such reception sharply limits adaptive system characteristics and may turn out to be acceptable far from around the world. For instance, high inertial objects have long transient processes with greatly varying conditions of their running. The sea and cosmic rolling objects, some technological processes are related to such type of objects.

If consider a problem of adaptation in that stating, when stationarity of object in many modes is nonguaranted, and sometimes it is reasonable to adjust the contour of control to over reduced models, the problem of a displacing information about the object is suggested to decide as follows.

The time, including a period of activation of control by dynamic object and a period of determination of object in a installed condition, we shall name as a cycle of two-level identification. The logic of mechanism of decision making in the adaptive system will be assign such, that activations of control by object corresponds to an enabling a mode to strategic identifications, but stationary modes are accompany by working a contour of operative identification. Thus the control is synthesized by an operative estimate, and the recursion of a parametric estimation is periodically restored with a new strategic estimate.

The concept of two-level identification, developed by the authors, assumes realization by the contour of strategic identification of selection of operative and strategic estimations on the basis of a principle of ranking of estimations [6].

In accordance with this principle any parametric estimation can get a status of operative or strategic. For the explanation to logic of conferring estimation of certain status will pay attention to the following circumstance. Highdimensioned estimations carry the most full information about the dynamics of object, though not always are the best for the syntheses of control (for instance, when it less corresponds to the vary conditions operations, nontin plate new lowdimensioned model in conditions close to stationary). The lowdimensioned estimations, in turn, absolutely unfit for the syntheses of control at a period of their activations.

The mentioned reasons naturally correspond with a principle of a plurality of dynamic models [7, 8], which capable adequately reflect properties of controlled object or process in various conditions of its operation. Thereby, adaptive system must possess mechanisms of accumulation, analysis and choices of parametric estimations in accordance with mortgaged in her decision making rules, but performing the enumerate functions is is offered to contour of strategic identification.

#### **4. Structure of adaptive system with parametric and strategic identifiers**

The making of contour of strategic identification, working in the mode of normal system's operation, stipulates an adaptive system ability to self-training. Thereby, generalise adaptive system scheme with division of processes of identification on two contour stipulates an addition of self-tuning contour by the expert module [5].

The operation of expert module is based on:

- an update buffering the parametric estimations;
- ranking the estimations;
- an accompanying the estimations by ratings;
- a casting-out of insolvent estimations;

- choices of operative and strategic estimations;
- control by buffer of regulator's parameters.

Expert module can be realized in accordance with stated above ideology of strategic identification at a real-time. Distinctive singularity of the given structure is presence of the updated buffer of dynamic models and strategic identifier, ensuring selection of models and conflict-free interaction of functional modules of a system.

More economical tabular self-tuning is provided in scheme alongside with algorithmic self-tuning, for that in the scheme is included a buffer of parameters of regulator. The once synthesized parameters of the regulator are stored in the buffer, than necessity of repeated synthesis is eliminated, if parametric estimations, which they correspond have not left from the buffer of estimations of parameters for the reason losses of adequacy.

The development of a structure of the strategic identifier is an independent problem, at a solution of which it is necessary to have in view of, that if the parametric identifier is intended for realization of functions of the numerical analysis, the strategic identifier fulfills logic functions in the correspondence with included in it facts and rules. In a limit the facts and the rules can be formulated by the expert module during operation of a system. The example of practical organization of logic of the strategic identifier in the correspondence with an explained above principle of ranking of parametric estimations is described in work of the authors [6].

Programme realization of strategic identifier does not require the greater resources of operative memory. Authors have an old experience of development and testing the adaptive systems with the two-contour identification. The main difficulty consists of creation of interactive tools of organization of logic of the strategic identifier.

**Conclusions.** The explained concept of an adaptive control is oriented on a solution of problems, connected with unstationarity of controlled processes, and is based on the following moments:

- organizations of recursion of parametric estimation on the base of normal pseudosolution;
- using the principles of strategic identification in the mode of normal system's operation;
- division of information flow of parametric identification on two contours (contour of operative and contour of strategic identification);
- ranking the parametric estimations depending on their dimensionality;
- creation in the adaptive expert module system, ensuring in it conflict-free interaction of processes of control and identification.

Organization of processes of adaptation in accordance with stated principles supplies reliability of working the adaptive systems by unsteady objects in conditions of small-informative measurements.

### **References**

1. Eykhoff P., System Identification, Wiley, 1974.
2. Ядыкин И.Б., Шумский М.В., Овсепян Ф.А. Адаптивное управление непрерывными технологическими процессами. -М.: Энергоатомиздат, 1985. 240 с.
3. Беклемишев Д.В. Дополнительные главы линейной алгебры.-М.: Наука. Главная редакция физико-математической литературы, 1983. - 336 с.
4. R Isermann. Digital control systems. Springer - Verlag, Berlin, 1981.
5. Попов О.С., Балонин Н.А., Гусев С.А. Двухканальный идентификатор параметров динамической системы//Изв. вузов. Приборостроение. 1990. № 1. С. 90 - 92.
6. Балонин Н.А., Гусев С.А., Попов О.С. Элементы искусственного интеллекта в адаптивном управлении//АиТ. 1994. № 4. С. 114 - 123.
7. Справочник по теории автоматического управления/Под ред. А.А. Красовского. -М.: Наука. Гл. ред. физ.-мат. лит., 1987. - 712 с.
8. Lennart Ljung. System Identification: Theory for the User. University of Linkoping Sweden.

# IDENTIFICATION ALGORITHM OF TEMPERATURE-HUMIDITY MODES MATHEMATICAL MODEL PARAMETERS OF ARCHIVAL AND BOOKDEPOSITORY ROOMS.

L.V. Baskakov, G.A. Diduk

North-West Polytechnical Institute, St.Petersburg, Russia

E.N. Zigalenko

Russian Academy of Sciences, Centre of Ecological Fence, St.Petersburg, Russia

**Abstract:** The way of approximation for nonlinear dependences describing the dynamics of library book storage rooms temperature-humidity modes is offered. The algorithm is realized by linear dynamic models with definition their factors in real time with the help of identification procedures. The algorithm can be used in convective drying control system in mass restoration of damaged by water library and archives materials.

**Keywords:** Libraries, Bookdepositories, Storehouses; Temperature-humidity modes; Dynamic model of storehouse room; Automatic control; Safety, Safe keeping

At the moment, in connection with deterioration of economic conditions, the safety of library materials problems demand more strong requirements for air environments parameters, and also for temperature-humidity parameters of a mode in a working area of libraries and book storehouses rooms.

For system of automatic control of climatic parameters (first of all, temperature-humidity modes in a room of storehouse) characteristic is the presence of constant perturbation influence on the part of external environment, and also non-stationary sources both flowing of a heat and moisture in served rooms. In such conditions the decision of a problem of automatic control by temperature-humidity modes of archival and book storehouses is impossible without use of dynamic models, allowing to take into account change of parameters of a room at change of external conditions and other perturbation influence. Knowledge of models allows to estimate a degree of tightness of rooms, change of thermal-physical parameters of rooms, change of thermal-physical parameters of protections in time, degree of protection of materials from influence of the adverse external factors and etc.

In dynamic models peculiarities of operation of rooms and change of the characteristics in the time should be taken into account of nonlinearity of thermal-physical elements of designs.

The existing methods of account cannot ensure dynamic models, adequate the similar requirements. These methods are used basically, at design of inhabited and industrial buildings, determination of a number of empirical factors require and differ by the rather cumbersome calculation[1, 2].

Following approach to the decision of a problem of construction of climatic storehouses parameters dynamic model is offered. At the first stage approached mathematical model in space of conditions, describing dynamics of change of temperature and relative humidity of air in storehouses depending on change of temperature, relative humidity and wind

pressure of outside air is developed. At the second stage algorithm and software for identification of model parameters in real time is developed.

The dynamic model of temperature-humidity modes of storehouses rooms in space of conditions can be submitted as:

$$\dot{X}(t) = f [x(t), u(t)],$$

when  $t$  - time;

$x(t)$  - vector-column of condition variables of system (temperature, humidity content in air of storehouse);

$u(t)$  - vector - column of entrance variables (temperature, humidity content, wind pressure of outside air).

Passing from vector to the scalar form of record and carrying out linearisation operation, we receive dynamic model of climatic parameters of storage rooms as the following system differential and algebreic equations:

$$\begin{aligned} \dot{X}_1 &= a_{11}x_1 + a_{12}x_2 + b_{11}u_1 + b_{12}u_2 + b_{13}u_3 \\ \dot{X}_2 &= a_{22}x_2 + b_{22}u_2 \\ Y_1 &= c_{11}x_1 + c_{12}x_2 \end{aligned} \quad (1)$$

when  $x_1$  - change of humidity content in air of storehouse,

$x_2$  - change of temperature of air in storehouse,

$u_1$  - change of humidity content in outside air,

$u_2$  - change of temperature of outside air,

$u_3$  - change of wind pressure (perpendicular making speeds of a wind),

$y_1$  - change of relative humidity in storehouse.

The equations system's factors (1) depend on the thermal-physical characteristics of the protecting designs, deviation's size and quality, and other parameters. For example, factor

$$a_{11} = - (G_H / W)_0$$

when  $G_H$  - infiltration of the outside air to the storehouse room,  $\text{kg}/\text{h}$ ,

$W$  - mass of air in storehouse room,  $\text{kg}$ .

In its tuern, the infiltration of the outside air to the storehouse room  $G_H$  in consequence of dynamic pressure is expressed by following dependency

$$G_H = f \rho B v \quad (2)$$

when  $f$  - area of holeyness,  $\text{m}^2$

$\rho$  - specific weight of air,  $\text{kg}/\text{m}^3$

$B$  - factor of aerodynamics,

$v$  - speed of a wind,  $\text{m}/\text{s}$

As it is visible from the formula (2), the calculation of factor requires knowledge on the total holeyness area  $f$ , which practically can be defined only experimentally. It largely

concerns and to the other factors of system (1). Therefore the numerical meanings of system factors (1) are offered to be considered as approached beginning meanings, which are specified with the help of a procedure of parametrical identification.

The parametrical identification of dynamic system in the theory of control is usually understood as getting or specifying of mathematical model's parameters of process on experimental data [3].

The system of the differential equations in expression (1) can be considered as object and to be submitted in the matrix form

$$\dot{X} = A x + B u \quad (3)$$

The appropriate model will in this case have the following kind:

$$\dot{X}^m = A^m x^m + B^m u \quad (4)$$

It is necessary to carry out set-up of model, which consists in calculation of factors  $A^m$  and  $B^m$  so that expression

$$\Delta X = X - X^m \quad (5)$$

will have the minimum meaning.

Other words, it is necessary on the basis of experimental data to define system's factors (3), at which deviation of signals from gauges  $X$  and climatic parameters calculated on dynamic model of storage rooms (4)  $X^m$  will have the minimum meaning.

For determination (the identification) factors of matrixes  $A^m$  and  $B^m$  it seems perspective to be used operating non-search adaptive algorithm, based on minimization of integrated function of generalized work [3].

$$\dot{X}^m = f^m (x^m, u, a^m) \quad (6)$$

$$\dot{a}^m = -k \varepsilon \frac{\partial}{\partial x} Q_3 (x^m - x) \quad (7)$$

$$\varepsilon - \varepsilon \frac{\partial f}{\partial x} = \frac{\partial}{\partial a} f (x^m, u, a^m) \quad (8)$$

when  $\varepsilon = \frac{\partial x}{\partial a}$  - matrix of sensitivity (Jakoby matrix),

k vector-column of constants,  
 $Q_3$  criterion function.

The realization of identification algorithm is reduced to numerical integration of the model's differential equations (6), sensitivity's differential equations (8) and differential equations of setting-up (7) for determination of dynamic model's factors. Proceeding from structure of dynamic model (1) it is possible to define set of the differential equations of identification algorithm separately from the first and second differential equations of expression (1), as variable  $x_1$  is not included to the second equation.

Setting criterion function as positively determined square-law function of "non-equality"

$$Q_3 = 0,5 \beta_{22} (x_2 - x_2^m)^2$$

we receive algorithm of identification factors  $a_{22}^m$  and  $b_{22}^m$  in the following kind:

Equation of model

$$\dot{x}_2^m = a_{22}^m x_2^m + b_{22} u_2. \quad (9)$$

Equation of parameters (factors) setting-up of model  $a_{22}^m$  and  $b_{22}^m$

$$\begin{aligned} \dot{a}_{22}^m &= k_1 \beta_{22} \varepsilon_a (x_2 - x_2^m) \\ \dot{b}_{22}^m &= k_2 \beta_{22} \varepsilon_b (x_2 - x_2^m) \end{aligned} \quad (10)$$

Equation of model sensitivity

$$\begin{aligned} \dot{\varepsilon}_a - a_{22}^m \varepsilon_a &= x_2^m \\ \dot{\varepsilon}_b - a_{22}^m \varepsilon_b &= u_2 \end{aligned} \quad (11)$$

The set of the equations (9, 10, 11) forms non-search adaptive algorithm for identification of factors  $a_{22}^m$  and  $b_{22}^m$  of dynamic model of bookdepository rooms climatic parameters on the basis of experimental data.

The changes of outside air's temperature  $u_2$  and air's temperatures in storehouse  $x_2$  are removed from temperature gauges and are set as the table of initial data, which are substituted in the equations (9,10,11).

Variable  $x_2^m$  is calculated with the help of the equation (9) and is substituted in the equations (10, 11).

Thus, for determination of factors  $a_{22}^m$  and  $b_{22}^m$ , it is necessary to carry out the calculation of the equations system (9, 10, 11) in uniform time at the appropriate initial meanings of factors  $a_{22}^m$  and  $b_{22}^m$ . This implies, that the calculation of numerical meanings of the

equation system factors (1) is necessary only for the task of initial meanings in the identification equation and the accuracy of their calculation determines time of identification process.

The carried out researches have confirmed efficiency of the offered method of the climatic parameters dynamic model's description of storehouses rooms and define numerical meanings of this model's factors by methods of the identification theory.

### ***Conclusions.***

1. The offered way allows approximation of complex nonlinear dependences by linear dynamic models and definition their factors in real time with the help of identification procedures.

The inherent linearisation errors are compensated by definition of factors numerical meanings on physically measurable experimental data.

2. The received mathematical models can be used in adaptive systems of automatic control.

3. The developed mathematical models can be used in convective drying control system, which are used for mass restoration of library materials damaged by water (for example, at putting out of a fire).

### ***References***

1. Табунчиков Ю.А. Расчеты температурного режима и требуемой мощности для его отопления или охлаждения. - М. : Стройиздат, 1981.
2. Рымкевич А.Л. Математическая (термодинамическая) модель системы кондиционирования воздуха. - Л. : ЛИХП, 1979.
3. Красовский А.А. Оптимальные алгоритмы в задаче идентификации с адаптивной моделью. - М. : Наука, 1976.

# IDENTIFICATION OF THE LINEAR TIME -INVARIANT DYNAMICAL SYSTEMS IN CONDITIONS OF INCOMPLETE INFORMATION ACCORDING TO ITS STATE

Olga Volobueva, Ph. D.  
Kazakh National Technical University,  
22, Satpaev Str.  
Almaty

**Abstract.** At present time the problem of identification becomes the most important because of high demands in processes of control in different fields of technics. It is impossible to supply the qualitative control of system if its mathematical model is unknown with sufficient precision.

There are the results of scientific research of the linear time-invariant dynamical systems in conditions incomplete information according its state.

**Keywords.** Identification, controllable and observable, adaptive observer, stability from Lyapunov's, mathematical and algorithmical supply.

In this article the results of scientific research in field of the identification of the linear time-invariant dynamical systems are shown.

The main stages in the treatment of the results of technical experiment with the goal of building mathematical model are:

- survey of the information and forming in PC database of first observations ;
- initial treatment that includes the standardization of data, filtering, statistical treatment estimating degree of confidence and, etc.;
- results of data with the aim of building research characteristics of studied object.

First two stages are performed by standard methods.

The digital filtering is free linear operation for treated data. The signal can consist in operations of differentiation and integration, summarizing and calculation of diversion, amendment and forecasting, exploration for periodicity and then clean off noises. So all linear operations are equivalents of filtering.

In this work the functions of the recursive and non-recursive digital filters (DF) are used [ 1 ].

Recursive DF is described by the equation:

$$y_n = \sum_{k=0}^{N-1} c_k x_{n-k} + \sum_{k=1}^N d_k y_{n-k}, n = 0, \dots, N-1, \quad (1)$$

where  $x_n = \exp(i\omega k)$  - input information succession;  $i = \sqrt{-1}$ ;  $y_n$  - output succession DF;  $c_k, d_k$  - coefficients of succession Fourier;  $N$  - size succession.

If all  $d_k = 0$  that equation (1) describes non-recursive DF. In the work the following recursive DF-s are explored: all aspect (1), integrely numeration, exponential amend (RC - filter), Batterworts - filter (in frequency field).

During the filtering signals we have pulsating signals in points of the rupture transmissioncity function (by the fragment succession Fourier). It is accurence of Gibbs. The amending of the pulsating is made by the multiplication of  $\sigma_k$  - factor (window of Lanczos) [1]. In this work double amending is used.

Input - output variables of the exploratory processes can be distributed according to different laws. Most often they are random and normal, exponential and  $\gamma$  - distribution. The identification of parameters of mathematical model for a given structure (the statical object) is made with the help of current regression analysis [3].

If matrix of measured input data  $X$  is determined badly, the more reliable method for calculation of coefficients is the method of least squares (MIS). It is based on matrixal factoring called singular value decomposition.

We can take matrix  $\Sigma$  for any matrix  $X$  and two orthogonal matrixes  $U$  and  $V$  that  $\Sigma = U'XV$ .

We can select matrixes  $U$  and  $V$  so that majority of element  $\sigma_{ij}$  are formed into the zeros. Furthermore, we can make matrix  $\Sigma$  as diagonal matrix with non-negative diagonal elements.

Hence, we have the factoring real matrix observing  $X$

$$X = U \Sigma V'$$

that is according to its singular value decomposition.

The singular value decomposition (SVD) allows to find numbers of the initial matrix  $X$  and establish singular or non-singular solution and identify the estimation of coefficients.

The problem of the identification of characteristics of systems can be considered as a dual one according to the problem of control system. It is impossible to control a system if is not identified beforehand or is in a process of controlling.

This work used the method of stochastical approximative which is applied for identification of the linear and non-linear time-invariant systems. It is the method of the successional directional search. The directional methods of search can be considered as development of the classical Newton's method for calculation of roots equation. These methods were used successfully by Robbins and Monro, Kiefer and Wolfowitz based on Dvoretzky's likenessing theorem [3], [4], [5].

If structure system is given by equations:

$$\begin{aligned} X_{k+1} &= AX_k + BU_k \\ Y_k &= C'X_k, \end{aligned} \quad (2)$$

where  $X \in R^n, U \in R^m, y \in R^1, n \geq m, X$  - vector state variable,  $k = 1, 2, \dots$ , then estimation of the parameters  $\tilde{P} = \{\tilde{A}, \tilde{B}\}$  as [2], [5]:

$$\tilde{P}_{k+1} = \tilde{P}_k - \gamma_k (\varphi(U_k, \tilde{P}_k) - y_k) \frac{\partial \varphi(U_k, \tilde{P}_k)}{\partial \tilde{P}_k}$$

where  $\varphi(U_k, \tilde{P}_k)$  is right part of the equation (2).

The squarely criteria of the minimum declension is a measure of the likeness of a mathematical model with a real dynamical system.

The identification of the linear time-invariant dynamical objects in conditions of incomplete information is made according to its state. The complete vector of state variable isn't measured but, is measured partly, with big errors. The knowledge of complete vector state variable system is necessary to solve the problem of parametery identification.

Also, the complete observation is necessary for identification, but the complete identification is necessary to control system completely.

The linear time-invariant discrete system has described by the equations (2), where  $X \in R^n, U \in R^1, y \in R^1$ . The matrix  $(A, b, C)$  are completely controllable and observable.

The adaptive observer which has to identify the parameters and to estimate the state of the object [6] is

$$\begin{aligned}\tilde{X}_{k+1} &= H\tilde{X}_k + (h - \tilde{a}_{k+1})y_k + \tilde{b}_{k+1}U_k \\ \tilde{y}_{k+1} &= C'\tilde{X}_k,\end{aligned}$$

where  $\tilde{X} \in R^n, \tilde{y} \in R^1, H = \begin{bmatrix} -h & \frac{I_{n-1}}{0'} \end{bmatrix}$ ,  $H$  is a stable matrix.

The adaptive law of the parameters [6] is

$$\tilde{a}_{k+1} = \tilde{a}_k + \frac{T_a y_{k-1} \eta_k \sigma}{r'_{k-1} r_{k-1} \lambda_{\max}}, \quad \tilde{b}_{k+1} = \tilde{b}_k - \frac{T_b U_{k-1} \eta_k \sigma}{r'_{k-1} r_{k-1} \lambda_{\max}},$$

where  $T_a = \text{diag}(\lambda^1 \dots \lambda^n), T_b = \text{diag}(\lambda^{n+1} \dots \lambda^{2n}), \lambda_{\max} = \max(\lambda^i), \forall i = 1, \dots, 2n; \eta_k = \theta'_k r_{k-1};$   
 $r_k = (y_k, \dots, y_{k+1-n} u_k, \dots, u_{k+1-n})'$  is vector of input-output signals;  $\theta_k = (\phi_k^1, \dots, \phi_k^n, \psi_k^1, \dots, \psi_k^n)'$   
 is vector of the parameters of errors, where  $\phi_k = a - \tilde{a}_k, \psi_k = \tilde{b}_k - b; \quad 0 < \sigma < 2$  is scalar that received and provided from Lyapunov's stability theorem.

The solution of the problem is defining according to predicted precision  $\delta_j$  if it is made in following:

$$|a - \tilde{a}_k| \leq \delta_1, |\tilde{b}_k - b| \leq \delta_1, |y - \tilde{y}_k| \leq \delta_2 (k \rightarrow \infty).$$

The software are from the algorithmical language TURBO-PASCAL; IBM PC XT/AT OS MS DOS; volume memory 1.2 mb; time calculation is not more than 30 seconds. The results of calculations are provided in illustrative grafical and table forms.

**Conclusions.** The results of scientific research have been inculcated in different automatized control of systems on the territory of the Republic of Kazakhstan.

We can consider them as automatized working place explorer because they include the practical methods and instrumental means of dialogical treatment experimental data.

## References

1. Hamming R. (1980) Digital filters. Sovjet radio, Moscow.
2. Eykhoff P. (1985) System identification. J. Wiley, London.
3. Graupe D. (1979) Identification of systems. J. Wiley, New York.
4. Isermann R. (1984) Digital control systems. McGraw-Hill, New York.
5. Zypkin J.S. (1984) Informatical theory of system identification. Science, Moscow.
6. S. Tamaki, S. Omati, A. Kikuch and S. Takasi. (1981) Design of a direct adaptive observer based on Lyapunov's direct method. // INT. J. Systems Sci., vol. 12, NO. 4, pp. 473-484.

**CHAPTER XIII:**

***SAFETY CONTROL IN COMPLEX SYSTEMS***

THIS CHAPTER INCLUDES PAPERS  
PRESENTED AT THE CONFERENCE SESSION:  
SAFETY CONTROL IN COMPLEX SYSTEMS

Organized by: *Prof. Igor A. Ryabinin,*  
*Dr. Galina I. Miasnikova.*

# LOGIC-PROBABILISTIC THEORY OF SAFETY OF COMPLEX SYSTEMS

Igor A. Ryabinin  
Kuznechov's Navel Academy  
tel: 239-94-82, E-mail: sol@sapr.ipme.ru

**Abstract.** This paper attempts to develop logic-probabilistic methods for the safety theory in directions of taking into consideration not only their functioning standard conditions, but also taking into account any possible non-standard destroying effects. The prime tasks of such theory of complex systems safety should be methods of development of quantitative valuation of the risk of one or another break-down occurrence and its possible consequences prediction. So under logic-probabilistic theory of safety is understood risk evaluations based on dangerous condition development logic representation and mathematical methods of logic algebra functions validity calculation. A table of main concepts of safety is developed.

**Keywords.** Reliability, survivability, safety, damage, risk, logic-probability methods.

Here and below we shall consider reliability, survivability and safety [ 1, 2, 3 ] as follows:

THE RELIABILITY is a system property to execute the given purpose during a required period under daily conditions of operation;

THE SURVIVABILITY is a system property to execute the given purpose (even at lowered efficiency) under the effect of the force-major forces to a system;

THE SAFETY is a system property to function, without passing to dangerous conditions menacing to health and life of people or inflicting another damage in large scales.

Logic - probability methods ( LPM ), the mathematical essence of which consists of using the functions of algebra of logic ( FAL ) for analytical record of conditions of serviceability (or failure) of systems and in development of strict ways of transition from FAL to probability functions ( PF ), objectively expressing the unfailure (failure) of this system. The direct transition to the true probability of functions of algebra of logic is not simple for the complex tasks and structures described by FAL of any form. The main results of domestic and foreign scientists, as well as our own results, received to that time are stated in monograph [ 4 ] translated into Japan in 1987. Some ideas of these methods may be constituted after reading p.24- 39, 282-324, 465-497 of the monograph [1], p.3-114 of [4] and p.58-104 of [5]. However in the last years the LPM have received further development in different directions stated, as a rule, in journal articles, preprints and other editions not easily available.

For the important objects and technical systems and for the cases of break-downs and catastrophes with serious consequences it is expedient to develop the safety theory in directions of taking into consideration not only their functioning standard (design) conditions, but also taking into account any possible non-standard (beyond the design)destroying effects and rough (inadvertent and deliberate) violations of their operation rules.

The prime tasks of such a theory of complex systems safety should be methods of development of quantitative valuation of the risk of one or another break-down occurrence and its possible consequences prediction. The most difficult obstacle on the indicated theory

creation way is a wide spread opinion about the impossibility of accounting all situations which may lead a system into a dangerous state. To overcome this obstacle is possible with the help of a number of measures.

First, it is necessary to define precisely and have an exact idea of a dangerous condition. If it is necessary, for example, to evaluate a break-down - explosion risk all possible explosion «chemistry» (gases contents, their parameters) should be known, if a vessel wreck risk is evaluated it is necessary to be an expert on all sciences, connected with shipbuilding and navigation. Defining and differentiating a researched dangerous condition ( explosion, fire, affection by current etc.), we narrow essentially a set of possible system conditions.

Second, it is absolutely necessary to limit a research object to reasonable limits. Not only space borders (building walls, compartment bulkheads, region borders) are meant, but to a still greater degree, system's division to its elements (it is necessary to stop somewhere!).

Third, strict logic and discipline is required to consider every possible situations making the script of events development causing the system dangerous condition. It is necessary to move not «from below upwards» (from one or another breakage, failure, of undesirable initiating event), but «from above downwards» (from a researched dangerous condition to the reasons which can cause it). Emphasizing the necessity of development in the first place of the logic part of the safety theory, we should not neglect all the other knowledge fields (physics, chemistry, mechanics, electronics etc.) which are certainly present in each branch of events script causal relations. It is especially difficult to consider the so called «cross» relations unlike «longitudinal» ones following from a system dangerous condition to a particular initiating event). The cross connection are formed as a result of interaction through various obvious and implicit crossings (fields, structures ), which are often neglected because of their absence or unimportance. However it is not so.

Fourth, the above indicated task should be formalized in a convenient mathematical language (graph theory, logic algebra etc. ) and computerized.

Fundamental concept in safety LPT is a concept of system dangerous state (SDS) and the appropriate system danger logic function (SDF). Similar to the reliability theory , where everything begins with system serviceability concept, in the safety theory in each particular case it is required to give analytical description of the SDS, which can cause people's death or any other serious damage (stipulated beforehand as unacceptable one in this case). In LPT such description begins with dangerous condition script making (DCS) by considering every possible condition of a system « from above downwards »: from a particular researched SDS to those reasons which can cause it, that is, to the so called initiating conditions (IC) ( failures, breakages, service regulations violations etc.).After heuristic creative SDS drawing up and its approbation among experts it is necessary to begin drawing up SDF with the help of dangerous functioning shortest ways (DFSW), or with the help of danger prevention minimum sections (DPMS).

The dangerous functioning shortest way represents such initiating conditions ( $Z_i$ ) conjunction from which it is impossible to withdraw a component without violating system dangerous functioning. Such a conjunction can be recorded in the form of logic algebra function (LAF):

$$\varphi_l = \bigwedge_{i \in K_{\varphi_l}} Z_i, \quad (1)$$

10	Criterion of danger	It is measured by size of risk of getting a system to dangerous condition	It is measured by size of some of objective criterion guaranteeing the preservation of safety ( speed, voltage, force, percentage, etc. )	It is characterized by size of money or temporal damage and also by loss of health
11	Ways of determination of criterion	Calculation on mathematical model or treatment of the statistical material by similar incidents	Basically by experimentally or by simulation on a computer with total exhaustion of all possible situations	By ways of valuation of losses.
12	Role of subjective factors in the safety problem	Subjective fixing of size of « large scale » damage for a concrete system permits to objectively evaluate the whole diapason of possible situations: from a collapse up to an ordinary failure	It is subjectively coded absolutely all in exhaustive models of safety valuation by contrast to « black - white » situations in mathematical - models of risk valuation	Subjective factor dominates on all stages of valuation of damages, determination of culprits, etc.
13	Normalizing of danger criterion	It is not normalized for the present, but it is compared the calculated risk with backgroundly risk ( fixed in any field ) or with risk reached in the past	Physico-chemical criterions of danger exist and they are constantly specified	It is established by comparison of damaging with the cost (income) of researched system
14	Expected benefit from the safety theory	More deep study of system functioning taking into account of not only daily failures of equipment, but also adverse external and internal effects. Objective determination of a role of each initiating condition to system safety and measures of its protection	Establishment of objective and measurable criterions of safety and measures of protection of a system from failures of equipment development of recommendations by increase of accident - free	Unreasonable and naive expectation of miracle from science, i.e guaranteeing of absolute safety and if it does not occur than total disappointment in it
15	Top priority task.	Learn to make the script of system transition to dangerous condition	Substantiate scientifically the concepts of "dangerous" failures of equipment	Correctly understand what is possible and necessary to require from the safety theory and what is impossible

or probability of its safe condition

$$B_c = P\{\bar{y}(\bar{Z}_m) = 1\} = f_2(O_{z_i}, B_{z_i}) = 1 - O_c, \quad (7)$$

where:

$$O_{z_i} = P\{z_i = 1\} \text{ and } B_{z_i} = P\{\bar{z} = 1\}. \quad (8)$$

For complex tasks and structures described by LAF of any form the direct transition to logic algebra function validity probability under formulas (6) and (7) is not simple.

So under logic - probabilistic theory (LPT) of safety is understood as main knowledge on complex systems structure break-down and catastrophe occurrence risk evaluations based on dangerous condition development logic representation and mathematical methods of logic algebra functions validity calculation. The logic-probabilistic methods of safety research permit to objectively reveal the most dangerous places, reasons and initiating conditions; they form different researcher's point of view and make the experts concentrate efforts on the paramount tasks solution. The advantage of LPT is their serviceability and the absence of initiating events initial probabilities, that, as a rule, is basic difficulty at rare events risk quantitative valuation (because of the lack of frequencies stabilities of many initiating events, for example, such, as touch to electric current equipment, approach to a power-station's case, smoking when a storage battery charging and etc.).

The determined logic model permits to reveal danger prevention minimum sections, i.e. the most expedient combinations of initiating conditions denials which protect a system from getting into a dangerous condition. Moreover with the help of this model it is possible to estimate objectively the importance of any initiating condition and their any arbitrary combinations (by two, etc.).

Determination of elements (parts of a system), the characteristic of which is the most essential one from the point of view of unfailure, survivability or safety of a system as a whole is practically important at research and maintenance of these characteristics for systems of various purpose. The structure of indicated systems (power, communication, information, connection etc.) presents oriented or non-oriented graphs of a network type, according to which certain flows run from one unit to another. Some community of the named systems permits to build practically identical mathematical models and to produce new approaches to synthesis of structures of such systems. Taking into account the difficulties of transformation LAF into PF and the necessity of probabilistic evaluations at risk estimation, for example, for nuclear power stations the question arises how the specialists have done so far.

In the technique of the systems probabilistic analysis (PSA) or technique to risk probabilistic valuation (PRA) a real network (cyclic) structure LAF is replaced with a simple « events tree », i.e. reduced to the known theorem hypotheses. Therefore it seems to us expedient not to simplify the dangerous condition script but on the contrary to formalize the events development logic more deeply and exactly. The mathematical difficulties can be overcome with the COMPUTER help using all LPM results and their further development including logic-statistic simulation methods.

Below suggest a table of main concepts of safety for specialists mutual understanding.

Table 1 Main Concepts of Safety

N	Concept, terms, tasks	A R E A S   O F   U S E		
		fundamental sciences	applied sciences	practice
1	2	3	4	5
1	Damage	It is considered only «large scale» damage established beforehand as inadmissible one in any circumstances. This norm assists to evaluate its danger in spite of its subjective character	Sizes of possible damage are not beforehand interested as any damage harms	Damages of any scale
2	Dangerous condition	Synonym of force-major condition or collapse described by «large scale» damage	Supernumerary condition bringing to damage of permissible scale ( repair, replacement, etc. )	Expectations of threat, presentiment of it
3	Danger	Properties of a system to go to dangerous condition	Antipode of safety	Threats of any bad, any accident
4	Safe condition	Antipode to dangerous condition, i.e. condition described by functioning of a system without damages or with «small scale» damages	Condition of a system without accidents, catastrophe and forcemajor condition	Condition of a system, not connected with expectation of any damage
5	Safety	Properties of a system to function not going to dangerous condition	Properties of a system to preserve a safe condition during realization of given functions	Properties of a system to function  without any damages
6	Accident	Incident with equipment inflicting a harm of permissible scale	Unforeseen failure, destruction or wreck of a system	Incident requiring out-of-plan repair, replacement of damaged equipment
7	Accident - free	Abilities of a system to function without failures	Synonym to the term of safety	Abilities of a system to function without accidents
8	Catastrophe	Incident accompanied by wreck or loss of people	Accident with tragical consequences	Accidents accompanied sacrifice
9	Failure	The term is not used, as it is synonym to a dangerous condition with respect to a system	It is considered dangerous and non - dangerous failures of equipment which are established subjectively	Equipment faultiness connected with loss of efficiency

where:  $K_{\varphi_l}$  is a set of IC numbers, corresponding to the given DFSW l.

IC accepts one of two values:

$$z_i = \begin{cases} 1, & \text{if the condition i has taken place;} \\ 0, & \text{if the condition i was not executed.} \end{cases} \quad (2)$$

In other words DFSW describes one of the possible independent variants of getting a system into a dangerous condition with the help of a minimum set of initiating conditions absolutely necessary for its realization, that is, the given variant of explosion, fire, flooding or other SDS. The danger prevention minimum section represents such conjunction of initiating condition  $\bar{z}_i$  denials from which it is impossible to withdraw a component without violating system safe functioning conditions. Such conjunction can be recorded in the form of the following LAF:

$$\psi_j = \bigwedge_{i \in K_{\psi_j}} \bar{z}_i, \quad (3)$$

where  $K_{\psi_j}$  is a set of numbers corresponding to the given DPMSj.

In other words the danger prevention minimum section describes one of possible ways of violating dangerous functioning with the help of a minimum set of forbidden conditions  $\bar{z}_i$ . Each real technical system has final (!) number DFSW (l = 1, 2, ..., d) and DFSW (j = 1, 2, ..., n).

Using these concepts it is possible to record a system dangerous state conditions in different ways:

a) in the form of disjunction of all present DFSW

$$y(z_1, \dots, z_m) = y(\bar{Z}_m) = \bigvee_{l=1}^d \varphi_l = \bigvee_{l=1}^d \left[ \bigwedge_{i \in K_{\varphi_l}} z_i \right], \quad (4)$$

b) by the conjunction of denials of all DPMS

$$y(z_1, \dots, z_m) = y(\bar{Z}_m) = \bigwedge_{j=1}^n \bar{\psi}_j = \bigwedge_{j=1}^n \left[ \bigvee_{i \in K_{\psi_j}} z_i \right], \quad (5)$$

Thus the dangerous state condition of a real system can be presented in the form of dangerous functioning conditions of a certain equivalent (in a safety sense) system the structure of which represents parallel connection of DFSW, or other equivalent system, the structure of which represents series connection of DPMS denials.

Let us record in a common form the expression for system dangerous condition probability determination

$$O_c = P\{y(\bar{Z}_m) = 1\} = f_1(O_{z_i}, E_{z_i}), \quad (6)$$

**Conclusions.** 1. Confusion of scientific concepts of technical systems reliability, survivability and safety, replacements them with others ( for example, accident rate, failure-stability, guarantability etc.) hinder essentially the dialogue between experts, deform priorities, create illusion of prosperity in science and practice. Deliberate inclusion of survivability and safety into a wider reliability interpretation is useless for reliability not giving any dividend for survivability and safety. Establishment of the uniform interpretation will certainly promote progress in the complex systems survivability and safety field.

2. Saturation of modern technical complexes with dangerous engineering systems increased power scales of destroying effects requires qualitatively new, complex approach to their survivability maintenance problem. Alongside with abstract destruction models it is necessary to develop detailed physical-mathematical processes models of processes proceeding at failures. This is connected with the task of authentic information about utmost conditions for SCS elements serviceability.

3. No theory will enable to receive the objective failure development script without participation of particular systems experts. The development and visualization of scripts of system into a dangerous condition and hence the appropriate logic functions (SDF) will help designers to find dangerous transition ways to a critical condition ( and danger prevention minimum sections ), and experts - exploiter - not to admit such transition, i.e. to protect a system.

4. In connection with the great importance of complex systems survivability and the safety problems it is necessary to support actively all efforts in this direction. We must to spare no expenses on complex systems survivability and safety research, more accomplished methods development, because these charges make infinitesimal share of possible losses. Let us remember the words of academician B.Paton: " The knowledge costs much, but the ignorance comes to much more".

### **References**

1. Ryabinin, I.A. ( 1976 ). Reliability of Engineering Systems. Principles and Analysis. MII Publishers, Moscow , p.532 .
2. Ryabinin, I.A. (1977 ). Reliability of ship electric-power system. Sea collection, 1, 79-82.
3. Ryabinin, I.A. (1987 ). Reliability, survivability and safety of ships. Sea collection, 8, 62-65.
4. Ryabinin, I.A., G.N. Cherkosov ( 1981 ). Logic-probabilistic methods of research of structure-difficult systems reliability. Moscow, Radio and communication, p. 238 (is translated on Japan in 1987).
5. Ryabinin, I.A. ( 1988 ). Calculation of reliability of systems with structural redundancy . Engineering, Directory in ten volumes "Reliability and efficiency in an engineering ", volum 5, pp. 58-101.

# A DIAGNOSTIC ENGINE FOR POWER TRANSMISSION NETWORKS

Pietro Baroni<sup>†</sup>, Gianfranco Lamperti<sup>†</sup>, Paolo Pogliano<sup>‡</sup>,  
Giorgio Tornielli<sup>‡</sup> and Marina Zanella<sup>†</sup>

<sup>†</sup> Dipartimento di Elettronica per l'Automazione, Università degli Studi di Brescia, Italy  
e-mail: {baroni, lamperti, zanella}@bsing.ing.unibs.it

<sup>‡</sup> CISE s.p.a., Milano, Italy  
e-mail: {paolo, gt}@sia.cise.it

**Abstract.** This paper presents a technique for the diagnosis of short circuits in power transmission networks as an instantiation of a general diagnostic approach which is applicable to discrete, dynamic systems. A three-layers diagnostic engine is formally defined, including *local interpretation*, *global interpretation*, and *heuristic interpretation*. The local interpretation focuses on the behavior of single protection components that are distributed over the power network and operate when a short circuit occur. The global interpretation provides a global behavior of the protection system by combining consistent local behaviors. Finally, the heuristic interpretation is meant both to eliminate a number of spurious global behaviors on the basis of application-dependent *heuristic constraints*, and to eventually localize the short circuit and possibly faulty protection components within the transmission network. The implementation of the proposed technique is under way. The resulting system will be tested by ENEL, the Italian electricity board, using the transmission network of part of northern Italy.

**Keywords.** Fault diagnosis, model-based reasoning, power transmission networks, short circuit localization, knowledge representation.

**Introduction.** The protection system of a power transmission network is composed of a number of protections distributed over the network and is aimed to avoid service interruptions which may be determined by short circuits. To this end, the protection system aims to detect dangerous conditions, to disconnect a component (such as a line, a bus, a transformer or a generation group) as soon as it begins to operate in a dangerous way, and to keep in operation non-faulty components as much as possible, in order to avoid a black-out. This is achieved by tripping the circuit breaker associated with each protection. Each protection has to protect mainly one component, but must also operate as a backup to other protections nearby.

When reacting to a short circuit, all the protection components send logical signals (messages) to a *Regional Control Center* (RCC). These messages consist of a unique address of the event source, an event code, and possibly a timestamp. Operators of the RCC have to decide within one minute where the fault is located and what recovery actions have to be applied. Normally, due to time constraint reasons, only the final status of the breakers is considered, but this is not generally sufficient for fault localization. It is recognized that as a result of this an automated tool for fault diagnosis is an important requirement.

Several attempts to develop automated support tools for fault diagnosis in power transmission networks are reported in the literature. Somewhat surprisingly, they are based on a variety of different technological approaches including neural networks [1], fuzzy expert systems, [2], Petri nets [3], model-based diagnosis [4], and temporal reasoning techniques [5]. However, all these approaches present weak points.

In [1] a neural network is trained to recognize typical fault patterns, by processing in input the values of voltage and current measured in a given substation. Even within this limited context, a major difficulty of this approach is related to the huge amount of preliminary work required, since the neural network must be trained on a wide number of different cases each time it has to be applied to a different substation configuration.

The sagittal diagrams-based approach of [2], not including an explicit model of the diagnosed system, suffers from all the well-known limitations of first-generation, rule-based, diagnostic systems (see for instance a discussion in [6]).

An alternative approach [3] resorts to Petri nets in order to build a model of the protection system behavior. However this model is over-simplified: temporal aspects are not considered and the overall protection system behavior is modeled as a simple two steps activity.

More detailed models are used in [4, 6], where a classical model-based diagnosis approach is adopted, using the GDE+ diagnostic engine [7]. However, the major shortcoming of these approaches is that the component models adopted, being quite abstract and not very detailed, do not take into account the rather complex behavior of the protection system and are not able to produce detailed diagnoses about its malfunctions.

Detailed models of component behaviors, including temporal features, are used in [5], where an original diagnostic algorithm for time-varying systems is proposed and applied to power networks. This algorithm is able to produce a detailed temporal reconstruction of the events occurred in the network, but it relies on the assumption that the timestamp of each message received is available, that is not always the case in practice.

An approach whose rationale has some significant similarities with the work presented here has been developed in the frame of diagnosis of communication protocols [8]. Even though the nature of the diagnostic problem is rather different, it shares the use of automata-based techniques, involving observation matching, propagation, and interpretation, in order to deal with complex diagnostic problems of time-varying systems.

The technique proposed in this paper differs from other approaches proposed in literature for diagnosis of dynamic systems, a topic that has been addressed by many researchers in recent years [9-14].

As a matter of fact, even though the importance of dynamic system diagnosis and the need of specific techniques for this problem was recognised early [15-16], the *classical* theory of diagnosis [17-18] has been conceived for static systems only. Subsequent attempts to overcome this limitation have been, however, strongly influenced by the approach initially adopted for static systems. In fact, most of these approaches [9, 11-14, 19] share a common rationale: Reiter's algorithm is simply applied iteratively to subsequent instantaneous snapshots of the behavior of the system, generated through a suitable dynamic model. This method presents however some significant drawbacks, since it is unable to deal with faults whose manifestation involves several time instants and does not exploit efficiently knowledge about system dynamic behavior. An approach closer to our ideas is presented in [10] where the diagnostic activity for a dynamic system is seen as the task of reconstructing the history of the system starting from some temporally located information about system attributes and using a model of system behavior. However the behavioral representation adopted is made up of *if-then* rules including temporal information and suffers therefore from limited expressiveness.

In our approach, on the basis of a formal model of protection components, the diagnostic task can be viewed as an incremental interpretation process of observations, at the end of which relevant diagnostic information can be gathered to generate the diagnosis.

**Reactive models.** Each protection component is modeled by means of a finite state machine (FSM), called *reactive model*, which is driven by a series of *input events* and generates some other *output events* when transitions between *reactive states* occur. For example, a distance protection  $P$  is started when the impedance on the relevant line goes under a certain threshold (input event). After that,  $P$  is expected to change state at fixed time intervals (clock input events) and, possibly, to trip the associated breaker  $B$  (tripping output event). Note that the tripping output event of  $P$  generates a tripping input event for  $B$ : we say that the output event is *exported* by  $P$  and is *imported* by  $B$ .

Reactive states can be either *steady* or *unsteady*. This classification improves the semantics of reactive models and indirectly poses a number of constraints on the behavior of protection components which can be conveniently exploited by the diagnostic engine. A *null event*  $\epsilon$  is a formal notation to specify the absence of events. Formally, a reactive model  $M$  is a record of four elements:

$$M = (\Sigma, I, O, \delta) \quad (1)$$

where  $\Sigma$  is the set of reactive states,  $I$  is the set of input events,  $O$  is the set of output events, and  $\delta$  is the transition function:

$$\delta : \Sigma \times (I \cup \epsilon) \times (2^O \cup \epsilon) \rightarrow \Sigma \quad (2)$$

where  $2^O$  is the power-set of  $O$ . This means that a transition  $T$  from state  $S_1$  to state  $S_2$  is in general triggered by an input event  $i$  and generates, before changing state, the list of output events  $\langle o_1, o_2, \dots, o_n \rangle$ . This is denoted by  $T = S_1 \xrightarrow{i|o_1, o_2, \dots, o_n} S_2$ . Furthermore:

$$\begin{cases} \Sigma = \Sigma_s \cup \Sigma_u \\ \Sigma_s \cap \Sigma_u = \emptyset \end{cases} \quad (3)$$

where  $\Sigma_s$  and  $\Sigma_u$  denote the set of steady and unsteady states respectively.

Each reactive model describes both the correct and faulty behavior of a class of protection components. In addition, the model allows for uncertainty due to possible loss of messages. Specifically, if during a transition  $T_1 = S_1 \xrightarrow{i|m} S_2$ , message  $m$  may be lost, then the  $\delta$  function will include an additional transition  $T_2 = S_1 \xrightarrow{i|\epsilon} S_2$ .

The specification of the reactive model of each type of protection component and the instantiation of such classes of components into a given network topology yields the whole model of the protection system.

**Local histories.** When a short circuit occurs, we say there is a *misbehavior*  $\mu$  of the transmission network. The set of protection components reacting to a misbehavior  $\mu$  is called the *misbehavior extent* of  $\mu$ , and is denoted by  $extent(\mu)$ . After the specification of each reactive model, it is possible to interpret a given sequence of observations, the messages, in order to eventually find out the sequence of transitions covered by the involved protection components during  $\mu$ . When a short circuit occurs, each actual protection component is expected to react in a way that corresponds to an instantiation of the model, called the *local history* of the component. As such, a local history is a sequence of transitions within the reactive model. The initial and final states of the history are required to be steady. The local history is derived on the basis of the *local observation* of the component  $P$ ,  $obs(P)$ , namely a list of messages,  $obs(P) = \langle m_1, m_2, \dots, m_n \rangle$ . A local observation which is empty is called a *null observation* and is denoted by  $\epsilon$ .

**Definition 1** (*local history*) Let  $M = (\Sigma, I, O, \delta)$  be a reactive model of a protection component. A *local history* of  $M$  is a (possibly empty) sequence  $h = \langle T_1, T_2, \dots, T_n \rangle$  of transitions in  $M$  so that  $h$  conforms to the following *morphology constraints*:

1. *Determinism.* Each transition  $T_i, i = 1..n$ , is adorned with at most one allowed input event,
2. *Contiguity.* For each pair of contiguous transitions  $T_i, T_{i+1}$  in  $h$ , the final state of  $T_i$  coincides with the initial state of  $T_{i+1}$ , and
3. *Stability.* Both the initial state of  $T_1$  and the final state of  $T_n$  are in  $\Sigma_s$ .

If the sequence of transitions is empty,  $h$  is called a *null history*, and is denoted by  $\epsilon$ .

**Definition 2** (*local observation*) Let  $M$  be a reactive model of a protection component  $P$ . A *local observation*  $obs$  of  $P$ ,  $obs(P) = \langle o_1, o_2, \dots, o_n \rangle$ , is a sequence of temporally ordered observable output events, generated by a local history of  $M$ .

**Local interpretation.** The first step towards the detection of the short circuit is represented by the local interpretation of every local observation. A local interpretation algorithm (*lia*) has been defined. The algorithm takes in input a local observation  $obs(P)$  and generates a set of consistent local histories on the basis of the relevant reactive model. In general, a local interpretation gives rise to several local histories, namely:

$$lia(obs(P)) = [h_1, h_2, \dots, h_n] \quad (4)$$

all of them being consistent with  $obs(P)$ .

In our approach, a reactive model  $M$  of  $P$  is formally interpreted as the grammar of a regular language  $L$  where an observation  $obs(P)$  represents a phrase of  $L$  (for an introduction to automata theory and languages, see for example, [20]). Consequently, the problem of determining the local interpretation of  $obs(P)$  corresponds to the recognition of a string of tokens (the messages) as a phrase  $f \in L$ . What is essential in the local interpretation task is to keep track of the sequence  $h$  of transitions which are relevant to the recognition of  $f$ .

However, a model  $M$  may be non-deterministic, involving transitions which are not associated to any observable event. These are called *silent transitions*. Moreover, silent transitions may form *silent cycles* so that  $lia(obs(P))$  may include an unlimited number of local histories, all of them being consistent with  $obs(P)$ . For example, if  $h_1 = S_1 \xrightarrow{i_1|m_1} S_2 \xrightarrow{i_2|m_2} S_1$  is consistent with  $obs(P)$ , and the model of  $P$  includes the silent transition  $S_2 \xrightarrow{i|\epsilon} S_2$ , then  $h_2 = S_1 \xrightarrow{i_1|m_1} S_2 \xrightarrow{i|\epsilon} S_2 \xrightarrow{i_2|m_2} S_1$  is consistent with  $obs(P)$  as well, since the silent transition is immaterial for the observation.

In the *lia* algorithm, a non-deterministic reactive model  $M_n$  is first translated into a deterministic model  $M_d$ , in which a connected subgraph of  $M$  involving only silent transitions (called a *silent subgraph*) is collapsed into a new reactive state. The interpretation of  $obs(P)$  is therefore performed on the basis of the new deterministic model  $M_d$ , instead of the original non-deterministic model  $M_n$ . However, once a local history  $h$  is determined, for example  $h = S_1 \xrightarrow{i_1|m_1} S \xrightarrow{i_2|m_2} S_2$ , if  $S$  corresponds to a collapsed silent subgraph, then the actual local history relevant to the original model  $M_n$  will include a so-called *hypertransition*, namely the totality of the paths in  $S$  which start at an initial state  $S_i$

and end at a final state  $S_f$  such that  $S_1 \xrightarrow{i_1|m_1} S_i$  and  $S_f \xrightarrow{i_2|m_2} S_2$  are legal transitions in  $M_n$ . A hypertransition from  $S_i$  to  $S_f$  is denoted by  $S_i \Rightarrow S_f$ . A local history involving a hypertransition is called an *hyperhistory*. In the above example, the hyperhistory will be:  $S_1 \xrightarrow{i_1|m_1} S_i \Rightarrow S_f \xrightarrow{i_2|m_2} S_2$ . An *hyperhistory graph* is the graph representation of the hyperhistory and is obtained by connecting the graph representation of the involved (hyper) transitions.

**Global interpretation.** Once observations of protection components reacting to a misbehavior  $\mu$  are interpreted by *lia*, the set of consistent global histories must be generated.

**Definition 3 (global history)** Let  $[P_1, P_2, \dots, P_m] = \text{extent}(\mu)$ . A *global history*  $H$  of  $\mu$ ,  $H(\mu) = (h_1, h_2, \dots, h_m)$ , is the aggregation of the local histories  $h_1, h_2, \dots, h_m$  relevant to  $P_1, P_2, \dots, P_m$  respectively.

Informally, given two connected protection components  $P_1$  and  $P_2$ , the consistency of two local histories  $h_1$  of  $P_1$  and  $h_2$  of  $P_2$  involved in a global history  $H$  corresponds to the balancing of the interface events involved in  $h_1$  and  $h_2$ . Specifically, each event of  $h_1$  exported by  $P_1$  to  $P_2$  is required to be imported by  $h_2$  and vice versa.

**Definition 4 (global history domain)** Let  $[\text{obs}(P_1), \text{obs}(P_2), \dots, \text{obs}(P_n)]$  be the set of local observations relevant to  $\text{extent}(\mu)$ . The *global history domain* of  $\mu$ ,  $H_d(\mu)$ , is the cartesian product of the local interpretations of  $P_1, P_2, \dots, P_n$ , namely:

$$H_d(\mu) = \text{lia}(\text{obs}(P_1)) \times \text{lia}(\text{obs}(P_2)) \times \dots \times \text{lia}(\text{obs}(P_n)) \quad (5)$$

If  $H \in H_d(\mu)$ ,  $H$  is called a *candidate global history*.

**Definition 5 (global interpretation)** The global interpretation of  $\mu$ ,  $H_g(\mu)$ , is a relation among the local interpretations  $[\text{lia}(\text{obs}(P_1)), \text{lia}(\text{obs}(P_2)), \dots, \text{lia}(\text{obs}(P_n))]$ , namely a subset of  $H_d(\mu)$ :

$$H_g(\mu) \subseteq (H_d(\mu) = \text{lia}(\text{obs}(P_1)) \times \text{lia}(\text{obs}(P_2)) \times \dots \times \text{lia}(\text{obs}(P_n))) \quad (6)$$

so that the following conditions hold:

1.  $\forall H = (h_1, h_2, \dots, h_n) \in H_g(\mu)$ ,  $H$  is globally consistent with respect to the interface constraints, and
2.  $\neg \exists$  a globally consistent  $H' \in H_d(\mu)$  such that  $H' \notin H_g(\mu)$ .

A systematic approach for selecting non spurious global histories is to apply a  $\beta$  function (called *balance*) to every pair of local histories  $(h_1, h_2)$ , where  $h_1 \in \text{lia}(\text{obs}(P_1))$ ,  $h_2 \in \text{lia}(\text{obs}(P_2))$ , and  $P_1$  is connected to  $P_2$ . In the simplest case in which both  $h_1$  and  $h_2$  are plain local histories (not hyperhistories),  $\beta$  is a boolean function checking the consistency of  $h_1$  and  $h_2$  with respect to the interface constraints relevant to  $h_1$  and  $h_2$  only. Informally, each event of  $h_1$  exported by  $P_1$  to  $P_2$  is required to be imported by  $h_2$  and vice versa.

To this end, a *consistency matrix* is used, where the result of each application of  $\beta$  is recorded. A consistency matrix  $\Xi_\mu$  is relevant to a misbehavior  $\mu$  and is composed of rows and columns associated with local interpretations relevant to  $\text{extent}(\mu)$ . Thus,  $\Xi_\mu(h_1, h_2)$  denotes the element of the matrix which corresponds to histories  $h_1$  and  $h_2$ . The *inconsistency set* of  $\Xi_\mu$ , denoted by  $\mathfrak{I}(\Xi_\mu)$  is the set composed of those local history

pairs  $(h_1, h_2)$  for which  $\neg\beta(h_1, h_2)$ . On the basis of the inconsistency set we are allowed to remove the inconsistent global histories from the global history domain  $H_d(\mu)$ . This operation corresponds to the deletion of the global histories which include a pair of the inconsistency set. Therefore,  $H \in H_d(\mu)$  is a consistent global history if and only if the following condition holds:

$$\neg\exists(h_1, h_2) \in \mathfrak{S}(\Xi_\mu) \text{ so that } (h_1, h_2) \subseteq H \quad (7)$$

However, the trouble arises when the approach based on the consistency matrix is extended to deal with hyperhistories: in this case,  $\beta$  is expected to return a more complex information than a boolean value. For example, if  $h$  and  $h^*$  denote a local history and a hyperhistory respectively, three different cases are possible for  $\beta(h, h^*)$ , namely:

1.  $\beta(h, h^*) = \emptyset$ . This is the easiest case since it establishes that every candidate global history involving both  $h$  and  $h^*$  is to be discarded.  $\Xi_\mu(h, h^*)$  is marked with the empty set symbol.
2.  $\beta(h, h^*) = [h_1, h_2, \dots, h_n]$ . In that case the hyperhistory is *resolved*, namely replaced by a finite number of local histories.  $\Xi_\mu(h, h^*)$  is marked with a reference to this set.
3.  $\beta(h, h^*) = h'^* = [h_1, h_2, \dots, h_n, h_{n+1}, \dots], h'^* \subseteq h^*$ . The hyperhistory is *reduced* but not resolved. A reference to the hyperhistory graph of  $h'^*$  is put in  $\Xi_\mu(h, h^*)$ .

But the real problem is how to extend the global history generation process once the  $\Xi_\mu$  has been filled with this information. When all the local interpretations are composed of a limited number of local histories, we are able to enumerate all the candidate global histories  $H \in H_d(\mu)$ . In that case we say that we have an *extensional* representation of histories. By contrast, the possible inclusion of hyperhistories in the local interpretation forces us to maintain a so called *intensional* representation of histories, by means of the hyperhistory graph.

**Definition 6** (*hyper global history*) Let  $H \in H_d(\mu)$  be a candidate global history. If there exists a hyperhistory  $h^* \in H$ , then  $H$  is called an *hyper global history*.

Extending the scope of  $\beta$  to cope with hyperhistories, requires  $\Xi_\mu$  to record a more complex set of information. Specifically, the codomain of  $\beta$  must be represented by a set  $\nabla$  including elements which can be classified as follows ( $h^*$  denotes either a hyperhistory or a finite set of local histories):

$$\nabla = [\text{true}, \text{false}, \emptyset, h^*, (\emptyset, \emptyset), (h^*, \emptyset), (\emptyset, h^*), (h_1^*, h_2^*)] \quad (8)$$

which in turn can be seen as the union of three parts as follows:

$$\begin{cases} \nabla = \nabla_{h, h'} \cup \nabla_{h, h^*} \cup \nabla_{h^*, h'^*} \\ \nabla_{h, h'} = [\text{true}, \text{false}] \\ \nabla_{h, h^*} = [\emptyset, h^*] \\ \nabla_{h^*, h'^*} = [(\emptyset, \emptyset), (h^*, \emptyset), (\emptyset, h^*), (h_1^*, h_2^*)] \end{cases} \quad (9)$$

corresponding respectively to the domain of two local histories, a local history and a hyperhistory, and two hyperhistories.

**Definition 7** (*resolvable hyper global history*) Let  $H^*$  be a candidate hyper global history involving a number of hyperhistories:

$$\begin{aligned} H^* &= (h_1, h_2, \dots, h_k, h_{k+1}^*, h_{k+2}^*, \dots, h_n^*) \\ &= (h_1, h_2, \dots, h_k) \times h_{k+1}^* \times h_{k+2}^* \times \dots \times h_n^* \\ &= [H_1, H_2, \dots, H_i, H_{i+1}, \dots]. \end{aligned} \quad (10)$$

If it is possible to enforce the interface constraints so that  $H^*$  is reduced to a finite set:

$$H^+ = [H'_1, H'_2, \dots, H'_p] \subset H^* \quad (11)$$

then  $H^*$  is called a *resolvable hyper global history*.

**Definition 8** (*symbolic intersection*) Let  $h_1^*$  and  $h_2^*$  be two hyperhistories relevant to the same local observation  $obs(P)$ . The hyperhistory graph corresponding to  $h^* = h_1^* \cap h_2^*$  is called the *intersection graph* of  $h_1^*$  and  $h_2^*$ . The process of deriving the intersection graph is called *symbolic intersection*.

**Definition 9** (*residue*) Let  $H^*$  be a candidate hyper global history. Let  $h^* \in H^*$  be a hyperhistory embraced by  $H^*$ . Let  $h_1^*, h_2^*, \dots, h_n^*$  be the list of reduced hyperhistories relevant to  $\beta(h_i^*, h^*)$ ,  $h_i^* \in H^*$ ,  $h_i^* \neq h^*$ . The *residue* of  $h^*$  in the context of  $H^*$ , denoted by  $\mathfrak{R}(h^*, H^*)$ , is the symbolic intersection of  $h_1^*, h_2^*, \dots, h_n^*$ , namely:

$$\mathfrak{R}(h^*, H^*) = h_1^* \cap h_2^* \cap \dots \cap h_n^* \quad (12)$$

**Global interpretation algorithm.** The *gia* global interpretation algorithm can be concisely described as a function having as input a global history domain  $H_d(\mu)$  and returning a global interpretation  $H^*(\mu) \subseteq H_d(\mu)$  by means of the following steps:

1.  $H^*(\mu) \leftarrow H_d(\mu)$ ;
2. create the consistency matrix  $\Xi_\mu$ ;
3. by applying  $\beta$ , associate with each element of  $\Xi_\mu$  a relevant instantiation of a symbol in  $\nabla$ ;
4. build the inconsistency set  $\mathfrak{I}(\Xi_\mu)$  as composed of the pairs of (possibly hyper) histories  $(h_i, h_j)$  for which:

$$\begin{aligned} \neg\beta(h_i, h_j) \vee \beta(h_i, h_j) = \emptyset \vee \beta(h_i, h_j) = (\emptyset, h_{j'}) \vee \\ \beta(h_i, h_j) = (h_{i'}, \emptyset) \vee \beta(h_i, h_j) = (\emptyset, \emptyset) \end{aligned} \quad (13)$$

5. remove from the global interpretation those global histories  $H$  including a pair  $(h_i, h_j) \in \mathfrak{I}(\Xi_\mu)$ , namely:

$$H^*(\mu) \leftarrow H^*(\mu) - [H \mid \exists (h_i, h_j) \subseteq H, (h_i, h_j) \in \mathfrak{I}(\Xi_\mu)] \quad (14)$$

6. for each hyperhistory  $h^*$  of every hyper global history  $H^* \in H^*(\mu)$ , replace  $h^*$  with the corresponding residue, namely:

$$\forall H^* \in H^*(\mu), \forall h^* \in H^*, h^* \leftarrow \mathfrak{R}(h^*, H^*) \quad (15)$$

**Heuristic interpretation.** The interpretation engine presented so far only concerns concepts which do not depend upon any particular application domain. Potentially, a specific application domain provides a number of additional constraints which can be conveniently enforced on the result of the global interpretation so as to further reduce the number of consistent global histories.

To fix the ideas, consider a short circuit occurring in a transmission network and causing the reaction of the distance protections as shown in Fig. 1.

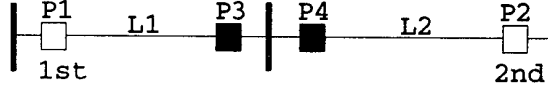


Fig. 1 The isolation produced by the occurrence of a short circuit

Vertical bars denote substations. Each transmission line is protected by a pair of distance protections, namely line  $L_1$  is protected by  $P_1$  and  $P_3$ , while  $L_2$  is protected by  $P_4$  and  $P_2$ . According to the picture, only breakers corresponding to  $P_1$  and  $P_2$  are open after the reaction. Protections  $P_1$  and  $P_2$  are assumed to have operated at first and second step respectively. Intuitively, the operation of a distance protection at a first step means that the protection realizes that the short circuit is on its line, while operating at the second step means that the short circuit is perceived in the next adjacent line (backup behavior).

To capture the faulty behavior of protection components, the corresponding model is enriched by means of *fault transitions*. In a fault transition  $\mathcal{T}$ , the list of output events may possibly include *diagnostic messages*. These messages are not observable, but rather reflect the application-specific knowledge on the faulty behavior of the protection system.

Considering a distance protection  $P$ , a diagnostic message is, for example,  $\phi = \text{delayed}$ . This means that, denoting with  $d_p$  and  $d_r$  the perceived and the real distance of the short circuit respectively, then  $d_r < d_p$ . Conversely,  $\phi = \text{untimely}$  means  $d_r > d_p$ . Obviously, in the correct behavior,  $d_p = d_r$ . Consequently, the completeness of the reactive model requires  $\delta$  to be extended to a new transition function  $\delta^\Phi$  as follows:

$$\delta^\Phi : \Sigma \times (I \cup \epsilon) \times (2^O \cup \epsilon) \times (2^\Phi \cup \epsilon) \rightarrow \Sigma \quad (16)$$

where  $\Phi$  is the set of diagnostic messages. Considering our example and disregarding for the sake of simplicity  $P_3$  and  $P_4$ , we assume:

$$\begin{cases} lia(obs(P_1)) = [h_1, h_1^u] \\ lia(obs(P_2)) = [h_2, h_2^d, h_2^u] \end{cases} \quad (17)$$

where  $h^d, h^u$  means *delayed* or *untimely* behavior respectively. Diagnostic messages lead to additional domain-dependent *distance constraints*. For example, considering a simplified model, the following implications hold:

$$\begin{cases} h_1^u \leadsto (d_r > (d_p = 1)) \\ h_2^d \leadsto (d_r < (d_p = 2)) \end{cases} \quad (18)$$

According to our method, since no interface constraints can be enforced among distance protections (the only interface constraints which should be considered are those between distance protections  $P_i$  and the breakers  $B_i$ ; however, we assume the unavailability of  $obs(B_i)$ ), so that the global interpretation coincides with the global history domain:

$$H_g(\mu) = H_d(\mu) = lia(obs(P_1)) \times lia(obs(P_2)) \quad (19)$$

A further restriction of the global interpretation can be achieved by enforcing the heuristic constraints defined below.

**Heuristic Constraint 1** (*focus of attention*) The short circuit is located within the *isolation*, this being the region delimited by the opened breakers.

**Heuristic Constraint 2** (*single short circuit*) The short circuit is unique within the time interval corresponding to the reaction of the protection system.

Considering our example, the *focus of attention* constraint requires the short circuit to be located either on  $L_1$  or  $L_2$ , but not on an external line. In addition, the *single short circuit* constraint requires the short circuit *not* to be located in *both* lines  $L_1$  and  $L_2$ .

**Definition 10** (*candidate location set*) Let  $\mu$  be a misbehavior triggering the reaction of a distance protection  $P$ . Let  $h$  be a local history of  $P$  relevant to  $\mu$ . The candidate location set of  $h$ , denoted by  $\Lambda(h)$ , is the set of all the possible shorted components consistent with the distance constraints and the *focus of attention* heuristic constraint.

Considering our example, the following location sets are yielded:

$$\begin{cases} \Lambda(h_1) = [L_1] \\ \Lambda(h_1^u) = [L_2] \\ \Lambda(h_2) = [L_1] \\ \Lambda(h_2^d) = [L_2] \\ \Lambda(h_2^u) = \emptyset \end{cases} \quad (20)$$

The application of the *single fault* heuristic constraint leads to intersect all the candidate location sets relevant to the local histories combined in a global history, so that the global history is consistent with respect to the heuristic constraints if and only if the resulting intersection evaluates a non empty set.

**Definition 11** (*location set*) Let  $\mu$  be a misbehavior. Let  $H = (h_1, h_2, \dots, h_n)$  be a global history for  $\mu$ . The *location set* of  $H$ , denoted by  $\Lambda(H)$ , is the intersection of all the candidate location sets relevant to  $h_1, h_2, \dots, h_n$ , namely:

$$\Lambda(H) = \Lambda(h_1) \cap \Lambda(h_2) \cap \dots \cap \Lambda(h_n) \quad (21)$$

The enforcement of the *single short circuit* heuristic constraint is equivalent to have a non empty location set, namely:

$$\Lambda(H) = \Lambda(h_1) \cap \Lambda(h_2) \cap \dots \cap \Lambda(h_n) \neq \emptyset \quad (22)$$

In our example, the non-empty location sets are the following:

$$\begin{cases} \Lambda(h_1, h_2) = [L_1] \\ \Lambda(h_1^u, h_2^d) = [L_2] \end{cases} \quad (23)$$

Thus, only these two global histories meet the heuristic constraints. Consequently, the final diagnosis  $\Delta$  of  $\mu$  will be as follows:

$$\begin{cases} \Delta(\mu) = [\delta_1(\mu), \delta_2(\mu)] \\ \delta_1(\mu) = [shorted(L_1)] \\ \delta_2(\mu) = [shorted(L_2), untimely(P_1), delayed(P_2)] \end{cases} \quad (24)$$

where  $\delta_1$  and  $\delta_2$  are the *diagnostic alternatives* of  $\Delta$ .

**Heuristic interpretation algorithm.** The heuristic interpretation algorithm is a function having as input a global interpretation  $H_g(\mu)$  and returning a subset of the global interpretation,  $H_h(\mu) \subseteq H_g(\mu) \subseteq H_d(\mu)$  by means of the following steps:

1.  $H_h(\mu) \leftarrow H_g(\mu)$ ;
2.  $\forall$  local history  $h$  relevant to a distance protection find the candidate location set  $\Lambda(h) = [\lambda_1, \lambda_2, \dots, \lambda_n]$  by enforcing the *focus of attention* heuristic constraint;
3. by enforcing the *single short circuit* heuristic constraint,  $\forall$  candidate global history  $H = (h_1, h_2, \dots, h_m) \in H_h(\mu)$ , yield the corresponding location set, namely:

$$\Lambda(H) = \Lambda(h_1) \cap \Lambda(h_2) \cap \dots \cap \Lambda(h_m) \quad (25)$$

4. remove from  $H_h(\mu)$  those global histories for which the location set evaluates empty, namely:

$$H_h(\mu) \leftarrow H_h(\mu) - [H \mid \Lambda(H) = \emptyset] \quad (26)$$

5.  $\forall H \in H_h(\mu)$ , extend  $H$  by means of the corresponding element of the location set and the relevant diagnostic messages.

**Conclusions.** In this paper an original approach to model-based diagnosis of event-based dynamic systems is presented and applied to the problem of short circuit localization in power transmission networks. However, to be useful, the method requires the availability of a set of observations relevant to the reacting components. Intuitively, the richer is the set of observations, the smaller is the number of diagnostic alternatives produced. In the ideal scenario, only a single diagnostic alternative is generated.

The implementation of the proposed technique is under way. The chosen programming language is C++. Moreover, as local interpretations might in principle be generated in parallel, a further implementation based on a parallel architecture is planned for the future.

The resulting system will be tested by ENEL, the Italian electricity board, using the transmission network of part of northern Italy

**Acknowledgments.** The present work is partially supported by the Commission of the European Union under the ongoing Esprit III Project 8491 *Timely* (Time-Constrained Integrated Management of Large-Scale Systems).

## References

1. Kezunovic, M., I. Rikalo, D.J. Sobajic (1994). Neural network applications to real-time and off-line fault analysis. Proc. ISAP 94 Int. Conf. on Intelligent System Application to Power Systems, Montpellier, F, pp. 29-36.
2. Cho, H.-J., J.-K. Park, H.-J Lee (1994). A fuzzy expert system for fault diagnosis of power systems. Proc. ISAP 94 Int. Conf. on Intelligent System Application to Power Systems, Montpellier, F, pp. 217-226.

3. Wang, J.P., J. Trecat (1994). A parallel fault diagnosis expert system for one dispatch center. Proc. ISAP 94 Int. Conf. on Intelligent System Application to Power Systems, Montpellier, F, pp. 201-208.
4. Tornielli, G., L. Capetta, S. Cermignani, A.S. Fabiano, R. Schinco (1994). An interval-based approach to fault diagnosis of power systems. Proc. ISAP 94 Int. Conf. on Intelligent System Application to Power Systems, Montpellier, F, pp. 809-816.
5. Baroni, P., U. Canzi, G. Guida (1995). SHORT: a knowledge-based system for fault diagnosis in power transmission networks, Proc. ISAI 95, 8th Int. Symp. on Artificial Intelligence, Monterrey, MX, pp. 199-208.
6. Beschta, A., O. Dressler, H. Freitag, M. Montag, P. Struss (1993). A model-based approach to fault localisation in power transmission networks. Intelligent Systems Engineering, pp. 3-14.
7. Struss, P., O. Dressler (1989). Physical negation-integrating fault models into the general diagnostic engine. Proc. IJCAI-89, Detroit, Michigan, pp. 1318-1323.
8. Riese, M. (1993). Diagnosis of communicating systems: dealing with incompleteness and uncertainty. Proc. IJCAI-93, Chambéry, F, pp. 1480-1485.
9. Dvorak, D., B. Kuipers (1989). Model-based monitoring of dynamic systems. Proc. IJCAI-89, Detroit, MI, pp. 1238-1243.
10. Gluckenbiehl, T., G. Schäfer-Richter (1990). SIDIA: Extending prediction based diagnosis to dynamic models. Working Notes of the 1st Int. Workshop on Principles of Diagnosis, Stanford, CA, pp. 74-82.
11. Hamscher, W. (1991). Modeling digital circuits for troubleshooting. Artificial Intelligence, 51 (1-3), 223-271.
12. Ng, H.T. (1991). Model-based, multiple-fault diagnosis of dynamic, continuous physical devices. IEEE Expert, 6(6), 38-43.
13. Lackinger, F., W. Nejdl (1991). Integrating model-based monitoring and diagnosis of complex dynamic systems. Proc. IJCAI-91, Sydney, Australia, pp. 1123-1128.
14. Dressler, O., H. Freitag. Prediction sharing across time and contexts. Proc. 12th Nat. Conf. on Artificial Intelligence AAAI-94, Seattle, WA, pp. 1136-1141.
15. Pan, J. Y.-C. (1984). Qualitative reasoning with deep-level mechanism models for diagnosis of mechanism failures. Proc. 1st IEEE Conf. on AI Applications, Denver, CO, pp. 295-301.
16. Hamscher, W., R. Davis (1984). Diagnosing circuits with state: an inherently under-constrained problem, Proc. 4th Nat. Conf. on Artificial Intelligence AAAI-84, Austin, TX, pp. 142-147.
17. Reiter, R. (1987). A theory of diagnosis from first principles. Artificial Intelligence, 32(1), 57-95.
18. de Kleer, J., B.C. Williams (1987). Diagnosing multiple faults. Artificial Intelligence, 32(1), 97-130.
19. Friedrich, G., F. Lackinger (1991). Diagnosing temporal misbehavior. Proc. IJCAI-91, Sydney, Australia, pp. 1116-1122.
20. Hopcroft J.E., J.D. Ullman (1979). Introduction to automata theory, languages and computation. Addison-Wesley, Reading, MA.

# DEVELOPMENT OF ALGORITHMICAL PROVISION FOR THE CALCULATION OF SYSTEMS' RELIABILITY AND SAFETY ON THE BASIS OF TUPLE ALGEBRA

Boris Kulik

Scientific & Production Association "Aurora"  
of the Ministry of Defence Industry of Russian Federation  
15, Karbysheva Street, 194021, St. Petersburg, Russia  
Telephone: +7(812)316-8244, Fax: +7(812)247-8061.

**Abstract.** The author suggests the use of the mathematical instruments of tuple algebra for solving algorithmical problems, which originate during the evaluation of systems with complicated structure. These instruments are able to reduce time-consuming calculations of difficult problems concerning logical control. Simultaneously, arises the possibility to expand the class of systems, in which efficient algorithms for reliability and safety evaluation can be realised. It is possible due to the incorporation of systems, which are described by indicative means of predicates and multivalued logic calculation.

**Keywords:** logical and probability methods, probability systems, algebra of multitudes, tuple algebra, reliability and safety evaluation algorithms.

**Introduction.** When solving the problem of evaluation related to the reliability and safety of structural complicated systems on the basis of logical and probabilistic methods (LPM) [1-3], a number of problems originates. These problems are connected with the time-consuming procedure of probability calculations, assigned as functional schemes or complicated logical formulae. Along with this, during the simulation of the systems, in which the number of states of several or all elements is more than two, emerges a problem of their adequate simulation. In this case, system decomposition is being used, i.e. its conversion into isostructural system with other composition of elements, where the number of states is not more than two. Such conversion is theoretically possible, but is very complicated for a number of systems, it requires time-consuming calculations and manual processing, besides, in some cases it is practically impossible to realise.

The author suggests to use mathematical instruments of tuple algebra (TA) [4-5] for solving these problems. These instruments were developed at the "Aurora" Corporation as a means for reducing the difficulties, related to algorithms problem-solving during the program realisation of intellectual support for decision-making in complicated control systems. In the process of further development and testing it became clear, that TA can be used for solving many other problems of logical control, in particular, during the simulation of probability systems [6-8]. The TA-based approach for probability systems' simulation enables the simplification of the solution of some complicated problems of LPM and, therefore, the problems related to the evaluation of structural complicated systems' reliability and safety.

**Theoretical Principles of Tuple Algebra.** The algebra of multitudes is the basic mathematical instrument of TA. Despite wide familiarity of the term "algebra of multitudes" (AM), its understanding and interpretation among experts is not ambiguous. Many of them identify AM with Boolean algebra, and in principle this is not true, though there is a close connection between these notions. The rules of AM and Boolean algebra are identical, but one can separate a feature in AM, which is not used in Boolean algebra and its applications. First of all,

this feature enables on the basis of AM, the transfer from statements' calculation patterns to more universal patterns of predicates' calculation, secondly, to simulate and develop an effective algorithmical basis for measurable (and, in particular probability) systems. This feature means, that AM is a specific case of algebra of rings and semirings, which serves as a basis for plotting practical, valid versions of the gage theory [9].

In the course of theoretical research it was determined, that the structures of algebra of multitudes are applicable for the simulation of many systems for logical control, which are being used and where indicative means of predicates are used. But, in order to provide this possibility, it was required to add traditional algebra of multitudes with other mathematical structures, which is very seldom used in "pure" algebra of multitudes. This structure is a Cartesian product of multitudes (CPM). Let us see, how the mathematical instrument of the algebra of multitudes expands, due to the employment of CPM. Assume that conjunction (AND) of predicates with different variables is assigned:

$$F=P(x) \& Q(y) \& \dots \& R(z). \quad (1)$$

Let  $S(F)$  be a finite or infinite multitude of all processing substitutions of (1) formula. Then, if  $P, Q, \dots, R$  — multitudes of processing substitutions of corresponding predicates  $P(x), Q(y), \dots, R(z)$ , equitably the ratio:

$$S(F) = P \times Q \times \dots \times R, \quad (2)$$

where  $P \times Q \times \dots \times R$  — CPM of corresponding multitudes.

In order to reflect CPM-based disjunction (OR) of the same predicates, let us enter  $X, Y, \dots, Z$  multitudes domains of values of corresponding variables  $x, y, \dots, z$  and universal set  $U_t = X \times Y \times \dots \times Z$ . Then, the multitude  $S(G)$  of processing substitutions of the formula

$$G = P(x) \vee Q(y) \vee \dots \vee R(z). \quad (3)$$

If to use the Morgan equation

$$P(x) \vee Q(y) \vee \dots \vee R(z) = \neg(\neg P(x) \& \neg Q(y) \& \dots \& \neg R(z)),$$

will be equal to the multitude of processing substitutions

$$S(G) = U_t \setminus (\bar{P} \times \bar{Q} \times \dots \times \bar{R}) \quad (4)$$

where  $\bar{P}, \bar{Q}, \bar{R}$  are equal correspondingly to  $X \setminus P, Y \setminus Q, \dots, Z \setminus R$ . This result can be received, without using operation of multitudes difference:

$$S(G) = (P \times Y \times \dots \times Z) \cup (X \times Q \times \dots \times Z) \cup \dots \cup (X \times Y \times \dots \times R) \quad (5)$$

The ratios (1)-(5) were taken as a basis in TA and it enabled the use of the instrument of algebra of multitudes, not only for operations and correlations with traditional finite or infinite multitudes, but with multitudes, which are presented in applied systems of logical control as

multiple predicates. The correlation between the TA indicative means with the indicative means of multi-sort calculation of predicates was determined. This correlation can be expressed by the following principle notions and structures.

1. **Co-ordinate** — random multitude  $X$  ; in logical patterns is interpreted as the domain of values of some variable  $x$  ;
2. **Co-ordinate operand** — random sub-multitude of co-ordinates; in logical patterns is interpreted as predicate ;
3. **Sort** - the class of different co-ordinates, elements, equal in composition ;
4. **Specific universal set** — co-ordinate or Cartesian production of some coherence of different co-ordinates ;
5. **Flexible universal set** — multitude of all possible specific universal sets in the exact logical system ;
6. **Tuple** — coherence of components, corresponding the coherence of co-ordinates of some specific universal set. The notion "tuple" in TA is identical to the notion of "vector", but this vector components' composition has not only individual values of "co-ordinates", but its random domains (points, multitudes of points, intervals and systems of intervals, in common case — sub-multitudes of "co-ordinates").

The notion of the component in TA approximately corresponds to the variable in traditional logical patterns, but instead of "variables", in TA, it is possible to make substitutions of single values and besides this, the substitution of random sub-multitudes, which are included in the domain of this "variable". It considerably expands the indicative and algorithmical capabilities of mathematical patterns, related to applied logical systems.

Three types of tuples are used in TA and two types of matrix structures. Let us first consider the types of tuples.

The first type is an **elementary tuple**, which is a coherence of elements from  $N$  of different multitudes, assigned as co-ordinates. Elementary tuple is interpreted as some element of  $N$ -space ratio or as a processing substitution of a logical formula, containing  $N$  of variables.

The second type — **C-tuple**, which is a coherence of random sub-multitudes from  $N$  different multitudes, assigned as co-ordinates.  $N$ -space ratio is an interpretation of C-tuple. This  $N$ -space ratio is equal to CPM of its components or to the multitude of formula processing substitutions, assigned as conjunction type (1). Elementary tuple is a specific case of C-tuple.

The third type — **D-tuple**, which as C-tuple is a coherence of co-ordinates' sub-multitudes, but its interpretation is a  $N$ -space ratio, which can be considered as a multitude of processing substitutions of disjunction type (3) and which can be developed with the use of ratios (4) or (5).

Matrix structures contain two types: **C-systems** — matrixes, in which C-tuples are the lines, simultaneously, each column have sub-multitudes of the same co-ordinate, and **D-systems** — matrixes, containing D-tuples at the same condition. C-systems are interpreted as a combination of incorporated C-tuples, and D-systems are interpreted as an intersection of D-tuples. At the same time, intersection of C-tuples is always equal to a single C-tuple or blank tuple. It means, that in some fixed universal set C-tuples is the algebra of semirings. It enables, comparatively easily, the immersion of TA structures (they are called TA-objects) into measurable and in particular, into probability systems. In predicates' calculation C-systems are

interpreted as some types of disjunction normal form (DNF) and D-systems as some types of conjunction normal form (CNF). C- and D-classes are alternative classes, for which the ratios of duality are true, similar to the ratios of duality between conjunctions and disjunctions of predicates, DNF and CNF.

The **schema of ratio** is an important notion of TA. It contains the name of the TA-object and the coherency of co-ordinates. The TA-object is assigned in the space of these co-ordinates. Schema of ratio corresponds to the formal designation of the multiple predicate. For example, the predicate  $P(x, y, z)$  is interpreted in TA as a TA-object  $P$ , assigned in co-ordinates  $X, Y, Z$ . If the coherence of co-ordinates in schemes of ratios of two TA-objects matches, they are considered as single-type, otherwise, different types. Two null components are entered in order to make operations and correlations of different type TA-objects. These null components are the multitude, equivalent to the corresponding co-ordinate and  $\emptyset$  is an empty set.

The component can be inserted in any C-tuple, if its schema of ratio does not contain the co-ordinate, which corresponds to it, and the component  $\emptyset$  can be inserted in any D-tuple under the same condition. In the case, when the TA-object contains two tuples and more, all null elements in one column must correspond to the same co-ordinate. If we assume, that component  $*$  and the universal set correspond to the constant True in the predicates' calculation, and the component  $\emptyset$  and the blank tuple correspond to constant False, we can prove, that any pair of TA-objects can be turned into a single-type scheme of ratio, by adding null components and rearranging co-ordinates. The logical equivalence will be preserved (for all the equivalence of ratios during these conversions is not preserved).

A simple and easily interpreted axiomatics is formed: three axioms are added to the axioms of algebra of multitudes:

**AS:** the system of sub-multitudes (components) for any co-ordinate is an algebra of multitudes;

**ACP:** CPM among which there is at least one blank multitude, is a blank multitude (the axiom of blank multitude);

**AFU:** all specific universal sets of TA belong to the same class of equivalency ( the axiom of flexible universal set).

The meaning of AFU is so, that for any specific universal set  $U_i$  of TA system, which is equal to CPM of some coherency of co-ordinates, in predicates' calculation one can correlate the arbitrary formula of general importance, where the composition of variables corresponds to the composition of co-ordinates of the specific universal set  $U_i$ . In other words, the class of all specific universal sets of TA random system in this axiom, is identified with the class of all formulae of general importance related to the corresponding pattern of the predicates' calculation.

More than 20 theorems are formulated and proved on the basis of these axioms, which constitute the main instrument of TA [6, 7]. In the calculation of statements and predicates, these theorems are corresponded by many (and often not seldom) widely-used ratios. Effective methods of random TA-objects' conversion into equivalent TA-objects of an alternative class are received on the basis of these theorems, along with that, it was determined that:

1. all operations and correlations with TA-objects are tabulated to algorithms, in which operations' coherence or correlations over multitudes, which are the components of these TA-objects are realised;
2. any pattern, which is built as a system of TA-objects is isomorphous to some algebra of multitudes; simultaneously, isomorphism is also preserved in cases, when some co-ordinates appear as multivariate, discrete, continuous or discrete-continuous spaces;
3. all indicative means of patterns' theory on the basis of predicates' calculation are realised in random system of TA-objects, which represents this or that logical pattern.
4. all logical axioms and rules for statements' derivation and predicates' calculations are derived from TA axioms on the basis of derivation rules, which are used in the algebra of multitudes.

Solution of logical equations, for example type  $F_i = F_k$  appear in TA to isomorphous conversion of formulae  $F_i$  and  $F_k$  into TA-objects  $S(F_i)$  and  $S(F_k)$  with single-type schemes of ratio, and to the calculation of TA-object class  $C$ -, equal to interlacing  $S(F_i) \cap S(F_k)$ .

Completeness and consistency of random TA-systems comes from isomorphism of these systems for the algebra of multitudes. However, logical systems, where alternative formulae  $F$  and  $\sim F$  are formed independently, can be carried over TA-system. Then, during the conversion of such formulae in TA-objects  $S(F)$  and  $S(\sim F)$  situations can occur, when:

$$1) S(F) \cap S(\sim F) \neq \emptyset \quad \text{and} \quad 2) S(F) \cup S(\sim F) \neq U.$$

Situation 1 corresponds variance — in this case, suggestion about the mutual addition of  $F$  and  $\sim F$  is not confirmed and the initial logical ratios must be adjusted. Situation 2 corresponds incompleteness of alternative formulae. In this case, not only adjustment is available, but the feasibility to determine the multitude  $U \setminus (S(F) \cup S(\sim F))$  as an intermediate (transient) multitude is possible, if this situation is reasonable in the simulated system (for example, transient state between normal and emergency situations).

*Presentation of Probability Systems with the Help of Tuple Algebra Structures.* Let us consider the enlarged pattern of the system with a finite number of elements, where each element of the system has finite number of states. At the same time, a wide range of variants is available for assigning the multitude of system elements' states:

1. the element has two states (for example, operative condition and failure) and a probability evaluation is known for each of these states;
2. the element has a finite number (more than two) of independent states with assigned probability evaluations;
3. the state of the element is assessed by the finite system of intervals on a continuous gage, and the probability distribution is known for this gage. It enables the evaluation of the probability of any state of the element, or logical ratios between some of its conditions (conjunction, disjunction, implication, etc.);
4. the state of the element is evaluated by several parameters, each of them relates to one of the previous variants.

The system must adequately represent all possible logical connections between different states of various elements. It is necessary to arrange accurate common pattern of the system. These logical connections are normally assigned as functional schemes, with the help of which

genetic relations between states of different elements are mapped in the form of logical formulae or in the form of the system of logical equations. If such a logical pattern is arranged, its presentation in TA structures is made as follows:

1. the space of states of each element is presented as an independent co-ordinate of some multitude of co-ordinates, in this case each selected state of the element is a component of the corresponding co-ordinate;
2. all logical ratios between states of the elements are converted into TA ratios (TA instruments present each pattern of statements or predicates' calculation as corresponding to TA-objects or some ratios between them (for example, as ratios of switching or equivalency).

We can ask, what advantages do we receive in the result of these unaccustomed, and in some cases not very simple conversions? During simulation of probabilistic systems two complicated problems appear: 1) the problem of systems' adequate simulation, for which the description of logical connections between states of the elements is not stacked in the frames of statements' calculation or Boolean algebra and requires attraction of means for calculation of predicates or multiple-valued logic, which are complicated for automatic realisation; 2) the problem of orthogonalization of logical ratios for the evaluation of the probability gage (measure) of this or that state of the whole system. These problems are more easily solved within TA, compared to the employment of traditional methods of logical simulation.

*Conclusion.* It is necessary to note, that now the solution of these problems, in general, is presented as a theoretical grounding, but not in the form of an application package. However, during theoretical research related to TA there was received a number of practical results, which confirmed the expediency of TA application for the simulation and PC-based realisation of many applied systems of logical control, in particular:

- new methods for the reduction of difficulties of algorithms related to the solution of NP-complete problems, which are often met in the application systems of logical control, have been developed and partially realised in PC-based programs;
- unified specification for different finite structures of data have been developed. It has enabled the replacement of ineffective listing structures of data for data structures of logical vectors and Boolean matrixes' type, during current intellectual tasks solving;
- the program of NP-complete problem quick solution "feasibility of CNF" has been developed for large formulae of statements' calculation. On the basis of this program there was developed a program of orthogonalization of statements' calculation, which is applicable for the calculation of reliability and safety values of complicated systems.

Programs, which have been developed during this research work have successfully passed examinations, led by leading experts on logical and probability methods, related to studies on reliability and safety of technical systems. Some results of these researches were used in scientific and research works (SRW) and experimental design works (EDW) during the development of ships' damage control systems.

## *References*

1. Rjabinin I.A., Cherkesov G.N. Logical and probability methods for the studies of structurally complicated systems. M., "Radio and Communication". (1981). Page 264.

2. Rjabinin I.A. The concept of the safety logical and probability theory. Devices and control systems. N 10. (1993). Pages 6-9.
3. Theory and information technology related to simulation of complicated systems' safety. Issue 1-5/ Edited by Rjabinin I.A. and Solozentsev E.D. St. Petersburg. (1994-1995). IPMASH( Institute of machinery problems) Russian Academy of Sciences. Page 413.
4. Kulik B.A. System of logical programming on the basis of tuple algebra. News of Russian Academy of Sciences. Technical cybernetics. (1993). N 3. Pages 226-239.
5. Kulik B.A. Mathematical pattern of deductive database based on tuple algebra. News of Russian Academy of Sciences. Technical cybernetics. (1994). N 2. Pages 161-169.
6. Kulik B.A. Logical and probability methods and tuple algebra. Theory and information technology of complicated systems' safety simulation. Issue 5/ Edited by Rjabinin I.A. and Solozentsev E.D. Preprint 123. St. Petersburg. (1995). IPMASH Russian Academy of Sciences. Pages 18-43.
7. Kulik B.A. Presentation of logical systems in the probability domain based on tuple algebra. The basis of tuple algebra. Automatics and telemechanics. (1997). N 1. Pages 126-136.
8. Kulik B.A., Naumov M.V. Presentation of logical systems in the probability domain based on tuple algebra. 2. Measurable logical systems. Automatics and telemechanics. (1997). N 2. Pages 169-179.
9. Kolmogorov A.N., Fomin S.V. Elements of functions and functional analysis theory. Nauka. (1972). Page 496.

# THE BINARY DYNAMIC SYSTEMS SAFETY CONTROL

Boris Victorovich Greck, Leonid Georgievich Tomashevsky  
Naval Academy  
73/1, Vyborgskaya Embankment  
St. Petersburg, Russia

**Abstract.** This article discusses the safety ensurance for various (by their physical nature) systems from different areas of human activities, which can be limited by the binary dynamic systems (BDS) class. The safety control methodology is suggested. It includes subject area formalizing, mathematical description of the object for the control and a binary model for threatening events, control criterion, problems statement, description of basic approaches to the algorithmization of safety control processes and methods for developing the control actions.

**Keywords.** Safety, control, binary dynamic systems, damage, catastrophe, accident, control criterion, model, equation, method, algorithm.

**Preliminary notes.** The safety concept is connected with consequences of object functioning as a result of external influences of environment, which are various by force and intensity. Safety characterizes the object ability to function without transforming into dangerous states threatening to people's health and life or causing a large-scale damage [1]. Therefore, the safety focus is a notion of a dangerous state or event which can lead to the large-scale dangerous consequences.

One of the safety ensurance trends represents competent personnel operations and control actions which are developed by automatic machines in accordance with specific algorithms.

The control actions can prevent undesirable development of events (with minimum expenditures and losses), avoid an accident and catastrophe or promote their happening. The existing theory of the control possesses a large store of mathematical means, methods and algorithms making possible to decide successfully various problems in different areas.

**The object for control.** The binary dynamic system (BDS) implies the object for the control, which consists of the set of elements  $E$  with junction relations  $T$  among them [2]:  $E = \{e_1, e_2, \dots\}$ .  $T$  - junction relations can be specified by vertex incidence matrices, incidence matrices, or lists of pairs being connected by an inputs/outputs of the elements:  $T = \{\dots, (e_i T e_j), \dots\}$ , where  $e_i$  and  $e_j$  - the elements which are in the junction relation. The elements of the system are characterized by binary parameters:

$$\Pi(e) = \{x(e), z(e), u(e), y_{\text{input}}(e), y_{\text{output}}(e)\},$$

where  $x(e)$ ,  $z(e)$  are binary variables of functioning and serviceability;  $u(e)$  - binary variable of control;  $y_{\text{input}}(e)$ ,  $y_{\text{output}}(e)$  - are binary variables of input/output.

Mathematical model of BDS is a finite Mealy automation with dynamics equations which include the Boolean equations of state and output:

$$X(t+1)=[AX(t)\oplus BU(t)]\cdot CZ(t),$$

$$Y_{\text{output}}(t)=F(X(t)) \cdot Y_{\text{input}}(t),$$
(1)

$X(t)$ ,  $U(t)$ ,  $Z(t)$  are respectively vectors of the state, control and serviceability of BDS at a discrete moment of time  $t$ ;

$F(X(t))$  - the Boolean function which represents a set of values of input  $Y_{\text{input}}(t)$  into a set of values of output  $Y_{\text{output}}(t)$  and is defined on the set  $X(t)$ ;  $A$ ,  $B$ ,  $C$  are diagonal matrices which are conforming the dimension of vectors  $X(t)$ ,  $U(t)$  and  $Z(t)$ ;

“ $\cdot$ ”, “ $\oplus$ ” are AND - operations and OR - operations by module 2.

**The Control Criterion.** The binary model of the event, occurrence of which leads to disastrous consequences, is entered on the set of binary outputs of BDS. For a example, we can give an explosion of a storage battery (SB), which took place aboard the submarine. The explosion scenario was made by A.N.Goncharuk [1]. On Fig.1 one can see the graphic representation of binary model of such explosion. On the same Figure, (at the bottom), the list of model elements is represented.

The output elements of BDS, which represent the control object, are shown at zero level. In this case

$$E = \{e^0_1, e^0_2, \dots, e^0_{10}\}; \quad T = \emptyset.$$

According to (1), the next equations show BDS elements functioning:

$$\begin{aligned} x^0_1(t+1) &= [x^0_1(t) \oplus u^0_1(t)] \cdot z^0_1(t), \\ x^0_2(t+1) &= [x^0_2(t) \oplus u^0_2(t)] \cdot z^0_2(t), \\ &\dots\dots\dots \\ x^0_{10}(t+1) &= [x^0_{10}(t) \oplus u^0_{10}(t)] \cdot z^0_{10}(t); \end{aligned}$$
(2)

$$\begin{aligned} y^0_{\text{output}1}(t) &= x^0_1(t) \cdot y^0_{\text{input}1}(t), \\ y^0_{\text{output}2}(t) &= x^0_2(t) \cdot y^0_{\text{input}2}(t), \\ &\dots\dots\dots \\ y^0_{\text{output}10}(t) &= x^0_{10}(t) \cdot y^0_{\text{input}10}(t). \end{aligned}$$
(3)

The binary model of a storage battery explosion has been developed by replacing its constituents with the binary variables. The occurrence of event is noted by the state variables written through the binary input variables:

$$\begin{aligned} x^1_1(t) &= y^1_{\text{input}11}(t) \cdot y^1_{\text{input}12}(t), \\ x^1_2(t) &= y^1_{\text{input}21}(t) \cdot y^1_{\text{input}22}(t), \\ x^1_3(t) &= y^1_{\text{input}31}(t) + y^1_{\text{input}32}(t) + y^1_{\text{input}33}(t), \\ x^2_1(t) &= y^2_{\text{input}11}(t) + y^2_{\text{input}12}(t), \\ x^2_2(t) &= y^2_{\text{input}21}(t) + y^2_{\text{input}22}(t), \\ x^3_1(t) &= y^3_{\text{input}11}(t) \cdot y^3_{\text{input}12}(t) \cdot y^3_{\text{input}13}(t) \cdot y^3_{\text{input}14}(t). \end{aligned}$$
(4)

The output variables of model elements are compared with the state variables:

$$\begin{aligned}
y^1_{\text{output } 1}(t) &= x^1_1(t), & y^2_{\text{output } 1}(t) &= x^2_1(t), \\
y^1_{\text{output } 2}(t) &= x^1_2(t), & y^2_{\text{output } 2}(t) &= x^2_2(t), \\
y^1_{\text{output } 3}(t) &= x^1_3(t), & y^3_{\text{output } 1}(t) &= x^3_1(t);
\end{aligned} \tag{5}$$

and the input variables of the model are compared with the output variables in accordance with junction relations (look at the Figure):

$$\begin{aligned}
y^1_{\text{input } 11}(t) &= y^0_{\text{output } 2}(t), & y^2_{\text{input } 11}(t) &= y^0_{\text{output } 1}(t), \\
y^1_{\text{input } 12}(t) &= y^0_{\text{output } 3}(t), & y^2_{\text{input } 12}(t) &= y^1_{\text{output } 1}(t), \\
y^1_{\text{input } 21}(t) &= y^0_{\text{output } 4}(t), & y^2_{\text{input } 21}(t) &= y^1_{\text{output } 2}(t), \\
y^1_{\text{input } 22}(t) &= y^0_{\text{output } 5}(t), & y^2_{\text{input } 22}(t) &= y^0_{\text{output } 6}(t), \\
y^1_{\text{input } 31}(t) &= y^0_{\text{output } 8}(t), & y^3_{\text{input } 11}(t) &= y^2_{\text{output } 1}(t), \\
y^1_{\text{input } 32}(t) &= y^0_{\text{output } 9}(t), & y^3_{\text{input } 12}(t) &= y^2_{\text{output } 2}(t), \\
y^1_{\text{input } 33}(t) &= y^0_{\text{output } 10}(t), & y^3_{\text{input } 13}(t) &= y^0_{\text{output } 7}(t), \\
& & y^3_{\text{input } 14}(t) &= y^1_{\text{output } 3}(t).
\end{aligned} \tag{6}$$

The safety control criterion is entered on a binary model of threatening event and represents sums (which had been ordered on the levels) of binary variables of the elements states:

$$K = (\sum_i x_i^r(t), \sum_i x_i^{r-1}(t), \dots, \sum_i x_i^0(t)). \tag{7}$$

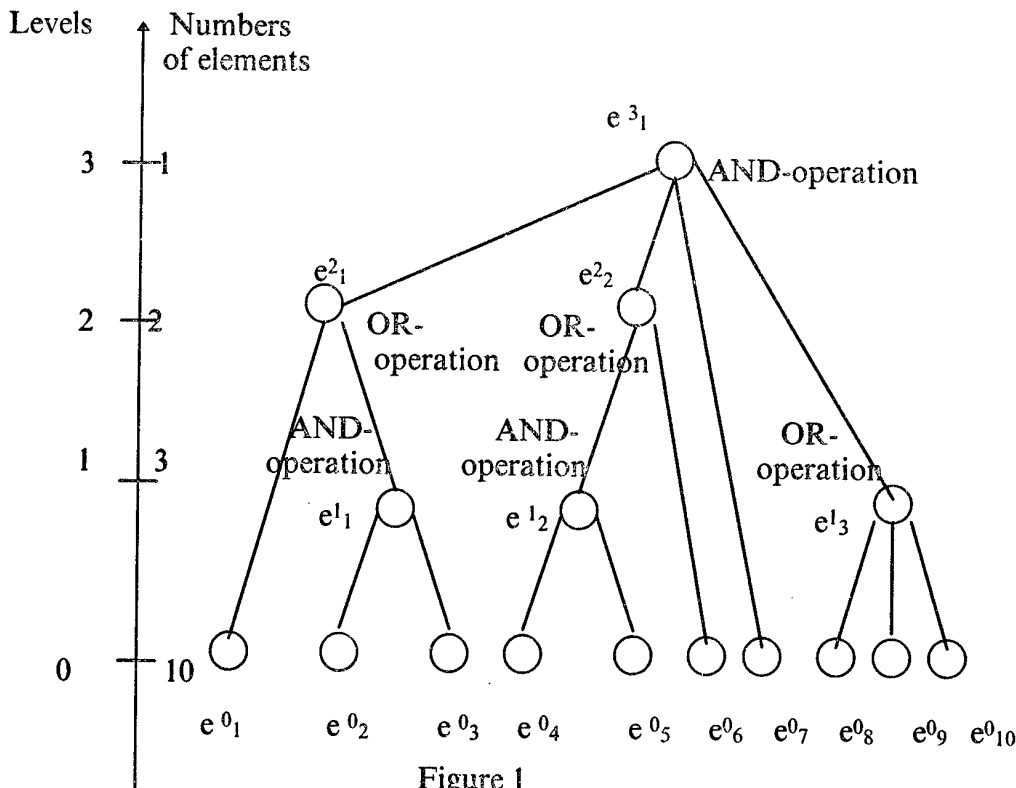


Figure 1  
Graphic representation of a storage battery explosion model

$e^0_1$  - personnel mistake;  $e^0_2$  - portable gas analyzer refusal;  $e^0_3$  - stationary gas analyzer refusal;  $e^0_4$  - re-heating automation refusal;  $e^0_5$  - gas - analyzer refusal;  $e^0_6$  - fan refusal;  $e^0_7$  - not manually-started-up fan;  $e^0_8$  - sparkling on the fan;  $e^0_9$  - sparkling at the bus;  $e^0_{10}$  - personnel smoking;  $e^1_1$  - gas analyzer refusal;  $e^1_2$  - automatic system refusal;  $e^1_3$  - ignition source availability;  $e^2_1$  - hydrogen control absence;  $e^2_2$  - not-automatically-started-up fan;  $e^3_1$  - hydrogen explosion.

The more is the safety criterion value, the higher is a danger of occurrence of a threatening event. The less is the safety criterion value, the lower is the level of danger. For our example, being considered here, the maximum and minimum values of the structurized safety criterion are represented below:

$$K_{\max} = (1, 2, 3, 10); \quad K_{\min} = (0, 0, 0, 0).$$

**The Control task.** The safety criterion depends on the states of X-elements of the model and state vector of BDS is a function of the controlling vector U. Therefore, according to the scale of values, a problem of safety control is formulated in the next optimum manner:

$$\hat{U} = \arg \min_{U \in \tilde{U}} K[(AX(t) \oplus BU(t)) \cdot CZ(t)] \quad (8)$$

where  $\hat{U}$  is the optimum control actions,  $\tilde{U}$  is a set of the admissible control actions. Limitation for a set of BDS states can be formulated additionally, because the optimum control is in agreement with the optimum condition of the system:

$$\hat{X} = \arg \min_{X \in \tilde{X}} K(x) \quad (9)$$

where  $\hat{X}$  is the optimum condition of BDS;  $\tilde{X}$  is a set of admissible states of BDS. When formulating a problem of the control, the input variables  $Y_{\text{input}}(t)$  of the control object, initial state vector  $X(t)$ , control vector  $U(t)$ , and the elements serviceability vector  $Z(t)$  are initial data.

With the specific values of these binary vectors BDS will trace out a trajectory in the space of the states; every point of this trajectory is characterized by specified value of safety criterion K. For a practical implementation of control the specific regions corresponding to "the hottest" measures for ensuring safety are predetermined on the set of criterion values (7).

**Developing the control actions.** The control actions are being developed according to the accepted methods, techniques, ways and algorithms of control. In this case, depending on time, which is at a disposal of a controlling system, the control divides into three parts: before-emergency control, alarming control, anticipatory control.

There exist two approaches to developing algorithms of processes.

The first approach (theoretically-multiple) assumes a search of possible states of a controlled object and the control actions matching for every of them, for the purpose of lowering the value of safety criterion.

The control method involves identification of current state of BDS by comparing with the recorded states of BDS in the memory of the controller (personal computer), determination of the control action corresponding to this state, and realization of this control action in a control object.

The use of this method is limited by the object dimension, that is, by a number of states which can be searched by the control designer.

The second approach (algorithmic) assumes availability of the rules system which equally processes any input information. These rules (algorithms) are recorded by the controller. The control method involves designing with the control actions algorithm every time all over again over input information. In such a way, a controlling system forms the control actions on a large set of the states of BDS and on various types of environmental influences.

The second direction of algorithmization is preferable. The difficulty of its realization involves determination of the control algorithm or system of rules and processing the input information.

For BDS the control actions development algorithm includes the next instructions:

1. State of the binary model of threatening events occurrence and significance of safety criterion are corrected according to the values of the vectors  $Y_{input}$ ,  $X$ ,  $U$ , and  $Z$ .
2. The control mode is determined in accordance with predicted changes of vector of serviceability of BDS. The level of danger of threatening events occurrence is determined by changing state of BDS through one, two or more elements.
3. Finite state of BDS, minimizing the value of safety criterion, and associated control vector are determined.
4. Forming trajectory of the optimum control actions and realization of this trajectory in BDS.

An example. Let's assume that in the considered object and in the storage battery explosion model de-energization took place. It is equivalent to assigning number "1" to variables of serviceability of elements 2, 3, 6 and 7. In such a way, vector  $Y^0_{output}$  will have the next evaluation:

$Y^0_{output}(t) = (0110011000)$ . The value of safety criterion  $K = (0, 2, 1, 4)$  will comply with it.

According to the accepted model of a storage battery explosion, a dangerous event occurrence takes place when changing a state of one of the next elements of BDS:

$e^0_8, e^0_9, e^0_{10}$ .

Besides, there exist two combinations of two, three and four elements. The control is antiemergency, and it is very important not to let changing the states of these elements, that is, to prevent an occurrence of a new source of fire in an accumulator compartment. Zero evaluation  $X(t)$  is a finite state of BDS. The control vector coincides with  $Y^0_{output}$ :  $U(t) = (0110011000)$ .

The trajectory of switching contains the control actions  $u^0_2(t)$ ,  $u^0_3(t)$ ,  $u^0_6(t)$ , and  $u^0_7(t)$ , taking values "1".

The order of using these control actions depends on duration of their realization.

The fastest one is the substitution of portable gas analyzer for a serviceable one, that is, the control action  $u^0_2(t)$ . The power supply resuming is longer, that is, the control actions  $u^0_3(t)$ ,  $u^0_6(t)$  and  $u^0_7(t)$ .

**Conclusions.** The discussed-above approach doesn't cover all the possibilities of the models, criteria, methods and algorithms of controlling BDS, which were considered here.

The results obtained are the methodological base for development of new methods of ensuring safety of different, by their physical nature, objects which can be represented by BDS.

### **References**

1. Рябинин И.А. Надежность, живучесть и безопасность кораблей. Морской сборник, МО СССР, 1987, вып.8. - Стр.62-65.
2. Бохман Д., Постхоф Х. Двоичные динамические системы.- М.: Энергия, 1986. - 400 с.

# THE ELECTROSTATIC FIELD PARAMETERS ASSOCIATED WITH OIL TANKS OPERATIONS

Victor L. Galka  
Konstantin B. Shchiglovskii  
Central Research Institute of Ship Electrical Engineering  
6, Blagodatnaya Street  
St. Petersburg, 196128, Russia

**Abstract.** This paper examines conditions which cause generation of an electrostatic field while filling tanks with hydrocarbon fuels. The field can cause serious explosions and catastrophes with grave consequences. The paper describes methods for analysis of electrostatic field which enable technological conditions for safe cargo operation to be derived and practical means for electrostatic safety to be formulated. Technical means are described for the maximum productivity of the filling operation with the object of shortening its duration while observing conditions of electrostatic safety.

**Keywords.** Electrostatic field, hydrocarbon fuels, tank filling, fire safety, analysis.

**Introduction.** The danger from electrostatic fields generated by filling of tanks with liquid hydrocarbon fuel has necessitated the investigation of electrostatic hazards in various branches of industry associated with the transportation and storage of oil products. Special interest in these investigations was demonstrated by the oil and petrochemical industries in the period between the two World Wars, when explosions of hydrocarbon vapours, dust, chemical powders etc. became more frequent [1].

Jet aviation development in the post-war period gave new impetus to the development of means for providing electrostatic safety during the refuelling of the aircraft themselves and auxiliary transport [2, 3].

This article is devoted to the statement of theoretical premises which have to be taken into consideration in the development of methods for calculating of the electrostatic field arising and to an account of practical results of efforts made to ensure the electrostatic safety.

**Conditions for an electrostatic hazard to arise.** The flow of hydrocarbon fuel along a pipeline gives rise to static electrification through double electrical layer deformation. The process of separating the electrical charges is accompanied by their transfer by the flow of the liquid. The electrical charges simultaneously carried away by the flow enter into relaxation under the influence of the electrostatic field they have created. Light oil products (gasoline, kerosene, etc.) are poor conductors, and therefore the majority of the charges are retained in the fuel for a relatively long time. This fact results in an increase in the total quantity of charges of the same sign entering the tank. The electrostatic field arising in this process in the liquid and in the vapour-air medium may reach a value which causes electrostatic discharges from the oil surface, these discharges having an energy sufficient for ignition of the mixture in the tank space above the fuel.

The ability of the liquid to generate and retain charges defines the rate of its electrostatic activity and is described quantitatively by the charging current  $I$ , and the time constant of

charge relaxation  $\tau = \frac{\epsilon}{\gamma}$  in the fluid concerned, where  $\epsilon$  and  $\gamma$  are the absolute dielectric permeability and the conductivity of the oil product respectively. The dielectric permeabilities of light oil products are relatively constant, and their values fall in range  $2 \cdot 10^{-11} \pm 20\%$  F/m, whereas their conductivities may diverge by several orders.

Investigations have shown that the tendency of liquid hydrocarbons to electrification appears in particular in conductivity range from  $10^{-13} - 10^{-9}$  S/m. For higher values of conductivity, charges have rapid relaxation and do not create a threat of ignition; for lower values there are few carriers in the liquid, and, though their relaxation proceeds slowly, the charge density is not sufficient to create a significant electrostatic field. Therefore, in the development of methods and techniques for estimating an electrification level, special emphasis is placed on the necessity to get reliable information on the conductivity values of the oil products concerned.

The charging current is determined both by physical and chemical properties of the boundary phases "liquid-pipeline walls" and by the liquid velocity. This means that the higher the operating productivity, the greater is the charge obtained by the liquid. Therefore, in the absence of means for control of electrostatic danger, limitation of the rate of receiving liquid hydrocarbons into the tanks is one efficient protective means, but results in a significant cost increase.

The volume charge of initial density  $\rho_0$  that has developed in the pipeline enters the tank and spreads through the volume of the liquid. The quantitative characteristic of this distribution is defined by convection transfer of the charge as well as by relaxation processes. The convection transfer is dependent on the properties of the fluid from the pipeline as well as on the velocity and direction of hydrodynamic flow within the tank.

**Electrostatic field in the tank.** One of the peculiarities of electrostatic fields developed in tanks is the fact that their electrical charges should not be considered as charges at rest, both on account of the finite fuel conductivities and because of movements of the liquids, i.e. in the strict sense the field formed by electrostatic charges has the properties of a stationary electric field. Therefore, to describe practical operating conditions, it is necessary:

1. to formulate equations and boundary conditions to which, with the required degree of accuracy, the distribution of charges, potentials, currents and field strengths in the tank complies;
2. to find conditions of applicability of static and stationary approximations. The general conditions are linearity, homogeneity and isotropy of the electrical properties of oil freights.

Let us consider the Maxwell equations and material relationships for moving phases:

$$\begin{aligned}
 \text{rot} \mathbf{H} &= \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}_{\text{pos}} + \rho \mathbf{v} & \mathbf{j}_{\text{pos}} &= \gamma (\mathbf{E} + [\mathbf{v} \cdot \mathbf{B}]) \\
 \text{rot} \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} & \mathbf{D} &= \epsilon \mathbf{E} - \left(\frac{1}{c^2} - \epsilon \mu\right) [\mathbf{v} \cdot \mathbf{B}] \\
 \text{div} \mathbf{B} &= 0 & \mathbf{B} &= \mu \mathbf{H} - \left(\frac{1}{c^2} - \epsilon \mu\right) [\mathbf{v} \cdot \mathbf{B}] \\
 \text{div} \mathbf{D} &= 0 & \frac{1}{c^2} &= \epsilon_0 \mu_0 \quad \epsilon = \epsilon_0 \epsilon_r \quad \mu = \mu_0 \mu_r \quad (1)
 \end{aligned}$$

and show the potential character of the electric field as well as the applicability of the simplest material relationships.

To take into account real values of various factors, let us introduce their characteristic values (with a root index "0") and some evident limitations.

1. The medium velocity of motion  $v_0 \ll 10^2$  m/s,  $\text{div} \mathbf{v} = 0$ .
2. The external magnetic field corresponds to the natural field of the Earth:

$$H_0 \sim 10^2 \text{ A/m}, \quad \text{rot} \mathbf{H}_0 = 0, \quad \frac{\partial \mathbf{H}_0}{\partial t} = 0. \quad (2)$$

3. The conductivity of the charged liquid is

$$\gamma_0 \sim 10^{-13} - 10^{-9} \text{ S/m}. \quad (3)$$

4. The media are neither ferroelectric nor ferromagnetic:

$$\epsilon \sim 10^{-11} \text{ F/m}, \quad \epsilon \mu \sim 10^{-17} \text{ s}^2/\text{m}^2. \quad (4)$$

5. The characteristic time is  $T_0 = \tau_0 = \frac{\epsilon}{\gamma_0}$ . (5)

6. The characteristic linear dimension  $l_0$  is chosen using the conditions:  $l_0 = \max\{L_0, \lambda_0\}$  or  $l_0 = \min\{L_0, \lambda_0\}$ , where  $\lambda_0 = v_0 \tau_0$ , and  $L_0 \leq 10^2$  m is the maximum dimension of the area.

7. Minimum values of the charge density and field strength are of interest if  $\rho_0 \geq 10^{-13} \text{ C/m}^3$ ,  $E_0 \geq 1 \text{ V/m}$ .

To compare the effects having an influence on formation of the electric field, let us use known methods of similitude theory, for which purpose we form dimensionless scalars, vectors and operators, denoting them by the superscript \*, and dimensionless complexes. So, for the first of the material relationships we obtain

$$\mathbf{j}_{\text{con}}^* = \gamma^* \left( \mathbf{E}^* + \frac{v_0 \mu H_0}{E_0} [\mathbf{v}^* \cdot \mathbf{B}^*] \right). \quad (6)$$

Since  $\frac{v_0 \mu H_0}{E_0} \sim 10^{-2}$ , the effects associated with medium movement in the magnetic field are no more than 1% of the minimum electrostatic field strength, and they may be neglected. Similar estimates for other material relationships show that relationships correct for stationary media may be adopted with no less accuracy.

To show the conservative nature of the electric field, let us represent its strength vector  $\mathbf{E}$  in terms of the scalar potential ( $U$ ) and vector potential ( $\mathbf{A}$ ):

$$\mathbf{E} = -\text{grad} U - \frac{\partial \mathbf{A}}{\partial t}, \quad (7)$$

related to each other by the Lorentz condition:

$$\text{div} \mathbf{A} + \mu \epsilon \frac{\partial U}{\partial t} + \mu \gamma U = 0 \quad (8)$$

or

$$\mathbf{E} = -\frac{U_0}{l_0} \text{grad}^* U^* - \frac{A_0}{T_0} \frac{\partial \mathbf{A}^*}{\partial t^*} \quad (9)$$

$$\frac{A_0}{T_0} \text{div}^* \mathbf{A}^* + \mu \epsilon \frac{U_0}{T_0} \frac{\partial U^*}{\partial t^*} + \mu \gamma_0 U_0 U^* = 0 \quad (10)$$

From the last equation, allowing for  $T_0 = \tau_0 = \frac{\epsilon}{\gamma_0}$ , it follows that  $A_0 \sim \mu \gamma_0 U_0 l_0$  which makes it possible to write

$$\mathbf{E} = -\frac{U_0}{l_0} \left( \text{grad} * U^* + \frac{\mu \varepsilon l_0^2}{\tau_0^2} \cdot \frac{\partial A^*}{\partial t} \right) \quad (11)$$

The maximum value of complex  $\frac{\mu \varepsilon l_0^2}{\tau_0^2}$  does not exceed  $10^{-5}$ , i.e. it may be accepted that

$$\mathbf{E} = -\text{grad} U \text{ and consequently } \text{rot} \mathbf{E} = 0.$$

In such a manner, the equations and material relationships for the case in point may be written as

$$\begin{aligned} \text{rot} \mathbf{H} &= \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}_{\text{pos.}} + \rho \mathbf{v}; \quad \text{rot} \mathbf{E} = 0; \\ \text{div} \mathbf{D} &= \rho; \quad \text{div} \mathbf{B} = 0; \\ \mathbf{D} &= \varepsilon \mathbf{E}; \quad \mathbf{j}_{\text{pos.}} = \gamma \mathbf{E}; \quad \mathbf{B} = \mu \mathbf{H}. \end{aligned} \quad (12)$$

These equations differ from the static, stationary and quasistatic approximations, and yet they are much simpler than the original system. Following the example of Parcell, such a field can be referred to as quasistatic.

Then the system of equations and boundary conditions describing the electric field of a moving fluid, taking account of the presence of the "fluid-vapour-gas space" boundary, is of the following kind:

equations:	boundary conditions:	
$\text{div} \mathbf{D} = \rho, \quad \mathbf{E} = -\text{grad} U$	$U_1 = U_2, \quad E_{2\tau} - E_{1\tau} = 0$	
$\mathbf{D} = \varepsilon \mathbf{E}, \quad \mathbf{j}_{\text{pos.}} = \gamma \mathbf{E}$	$\varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} = \sigma$	
	$\gamma_2 E_{2n} - \gamma_1 E_{1n} = -\frac{\partial \sigma}{\partial t}$	
transfer equation:		
$\frac{\partial \rho}{\partial t} + \frac{\rho}{\tau} + \mathbf{v} \text{grad} \rho = 0$	$\rho(x_0, y_0, z_0) = \rho_0$	(13)

Having carried out integration over all internal points of the area investigated, important integral relationships can also be obtained. In particular, the expression

$$\frac{dQ}{dt} + \oint_S \gamma \mathbf{E} d\mathbf{S} + \oint_S \rho \mathbf{v} d\mathbf{S} = 0$$

clearly shows that the variation in the total charge of the area is determined by the conduction current reaching its boundaries as well as by the influx and exit of the charges transported by the fluid through these boundaries. From the relationships given above, equations describing the field in a stationary liquid, as well as the static and stationary approximations, are easily obtainable. Regarding the latter, it is necessary to point out that it is correct only in the presence of external currents (in this case, in the presence of convective currents).

The field may be regarded as a static one if it is observed over a time interval significantly shorter than the charge relaxation time. In charge and discharge operations, as well as for non-stationary outside currents, the most common quasistatic relationship should be used.

**Electrostatic safety test.** Development of methods for estimating system safety and evaluating the means of control of electrostatic danger in the application of these systems has to be based on a test which makes it possible to form a firm conclusion regarding safety of the filling operation. The choice of test is associated with economic factors, since the provision of electro-

static safety requirements may result in a diminished rate of the filling operation and therefore in its increased cost. Therefore, the test should not be excessive and, in particular, should not exclude the possibility of the creation of charges with an energy insufficient to ignite the vapour-air medium.

It is known that, for ignition of the vapour-gas atmosphere of a reservoir with an electric spark, it is necessary for the energy released in the spark channel to be no less than a certain specific value for the particular mixture, ensuring the propagation of fire from a small area of the mixture over its entire volume. This value is referred to as the minimum ignition energy. Experimental research has determined reliable values for the vapours of organic fluids, which range from 0,1 to 1 mJ [6]. However, application, as a test, of the value of the total energy of electrostatic charges accumulated in a tank, for example, would result in unjustified restriction of the filling operation, because the total energy of the electrostatic field could not be released in a discharge.

Another fundamentally different approach is possible. For this purpose, it is necessary to choose a particular generalized parameter as a function of the factors defining electrostatic danger and to determine the correlation of each variation in the value of this parameter with the change in the degree of electrostatic safety. An analytical form of the test is defined in accordance with the solution of the equations describing the dynamic behaviour of operations with hydrocarbon fuel. With this approach, the maximum value of the electrical potential  $U_{kp}$  of the tank fluid surface, this value being an integral parameter associated quantitatively with the energy which can be released in the course of a spark discharge, will correspond to the necessary test to the greatest extent. This value may be associated with the conditions of generation of discharges leading to a fire.

In [7], a maximum value of fluid surface potential of 40 kV has been proposed as a test of electrostatic safety; at the symposium on "Testing for electrostatic ignition" held by a working group on static electrification of the London Institute of Physics, a value of  $U_{kp} = 35$  kV [8] was accepted; in domestic practice a value of  $U_{kp} = 25$  kV has been proposed [3], this value being used for estimating electrostatic safety of freight systems.

**Methods for estimating electrostatic safety.** Estimation of electrostatic safety of freight systems is reduced to the calculation of the electrical potential in the tank with the known totality of values  $\{I, \tau, w, h\}$  and to its comparison with a selected test. Here, for a safe system, the following condition must be fulfilled:

$$\forall \{I, \tau, w, h\} \Rightarrow \sup_{x, y, h \in \Gamma^*} |U| \leq U_{kp} \quad (14)$$

where  $w$  is the tank filling rate,  $h$  is the level in the tank, and  $\Gamma^*$  is the interface surface of the liquid in the tank.

Consider the theoretical model shown in Fig. 1, where  $A$ ,  $B$  and  $H$  are the overall dimensions of the tank,  $\Gamma$  are grounded walls of the tank,  $G_1$  and  $G_2$  are the areas of the tank filled with fuel and vapour/air respectively, and  $x_0, y_0, z_0$  are the coordinates of the charge origin.

To fill the tank slowly, in quasistatic approximation ( $\frac{T}{\tau} > 100$ , where  $T$  is the tank filling time), for the boundary conditions considered above, with account taken of the fact that  $\gamma_2 \ll \gamma_1$ , with an error lower than 4% we may assume that  $\frac{\partial \sigma}{\partial t} = 0$  and  $E_{1n}|_{\Gamma^*} = 0$ , where  $\sigma$

is the surface charge density, and  $E_{ln}$  is the normal constituent of the electrostatic field strength in the liquid medium.

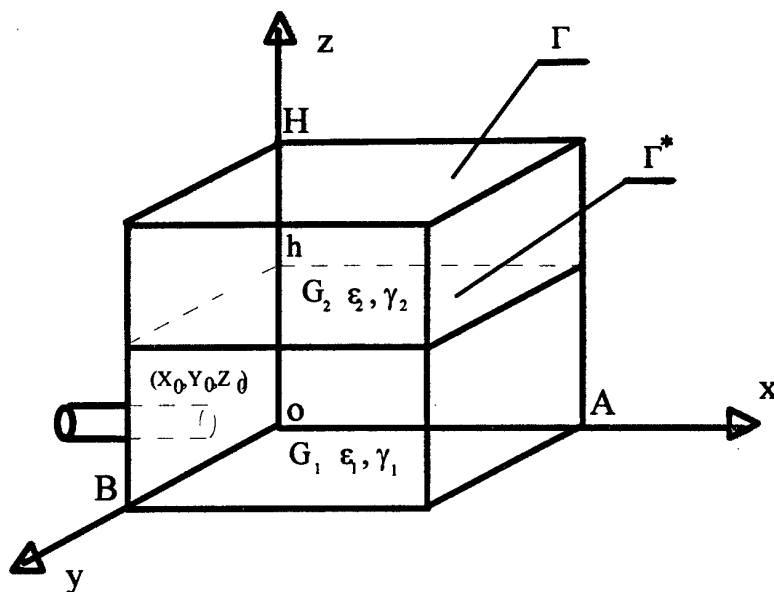


Fig. 1 Theoretical model

It should be noted that such an assumption results in an upper limit estimate of the electrostatic field.

Then determination of electrical potential is reduced to the solution of the following system of equations:

$$\begin{aligned} \Delta U_1 &= -\frac{\rho}{\epsilon_1 \epsilon_0}, & (x, y, z) \in G_1; \\ \Delta U_2 &= 0; & (x, y, z) \in G_2 \\ U_1|_r &= U_2|_r = 0; \\ U_1|_{r^*} &= U_2|_{r^*}; \\ \frac{\partial U_1}{\partial z}|_{r^*} &= 0. \end{aligned} \quad (15)$$

The volume charge density distribution is found from the solution of the transfer equation. However, firstly it is necessary to obtain the value of the velocity vector  $v$  of the filling flow spreading within the tank.

The velocity field of the liquid spreading within the tank can be found from the solution of hydrodynamic equations in a potential approximation.

For potential flows, the potential  $\psi$  of the velocity field ( $v = -\nabla\psi$ ) in a source-free area satisfies the Laplace equation  $\Delta\psi = 0$ . In this connection, the condition of imperviousness of the walls containing the fuel volume is obtained:

$$V_n|_r = -\frac{\partial\psi}{\partial n}|_r = 0 \quad (16)$$

as well as the liquid surface condition:

$$V_z|_{r^*} = -\frac{\partial \psi}{\partial z}|_{r^*} = \frac{w}{AB}. \quad (17)$$

Then, from the transfer equation, the volume charge density distribution  $\rho(x, y, z)$ , is determined. The final stage of the calculation is a search for the distribution of the electrostatic field potential of the liquid surface  $U(x, y, h)$ . Fig. 2 shows the solution of the problem considered using the grid method. There is naturally a maximum close to the input pipe, where the electrical charge "core" is concentrated. Based on a comparison of the maximum value of the potential with the test  $U_{kp}$ , a conclusion about the electrostatic safety of the filling operation can be drawn.

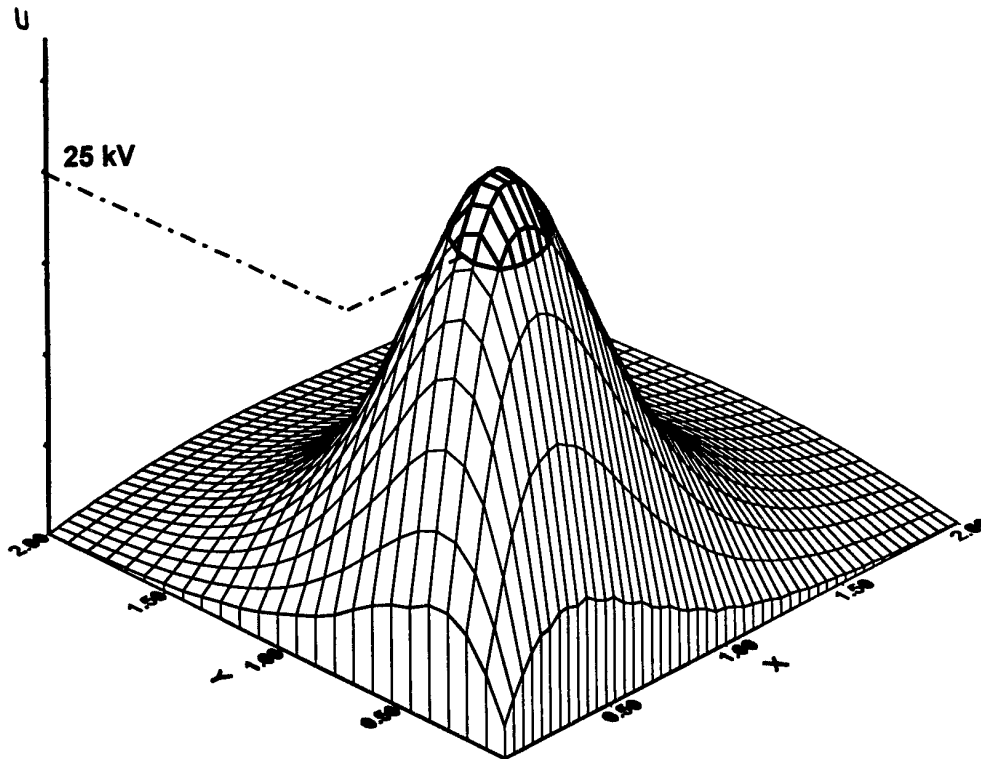


Fig. 2 Fluid surface potential distribution

**Ensuring electrostatic safety of the filling system.** Analysis of the factors defining the liquid surface potential value and, as shown above, the degree of electrostatic hazard makes it possible to distinguish three principal ways for its reduction:

- designing the system for rapid charge relaxation by fuel outflow from a pipeline to reduce the value of  $\rho_0$ ;
- arranging the hydrodynamic flow in such a way as to provide the maximum possible contact of the liquid with the tank bottom; choosing optimum conditions of system operation.

Rapid charge relaxation is achieved by making use of relaxation reservoirs, the theory of which has been described in [9].

The second area of development is provided by designing a hydrodynamic system; special fittings are used at the ends of the pipelines and also antistatic wells making it possible to close a significant part of the electrical induction on the tank bottom. As a result, the electrostatic field in the vapour-air medium is reduced, and the chance of igniting charges appearing is reduced (Fig. 3).

The choice of operating conditions is based on arranging the areas of efficiency of the system according to the methods proposed in [10]. They form the basis for development of filling schedules and a means of forecasting the duration of the filling operation. One should take into account that the operating areas are developed for the upper limits of the range of variation in oil product conductivity and that their charging currents are established experimentally. The actual values are significantly lower, and therefore, in system operation, there is always a safety factor, and the filling rate can therefore be increased. It is possible to ascertain this factor by technical means for electrostatic danger control.

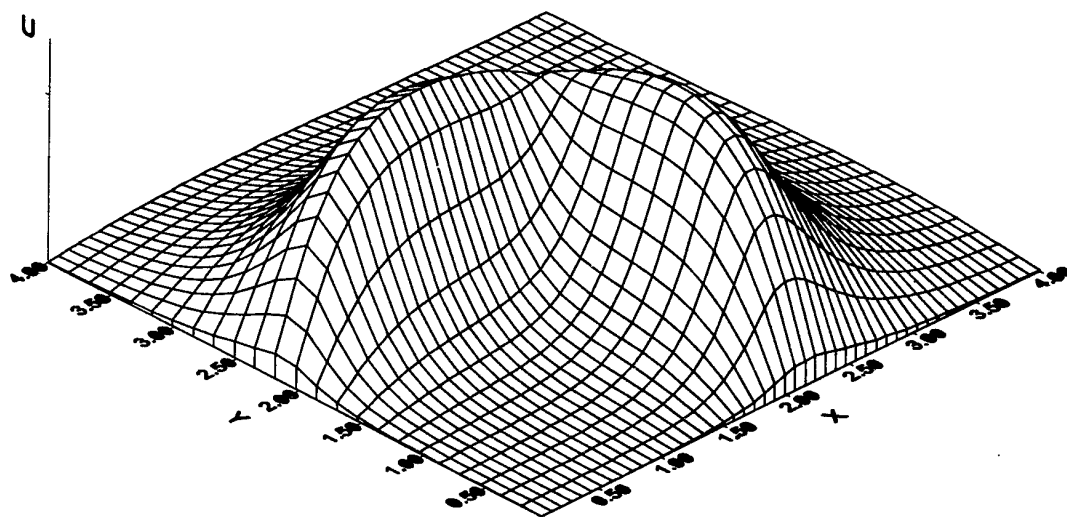


Fig. 3 Fluid surface potential distribution with protection

**Techniques for control of electrostatic danger.** The staff of the Central Scientific Research Institute of Electrical Engineering and Technology for Ships has been carrying out the development of devices for providing a diagnosis of electrostatic safety of filling systems. An electrostatic field strength meter, an electrophysical parameter meter for oil products, and a current meter for isolated areas of the system have been adopted by the Interdepartmental Joint Committee and have received a Metrological Certificate. These devices are used successfully for experimental research of the electrostatic field on test installations and on tankers, as well as for commissioning trials of oil tankers [11].

### Conclusions.

1. It has been shown that an electrostatic field arising in the process of electrification of poorly conducting fluids has to be considered as non-stationary or quasistatic; in its formation the conduction and convection currents are of considerable importance.
2. Investigations carried out have made it possible to construct a mathematical model of the process of electrostatic danger arising from filling a tank with oil products, to develop methods of estimating theoretically the degree of electrostatic safety and to use these techniques in tank filling system projects.
3. To provide electrostatic safety and to increase the filling system efficiency, it is necessary to employ technical means of electrostatic danger control. As experimental investigations have shown, this enables the tank filling time to be reduced significantly.

## References

1. Дроздов, Н. Г., (1949) Статическое электричество в промышленности. Энергоатомиздат, М.–Л.
2. Мальшев, В. В., Б. К. Максимов, В. Г. Петухов (1981) Электростатическая безопасность заправки топливом. КИИГА, Киев.
3. Максимов, Б. К., А. А. Обух, А. В. Тихонов (1989) Электростатическая безопасность при заполнении резервуаров нефтепродуктами. Энергоатомиздат, М.
4. Mills, G. S., R. S. Oldham (1983) Evaluation and prevention of electrostatic hazards associated with oil tanker operations. Trans. 9 Mar. E (TM), vol. 95, Paper 44.
5. Галка, В. Л., К. Б. Щигловский (1993) Проблемы электростатической защиты танкеров. «Судостроение», № 10.
6. Гавриленко, Н. Д. (1973) Минимальная энергия зажигания паровоздушных смесей при различных температурах. Экспрессинформация ВНИИ ТБХП, № 1.
7. Kramer, H., A. Asano, C. Schon (1977) Criteria for safe filling of tanks; theoretical and experimental studies to determine the electrical hazard. In 3-rd Inter. Congr. Stat. Electricity, Grenoble, Paper 31, April.
8. Butterworth, G. J., K. P. Brown (1982) Assessment of electrostatic ignition hazards during tank filling with the aid of computer modelling. J. Electrostatics, No. 13.
9. Галка, В. Л., К. Б. Щигловский (1994) Модель распределения электрического заряда при заполнении резервуаров. «Электричество», № 7.
10. Галка, В. Л. (1990) Метод построения зон работоспособности грузовой системы нефтеналивного судна. Сер. «Судовая электротехника и связь», вып. 11.
11. Белоус, Е. А., А. А. Воронин, В. Л. Галка, К. Б. Щигловский (1994) Комплекс приборов диагностического обеспечения электростатической безопасности эксплуатации танкеров. «Судостроение», № 6.

# NEW AREAS AND TASKS OF SIMULATION AND ANALYSES OF RISK BY LOGIC-PROBABILISTIC METHOD

Evgeniy Solojntsev  
Institute of Problems of Mechanical Engineering  
Russian Academy of Sciences  
Tel.: (812) 217-31-24; E\_mail: sol@sapr.ipme.ru

**Abstract.** The new perspective subject areas of simulation and analysis of risk by the logic-probabilistic method (LPM) are considered: credit risks, risks in business and risks of insurance. Main definitions of the risk logic-probabilistic theory are adduced. New tasks for systems of valuation of risk in technical and economic systems are formulated. Necessity of solving a complex of safety tasks (control, diagnosing and risk) and creation of an information technology for reduction of uncertainties is substantiated. The analysis of a role of knowledge and visualise at the decision of risk tasks is executed. For systems of valuation of credit risk of legal person and customer loan the statement of an identification task of risk model on statistical data is described.

**Keywords:** risk, technique, business, credit, insurance, logic, probabilistic, method, identification

**Introduction.** The structurally complex systems (SCS) usually consist of equipment, computers and software, actions of staff, including control, testing, repair and service. To simulate such the complex real world various models, criteria, data bases and knowledge are necessary. SCS include complex technical, ecological, organisational and industrial systems (nuclear power stations, chemical manufactures, vessels, bank, business, insurance).

There are the new perspective subject areas of simulation and analysis of the risk by LPM: credit risks, risks in business and insurance. The risks in banks, business and insurance are the normal phenomenon. There is not risk, there is no profit. Therefore, the necessity of the decision of new tasks, including identification of the risk model has arisen.

## 1. *New object areas of valuation risk*

Now for valuation of credit risk of customer loan some techniques are offered and investigated [1-4]: neural network, discriminate analysis and the CART-method. Comparison of these different techniques, conducted in the work [1], shows, that they have about identical qualities. The shares of faulty decisions on valuation of good and bad credits equal about 28 %.

These techniques use artificial, inadequate to essence of risk, the mathematical apparatus. They do not apply such natural and fundamental ideas for safety and risk, as casual events, errors, probability of event, logic and structural connections of events. On a substance, these techniques are more or less successful approximation, set-up by black box for answer on question: to give or do not give the credit. Because of inadequate to the risk task they do not permit to decide other tasks important for bank, namely:

- to calculate quantitative valuation of risk for each credit;
- to establish as far as risk of the credit to the allowable risk is close;
- justified to establish allowable risk and price for risk;

- to find contributions of characteristics of the credit to risk;
- to find average contributions of credits characteristics on set of all credits of bank in the average credit risk of bank and by this to establish the most repeated error of bank;
- to operate by credit risk of bank, reducing of loss of means and increasing profit of bank.

In an engineering for valuation of the safety and risk the other adequate to the risk task is used - the logic-probabilistic method (LPM) [5,6,7]. The task of use of LPM for valuation of risk in banking systems is not ordinary. For solution of the task it is necessary: to introduce the idea of casual event in credit activity; to develop structural, logic and probabilistic models of credit risk; to decide a task of identification (training) of risk LP-model; to overcome a problem of obscene of dimensions at addition of large number of events, connected by relations AND, OR and including large number of group of the incompatible events.

The insurance risk of the enterprises, life and properties based on statistical methods [8] and practice are evaluated. It is obviously, that the insurance risk is equal to risk of accident. Therefore, it is necessary to develop and research structural, logic and probabilistic models of accidents risk of different objects of insurance. Developed and investigated models of risk insurance (explosion in a premise, suicide of elder person, murder of the person by a current) show, that complexity of the models of risk of insurance equal to complexity of risk models of complex technical systems.

There are now qualitative methods of valuation of risk in business [9,10]. The first work on LP-technique of valuation of credit risk and frauds in business [11], despite simplicity of logic models, has allowed to formulate and to investigate new tasks for the problem of risk valuation. It considers logic-probabilistic models of the credit risk for legal persons and the risk of the fraud of the manager and hired worker, fraud with investments.

## ***2. Main definitions***

The apparatus of the logic-probabilistic theory of risk, including orthogonalisation of the risk logic functions in the disjunctive normal form, calculation of risk (probability of final event) and contributions of initiating events to risk is enough advanced and adduced in work [5,6,19]. But at development of the risk new subject areas by new experts there are difficulties with a terminology and comparison of different methods of risk valuation.

As in mechanic and economy the main concepts are entered and are used the fundamental laws of a nature (preservation, balance and transformation of energy, weight, pulse etc.) for the description and research of various condition and processes, as in the risk theory main concepts and the laws of risk should be formulated. Such an approach is being developed below concerning technical, economic and organisational systems, in which the important place is taken by the human factor.

***Definition 1.*** The risk is present a quantitative measure (probability) adverse event in a system, leading to losses.

*Definition 2.* The system can consist of people, of their relations, interests and motives, computers and programs, gauges, devices, machines, equipment etc. The systems can conditionally be separated on technical, technological, economic, ecological and organisational.

*Definition 3.* The general scientific knowledge for safety of systems are following:

- the errors in projects technical, technological, economic, ecological and organisational systems are inevitable;
- the zero risk in systems, possessing money, stocks nuclear, chemical, biological or other energy, is unattainable.
- the errors in valuation of motives and interests of people and circumstances in credit activity, insurance and business are inevitable;
- the risks in banks, business and insurance are the normal phenomenon; there is not risk, there is no profit;
- each able to do a fraud, if money and material means is bad count, if it is possible to hide a fact of plunder and if on the person press vital circumstances.
- the valuation tasks of probability successful and unsuccessful of functioning of a system are dual; structurally to be engaged in the task of unsuccess (failure, unreturn of credits etc.).

*Definition 4.* Casual elementary event in the risk theory is understood:

- in technical and technological systems: failure of an element, malfunction of an element, dangerous significance of parameter etc.;
- in credit risk: availability or absence of a error in valuation of importance of the characteristic of the credit or its gradation for success of crediting operation;
- in fraud in business: availability or absence of a error in valuation of importance of a symptom, associated with a fraud, for acceptance of the decision about judicial investigation;
- in insurance: the risk of insurance is equal for risk of failure ( accident ) for technical, ecological, economic systems, also life of the person. For a suicide risk of person the events associate with motives of suicide.

*Definition 5.* The elementary casual events are independent, can accept two (1/0) or some discrete significance, to belong to groups incompatible events or to be connected by conditional events.

*Definition 6.* In the risk new subject areas at mass operations of insurance, crediting etc. historically for valuation of risk used statistical methods or statistical data . It is possible simplify to say: the logic-probabilistic method of risk valuation presents a statistical method, in which events and logic associative connections between events are entered. The script of final event can be submitted as the graph of elementary and derivative events.

### **3. Concepts of safety systems construction**

There are various concepts of safety systems construction: 'zero risk', 'non-zero risk', mixed approach [12,13]. The mentioned systems of SCS safety maintenance from failures, also prospects of their development are based on the research-well-founded conceptual approaches of industrial risk accepted in developed countries that effect development of political, state, low and other bodies of these countries [12]. We shall consider the concepts of safety in detail .

**The "zero risk" concept.** The concept of "zero risk" is the basis for standards in nuclear and other branches of engineering. This concept recently was called the determinism approach to safety maintenance [12]. In this approach [14] "...it is erroneously considered that practically it is possible to exclude any danger to the population and environment if there are means for creation of safety systems and a high level of discipline. But even the use of most effective technical systems of safety and modern methods of control of technological processes does not provide and cannot provide absolute reliability of work, excluding emergencies. The zero probability of failure is reached only in systems deprived of money and interest of people, stored energy, chemical and biological active components. The concept of "absolute" safety is inadequate to internal laws of technocracy. These laws have probabilistic character and risk of accidents always exists".

According to the essence of determinism "zero risk" in industrial countries technical and organisational systems are created to prevent accidents and decrease their possible negative consequences. Essential elements of this safety system are the following data bases [12]: list of the dangerous manufactures and financial companies and data of its, accessible for all national organisations and interested persons; list of measures of providing manufacture and business by safety systems, inspection, training of staff etc.; list of measures of the state bodies, including control of legislative acts, conclusions on valuation of danger and licensing, inspections etc.; plans of measures in extreme situations with the purpose of decreasing damage.

**The concept of "non-zero risk".** The concept of allowable risk is based on numerical valuation of risk of frauds, credit risk, risk of insurance, risk of technical accidents. The probabilistic risk analysis allow practically to accept much new measures for increasing safety of operation of nuclear power stations and other potentially dangerous manufactures. The concept of allowable risk permits more reasonably with opened eyes to concentrate and distribute resources not only for accident precaution, but also for timely preparation for actions in extreme conditions. Measurement of risk means numerical determination of danger from one or another source for individual or group with the use of LPM's [14].

There is a question whether it is always possible to use methods of "non-zero risk" for the analysis and examination of SCS safety. Or in other words, whether it is always necessary to begin research and valuation of SCS safety with the construction of the script as cyclic graphs of connection (network, graph-model or simple trees with elements OR and AND). We believe, it is. The examples of research of accidents from different subject areas prove this: sinking of a ship, explosion in a submarine, striking a person with current aboard a ship, robbing of cash machine, non-returning banking credits, frauds, failure of an electrical network of a ship, failures in a complex of computers and organisation systems, failure on a nuclear power station etc. Note, that the concept of "non-zero risk" based on LPM, created by Russian scientists labours [5,6], was not admitted in our country for a long time, it was not given sufficient attention, though these books were translated into foreign languages.

**The mixed approach.** In mixed approach to safety maintenance both above mentioned approaches (their methods) are simultaneously used: determinism ("zero risk") and logic-probabilistic ("non-zero risk"). Determinism component of safety maintenance corresponds fixed, non-random and controllable factors. Logic-probabilistic component of safety maintenance corresponds to variable, random and controlled factors. Selection of two components of safety maintenance is stipulated by the following reasons: insuperable

complexity of real tasks of SCS safety maintenance; insufficiency of resources for maintenance of determinism "zero risk".

Division of roles of deterministic and probabilistic approaches in mixed approach of SCS safety maintenance will be considered on examples of different simulation with the use of concepts as data ("zero risk") and knowledge ("non-zero risk"):

- 1) Mathematical SCS simulation: data about boundary conditions; knowledge about the mathematical description of an object;
- 2) Simulation of SCS control [13]: data about precedents and parameters initiating failures; knowledge about technology of the control program based on of the complex object control scheme of with the movement on a given trajectory and correction by a deviation from it;
- 3) Simulation of SCS risk [13]: data about conditions of normal SCS functioning, established by laws and standards and controllable by special bodies; knowledge about scripts of accidents and LPM's of calculation of risk and significance of initiating factors.

The "non-zero risk" theory knowledge and the "zero risk" practice data are interconnected when solving a safety task, but the leading place in the system analysis of SCS safety belongs to the "non-zero risk" theory. Demarcation line between these approaches is established by consecutive application of the following methods [13]:

- 1) Construction of dangerous events scripts for researches SCS with connections of AND, OR, NOT type and calculation with the help of LPM of numerical valuations of the criterion of risk, individual and group significance of initiating factors. The most important factors should be transferred into factors of "zero" risk or the probability of their occurrence should be reduced essentially. The appropriate resources are necessary for it.
- 2) Simulation of expenses and resources distribution on the design decision on safety [15]. The resources can be spent both for expenses and for elimination of losses from failures. Design decisions are made on the object  $K$  with classes of initiating factors  $K_1, K_2, \dots, K_n$ . In its turn in each classes  $K_i, i=1, n$  there can be subclasses  $K_{i1}, K_{i2}, \dots$ . For each classes  $K_i, i=1, n$  and its components there are models  $F_i, i=1, n$  enabling to calculate: maximum expenses of resources  $Q_e$  for the design decisions on safety and maximum losses of resources  $Q_l$  if failure. Multicriteria task of optimisation is solved with use of an expert system and successive acceptance of decisions by graphic images before exhaustion of resources. Thus some of "non-zero risk" factors are transferred into "zero risk" factors.

#### **4. The complex of safety tasks**

One of complexities of the "non-zero risk" concept is necessity to know probabilities of initiating factors for calculation of absolute value of accident probability. Though, we remind again, that for determination of individual and group "weights" of the initiating factors (that is the most weak SCS places) in the first approximation using LPM one may not know absolute values of probabilities of initiating factors [13].

This complexity of the safety concept on the "non-zero risk" basis is most of overcome, if tasks of risk, diagnosing, control (tests) are solved simultaneously (in parallel). The matter is not that these tasks are also referred to safety and are solved on the same systematic and information basis. The main is that the solution of the tasks of diagnosing and control gives new information

about the danger of initiating factors and probabilities of failures and thus reduce uncertainties in "non-zero risk" data.

Let us consider these effects of tasks of diagnosing and control. When solving a diagnosing task based on construction of a graph-model of a diagnosed object in the parameters space [16] an effective set of diagnostic measurable parameters for recognition of defects is separated. Thus possible defects, their importance and their differentiation by measurable parameters are determined. The available statistical data on failures are used for establishing the significance relations of the "influences" type (edges of the graph between the tops-parameters).

When solving control task [13] the following procedures are made: failures forecasting, expenses-losses simulation, planning (development of the test program), execution of tests with decisions acceptance on elimination of errors in the project. The purpose of the "forecasting" procedure is revealing possible failures and removal of uncertainties in SCS functional possibilities. Simulation of SCS elements (strength, thermal and other calculation) by the Monte-Carl method on engineering program) to value parametric failures possibility and methodology of "critical questions" for revealing uncertainties are used. In the "simulation" procedure resources are distributed to test and measurable parameters with the purpose of removal uncertainties. The resources for control are usually insufficient. Both parametric failures probability and uncertainties in functioning that cannot be removed by tests because of the lack of resources are determined in the control task.

Thus, when solving the control tasks and diagnosing receive the new information on initiating factors causing failures is received. That is why the task of estimate of absolute value of failure probability using the "non-zero risk" concept should be solved after or in simulteneunslly with the solution of the control tasks and diagnosing.

### ***5. The information technology of simulation and analyses of risk***

Another complexity of the safety concept based on the "non-zero risk" methods is necessity of serious informational, linguistic, intelligent and program support of the logic-probabilistic methods. Thus the problem is not such usual means, as data base and programs of transformation of logic models into probabilistic ones and reception of numerical valuations of the safety criteria.

For the mass researches possibility and examination of a real SCS safety creation of the automated information technology on PC base is of not less significance; it is used for the following purposes:

- 1) construction of the SCS logic model based on creation of models of the so called second level, which are more clear and accessible to experts [13,17]: as a script in the productions' language; as a topological model (graph-model );
- 2) support of engineering models of SCS elements and economic models of the expenses-losses simulation and distribution of resources for the design decision.

These components of the information technology solve such important tasks as [18]: revealing of parameters of safety models; construction of the different levels safety models; visualise of simulation and control of SCS safety. Let us make some explanations.

The graph-model, constructed on a display screen, is automatically transformed into the logic-probabilistic safety model, suitable for the computing program. The expert analyses numerical valuations and can notice errors, forgotten initiating factors or enter other constructive decisions increasing safety. He can change the graph-model constructed by him and give a command to repeat calculations for reception of new safety valuations. Such process of cognition and the simulation of SCS safety is but a process of the selection knowledge about model of SCS safety from the professional [13,17].

Scripts of failures and accidents are easily translated into the language of the graph and them can be removed on a display screen for the visual analysis and control. Or it is even possible to execute these operations with the help of the graph theory methods. Thus not only the problem of knowledge selection from the expert-professional is solved, but completeness and non-contradiction of these knowledge [17] is easily checked up.

In information technology the following sequence of construction and use of SCS safety models is executed [13]: the contents description, the script, the graph-model, the logic and probabilistic models. Thus gradual transition from a figurative representation of an expert to a mathematical one, suitable for calculations is provided. When constructing the logic models for SCS elements the means of the engineering models support can be used. For calculation of the economic criteria and the resources' distributions the means of the economic models support are used.

In information technology the knowledge is the means for overcoming uncertainties in parameters, their relations, in models and decisions acceptance with the help of expert systems. Knowledge form the script of SCS safety in the productions' language or as the graph-model; they are applied as causal knowledge ( help system ) by precedents and scripts [13,17,18].

In information technology the visualise is a means for structural representation of the safety models as the graph, knowledge selection from the expert based on construction of a graph-model of safety on the display screen, visualise of simulation on electronic tables with demonstration of the conclusion mechanism, results check, decisions acceptance and control under conditions of uncertainties [13,17,18].

Any SCS safety maintenance requires large additional resources. Introduction of safety systems into branches of industry is impossible without the State regulation, that is, laws and standards on safety and administrative bodies for control of their execution. Because when creating new SCSa complex of safety tasks (risk, diagnosing, control) based on scientific methods is not solved. Therefore it is necessary to create new standards on SCS development and to provide their use. It will essentially reduce the probability of failures and accidents and respective losses. The SCS projects cost will increase. Valuations of possible increase of expenses for SCS projects are [ 13 ]: safety maintenance increases costs by 100 % ; control maintenance increases costs by 30 %; diagnosing maintenance increases costs by 50 %.

## ***6. Identification of the credit risk model***

We consider for an example the statement of the identification task of the credit risk model for consumer loan. We shall use actual data, which were applied in techniques /1-4/. It is information

about on 1000 credits, 300 of which were unsuccessful. Each proposal on the credit is evaluated 20 characteristics  $f1-f20$ , for each characteristic up to 2 gradation from 11 is used.

The credit risk is determined as  $P_c$  probability of unsuccess of crediting operation. The event is present availability or absence of an error of bank (1/0) in valuation of importance of the characteristic or its gradation for success of crediting operation:  $p, q=1-p$  - probability of availability or absence of an error. The  $z1-z20$  events correspond to  $f1-f20$  characteristics. The  $z21-z117$  events correspond to gradation of characteristics. For each characteristic they are group of incompatible events.

We record logic model of risk only with the  $z1-z20$  account of events and without their detailing on errors in gradations. Logic model of the credit risk in the disjunctive normal form (DNF) is :

$$Y = z1 \vee z2 \vee \dots \vee z20. \quad (1)$$

Logic model of absence of the credit risk in DNF is:

$$\overline{Y} = \overline{z1 \vee z2 \vee \dots \vee z20} = \overline{z1} \wedge \overline{z2} \wedge \dots \wedge \overline{z20}. \quad (2)$$

To logic equations (1) and (2) correspond the algebraic probabilistic polynomial [5,6,19,]. The task of identification consists in valuation of 96 probabilities of events, appropriate to gradation. Target function of a task of identification: maximise of the sum of bad and good credits, on which valuations of statistical data and the LP-techniques coincide. Algorithm and analytical methods of identification based on the 1000 logic equations (1) and (2) are offered.

### Conclusions

1. The new perspective subject areas of simulation and analysis of risk by the logic-probabilistic method (LPM) are considered: credit risks, risks in business and risks of insurance.
2. Neither "zero" nor "non-zero" risk concepts can provide the SCS safety; the mixed concept of SCS safety maintenance is necessary.
3. Differentiation between "zero" and "non-zero" risk concepts in the mixed approach of SCS safety maintenance should be defined with the use of logical-probabilistic "non-zero risk" methods and method of resources distribution to the design decisions on safety.
4. Necessity of simultaneous solution of a complex of safety tasks, including tasks of risk, control (tests) and diagnosing, for reduction uncertainties of "non-zero risk" is justified.
5. Necessity of creation of information technology for construction and support of safety models of all levels for maintenance of the mass use of the "non-zero risk" concept is justified.
6. The purposes of development on maintenance of safety are: creation of standards, laws, safety specifications and the control structures, standard program instruments for construction of the failures graphs-models and valuation of risk and significance of factors and resources distribution to the design decisions on safety; development of technology of safety tasks complex solution; the description of scripts of failures, accidents and precedents in industry and business.
7. For systems of valuation of credit risk of legal and natural persons statement of the identification task of the risk models on the statistical data is stated.

## References

1. Seitz, J., E. Stickel (1992). Consumer Loan Analysis Using Neural Network / Adaptive Intelligent Systems.- Proceed. of the Bankai Workshop, Brussels, Belgium, - p.177-189.
2. Fahrmeier, L., A. Hamerle (1984) Multivariate statistische Verfahren.- Berlin, New York.
3. Haussler, W.M. (1981). Punktebewertungen bei Kreditscoringsystemen.- Frankfurt .
4. Hofmann, H.J. (1990). Die Anwendung des CART-Verfahrens zur statistischen Bonitätsanalyse von Konsumentenkrediten. Zeitschrift für Betriebswirtschaft 60, H.9, pp.941-962 .
5. Ryabinin, I. G. Cherkosov (1981) The probability-logistic methods of reliability evaluation of complex system.- Moscow: Radio and Sviaz,- 238 p.
6. Ryabinin, I. (1976). Reliability of Engineering Systems. Principles and Analysis.- Moscow: Mir Publishers,-532 p.
7. Kafka, P. (1993). Probabilistic Safety Assessment (PSA) Technology - how it works, what does it do, where are the gaps.- Safety and Reliability Assessment.- an Integral Approach. (Proc. of the European Safety and Reliability Conference, Munich, Germany).- Amsterdam, New York-pp.393-403.
8. Harald Cramer. (1996) Collected works. Volume 1,2. / By Editor Anders Martin-Loef.- Springer-Verlag, Berlin Heidelberg, New York.
9. An Introduction to Credit Risk Management (1994)/Trans. with Eng.-Moscow: Price Waterhouse. - 334 p.
10. Albrecht W., G. Wernz, T. Williams (1995). FRAUD. Bringing Light to the Dark Side of Business: Trans. with Eng.- SPb, Piter.- 396 p.
11. Solojontsev E., V. Karasev, V. Solojontsev (1996). Logic-probabilistic valuation of banking risks and frauds in business. SPb.: Politechnika, 118 p.
12. Modern condition of a problem of safety in industrial advanced countries (1994) // Problems of safety at extreme situation / N. Mahutov, V. Petrov, P. Tartashov, G. Sergeev.-N4. - p.2-36.
13. Theory and information technology of simulation of safety complex systems (1994-95) / Edited by I. Ryabinin and E. Solojontsev. - Issue 1-5, - 400 p.
14. Ryabinin I. The concept of the logic-probabilistic theory of safety. (1993) - Devices and control system, N10. - p.6-9.
15. Solojontsev E., I. Korobitsin, D. Metyolkin (1993). An intelligent system for simulation in coordinates costs-loss with acceptance of decisions on graphic images. - Program products and systems, N3, p.30-36.
16. Dobrinin V., G. Myasnikova, E. Solojontsev (1994). Simulation process automation of diagnosing objects. // The theory and information technology of simulation of safety of complex systems. Issue 2 / Edited by I. Ryabinin. Preprint 104. SPb., IPM RAS.
17. Tarkhov A. A system of automated graphic construction and analysis of structural models (SAGCA SM) (1994) // The theory and information technology of simulation of safety of complex systems. Issue 3 // Edited by I. Ryabinin. Preprint 109. SPb., IPM RAS.
18. Solojontsev E., V. Solojontsev. (1994). Simulation of safety structurely-complex systems: parameters, model, knowledge, visualize // The theory and information technology of simulation of safety of complex systems. Issue 2 / Edited by I. Ryabinin. Preprint 104. SPb., IPM RAS.
19. Cherkosov, G., A. Mojaev. (1991). The logical-probabilistic methods of evaluation of structure-complex systems reliability.- In the book: Reliability and Quality of Products. -M.: Knowledge, pp.34-65.

# SAFETY AND STABILITY SECURING OF AUTOMATIC CONTROL SYSTEMS AT PARAMETERS VARIATIONS

Jury P. Petrov  
S. Petersburg State University

**Abstract.** Customary and conventional calculation methods of stability reserve can in some cases (that are not very rare !) lead to incorrect results and they can become causes of hard wrecks. In order to prevent them it is necessary to deeply investigate the rules of transformations that are equivalent in a classical sense but not equivalent in a widened sense.

**Keywords.** Safety, stability automatic control systems, parameters variations.

A save work of control systems is possible only when they preserve stability at their inevitable parameters variations during exploitation. Therefore the check of stability and its preservation at parameters variations and at the same time the check of the value of stability reserve are an important and inalienable step in the projection and calculation of control systems. Here a mistake in the calculation of stability reserve value is more dangerous than a mistake in the calculation of stability since the instability of a system will be at once apparent during tests but a mistake in the estimation of stability reserve will become apparent during tests only after great pains. This mistake can even remain unnoticed and later it can lead to a hard wreck.

Up to recent years it was considered that the preservation of stability can be carried out for any equivalent form of writing a mathematical model of an investigated system. At hand calculations equations of a system reduced to a real exit variables, to a form of "entry-exit" were usually applied. But while calculating by means of quick-operating calculation machines it is more convenient to use a normal Cauchy form when equations of an investigated control system are transformed to a system of first degree differential equations in relation to a full state vector. And it was always admitted that if applied transformations are equivalent, if all solutions of a transformed system coincide with all solutions of an initial system then the calculation of stability preservation at parameters variations carried out for a normal Cauchy form gives correct results.

Recently it was discovered that it was not so. It was discovered that equivalent systems with identical solutions could radically differ by a property of stability preservation at parameters variations.

Therefore customary and conventional calculation methods of stability reserve can in some cases / that are not very rare ! / lead to incorrect results and they can become causes of hard wrecks. There grounds to suppose that some well-known wrecks that occurred in recent years and that had not been fully explained were the result of mistakes in projection, of an excessive trust in conventional calculation methods and in equivalent transformation.

Example: while investigating the control of shipping objects perturbing interactions /with Rachkmanin-Firssov spectrum/ are presented as a result of passing "a white noise"  $\eta(t)$  through a forming filter, i.e. through a link described by an equation:

$$\frac{d^2x_1}{dt^2} + 2\alpha \frac{dx_1}{dt} + (\alpha^2 + \beta^2) x_1 = \eta(t) \quad /1/$$

For convenience of machine calculations an equation /1/ is reduced to a normal Cauchy form:

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ x_2 &= -(\alpha^2 + \beta^2) x_1 - 2\alpha x_2 + \eta(t) \end{aligned} \quad /2/$$

and variables  $x_1$  and  $x_2$  are added to a state vector of a controlled object. It is not difficult to verify that the transfer from /1/ to /2/ is an equivalent transformation. But at the same time the calculation of stability preservation at parameters variations by means of a full vector and by real exits leads to quite different results, as it was shown in [1; 2; 5]. A calculation by a vector of real exits gives a correct answer. Only quite recently a calculation by full vector that is convenient during machine calculation became a common method / more and more often / although it often gives a mistakable result and may become a cause of wrecks. This phenomenon occurs because the preservation of stability at parameters variations is in fact not a property of a system itself but a property of its surrounding. It is a property of systems that are close to the investigated one in a space of parameters but it is not identical to it. In the statement that a system preserves stability at parameters variations we understand that in the surrounding of an investigated system only stable systems lie.

Therefore it is clear that equivalent transformations that do not change solutions must not preserve a property of an investigated system surrounding. In order to prevent mistakes it is necessary to introduce a new conception - a conception of transformations that are equivalent in a widened sense when not only a set of solutions of an investigated system remains invariable / this is characteristic of transformations equivalent in a classical sense / but properties of system surroundings are preserved. There exist / although not often / transformations that are equivalent in a classical sense but not equivalent in a widened sense. Examples of such transformations can be found in works [1 - 5]. In order to prevent mistakes and as a results of them - wrecks, in order to secure control systems safety it is necessary to investigate properties of transformations that are equivalent in a classical but not in a widened sense. We must be able to distinguish them and avoid such transformations.

The problem of securing stability at parameters variations is closely connected with a general problem of calculations problems correctness. Since in practice parameters and coefficients of any real object are almost always known only with a limited exactness and cannot remain - constant then only correctly posed mathematical problems have any real sense. We means such problems in which small changes of solutions correspond to small variations of parameters and coefficients. Recently non correctly posed problems became an object of investigation / see [6] /. But these problems require quite other methods of solution.

Of late / before the publication of works [ 1 ] and [ 3 ] / equivalent transformations were considered such that they do not change correctness and therefore it is sufficient to check the correctness of initial equations. But in the works / [ 1 ] and [ 3 ] / it was shown that it was not

so. Let us discuss an example in which it is seen that correctness can be changed not only in control systems.

Let us consider a system of algebraic equations with a parameter

$$\begin{aligned} (2+\lambda) X_1 + X_2 &= 0 & X_2 - (2+\lambda) X_3 + X_4 &= 0 & / 3 / \\ X_1 - \lambda X_2 &= 0 & X_1 + 2X_2 + X_3 + X_4 &= 0 \end{aligned}$$

and let us put the following problem: it is necessary to find values of a parameter  $\lambda$  at which a system / 3 / has nonzero solutions.

Such problems often appear at calculating frequencies - small vibrations of mechanical and electromechanical systems. A successive exclusion of variables beginning from  $X_1$  is one of solution methods. After excluding  $X_1$ ;  $X_2$  and  $X_3$  by means of equivalent transformations /by multiplication and addition/ the following equation remains:

$$(\lambda^3 + 5\lambda^2 + 7\lambda + 3) X_4 = 0 \quad / 4 /$$

where we see that there are three values that we wanted to find. They are equal to  $\lambda_1 = -3$ ,  $\lambda_2 = \lambda_3 = -1$ . It is not difficult to verify that this problem is correctly posed and that small changes in system / 3 / coefficients will correspond to small changes in the found values  $\lambda_i$ . But let us consider equations obtained after exclusion of variables  $X_1$  and  $X_2$  from system /3/ in more detail. They are of the form

$$\begin{aligned} (\lambda^3 + 4\lambda^2 + 5\lambda + 2) X_3 + (\lambda^2 + 2\lambda + 1) X_4 &= 0 & / 5 / \\ (\lambda^2 + 4\lambda + 5) X_3 + (\lambda + 1) X_4 &= 0 \end{aligned}$$

Surely system / 5 / is equivalent to system / 3 / and it has the same values  $\lambda$  bringing nonzero solutions. If we exclude  $X_3$  from it we shall by all means come to equation / 4 /.

But system / 5 / is not correctly posed: it is sufficient to change some coefficients in system /5/ by some value  $\varepsilon$  of smallness / for example, if we make coefficients in  $\lambda^3$  equal not to 1 but equal to  $1 + \varepsilon$  / and after excluding  $X_3$  we shall obtain instead of equation / 4 / the following equation:

$$(-\varepsilon\lambda^4 + \lambda^3 + 5\lambda^2 + 7\lambda + 3) X_4 = 0$$

This equation already has not three but four roots  $\lambda$  and for small  $\varepsilon$  we shall approximately have

$$\lambda_4 = -\frac{1}{\varepsilon}$$

System / 3 / and / 5 / are equivalent in a classical sense but not equivalent in a widened sense. The exclusion of  $X_1$  and  $X_2$  from system / 3 / although it was carried out by lawful means - by multiplication and addition / is an example of transformations that are equivalent in a classical sense but not in a widened sense. If such transformations are used during calculations then a variation of any degree of smallness in coefficients or a mistake of any degree of smallness in rounding off will lead to a gross mistake in the results of calculation.

It must be underlined that the investigated phenomenon by no means leads to a well-known not complete equivalence of transformations, as a result of which "superfluous" roots appear. No, systems / 3 / and / 5 / are completely equivalent and if variations of coefficients or mistakes in rounding off are absent solutions of systems / 3 / and / 5 / are identical. This means that a new and dangerous phenomenon is discovered which can become a source of serious mistakes in calculations.

In order to prevent them it is necessary to deeply investigate the rules of transformations that are equivalent in a classical sense but equivalent in a widened sense. It is necessary to be able to distinguish them and to avoid such transformations especially during calculations on quick-operating calculation machines. Then engineer's experience and intuition can not be of any help to him. And then calculations must be founded on the faultlessness of the used algorithm. Therefore it is necessary to make a revision of mathematical foundations of algorithms used during machine calculations. It is necessary to exclude from them transformations that are not equivalent in a widened sense and other sources of mistakes. Only such a revision will secure the safety of control systems and other technique that is used nowadays.

#### References.

1. Petrov Ju.P. The synthesis of optimal control systems when perturbing forces are incompletely known. Leningrad University edition, 1987, 289 pp. ( in Russian).
2. Petrov Ju.P. Stability of linear systems at parameters variations. *Avtomatika i Telemekhanika*, 1994, N11, pp.186-189. ( in Russian).
3. Petrov Ju.P. On secrete dangers in conventional methods of stability check. *Izvestiya VUZ, Elektromekhanika*, 1991, N11, pp.106-108. ( in Russian).
4. Petrov Ju.P. Wreckage prevention in control system. *Izvestiya VUZ, Elektromekhanika*, 1994, N1-2, pp.37-40. ( in Russian).
5. Petrov Ju.P. Calculation of control systems preserving stability at parameters variations. S.Petersburg state university edition, 1992, 35 pp. ( in Russian).
6. Tihkonov A.N. Arssenin V.Ya. Solution methos of problems that are incorrectly posed. M., 1974. ( in Russian).

# TECHNICAL DIAGNOSTICS ASSOCIATED WITH TANK FILLING OPERATIONS

Victor L. Galka

Central Research Institute of Ship Electrical Engineering

6, Blagodatnaya Street

196128 St Petersburg, Russia

**Abstract.** The paper investigates a way of technical realization of the evaluation of electrostatic danger, arising during filling operations of transport means, which have a wide network of pipelines and reception tanks, tankers included [1, 2].

**Keywords.** Electrostatic danger. Electrostatic parameters, filling operations, quantitative estimation

**Introduction.** The necessity of the electrostatic danger evaluation is caused by real threat of explosions and fires, produced by static electricity charges, which proves to be true because of large catastrophes and accidents on tankers with numerous victims and irreparable damage to the environment.

**Environmental Impact.** Realization of the presented technique will help to prevent the irreparable damage to the environment.

Investigation of the reasons of hazards show, that filling operations on ships are carried out with utter lack of information about quantitative characteristics of danger and criteria of safe exploitation of cargo systems. On tanker «Ludwig Svoboda», blown up in 1985 in port Ventspils, the values of electrostatic parameters were exceeded: the potential of the electrical field on the surface of oil in 4.6 times, and the volume charge density of a stream, flowing into tanks, - in 14 times.

The realization of the control and the integrated evaluation of electrostatic danger on tankers will ensure not only safety of cargo systems exploitation, but also choice of an optimum technological mode of filling operation with as much as possible acceptable productivity. For designing of such means of control the mathematical apparatus and the appropriate diagnostic maintenance should be developed, which are the subject of this paper.

**Conditions of the electrostatic danger on tankers.** It is known, that when hydrocarbon fuels flow down the pipes they gain static charges, which is caused by division of a double electrical layer on oil-washed surface [3]. With the low specific volumetric electrical conductivity of a liquid the thickness of the diffusive part of the external facing of a double electrical layer grows considerably and becomes comparable with the thickness of a hydrodynamic boundary layer. Thus a part of the electrical charges of the external facing is carried along by the stream into the volume of liquid. The quantitative characteristic of the charges division and their entry into the stream is the current value of the charged liquid  $I_3$ .

The charges, which were divided, would try to unite and neutralize one another. The period of time, during which the charge is saved, is defined by time of its relaxation in the liquid and is quantitatively characterized by value of the time constant of relaxation  $\tau$ . This value is in direct dependence from permittivity of fuel  $\varepsilon_1$  and its specific electrical conductivity  $\gamma_1$  by a ratio

$$\tau = \varepsilon_1 / \gamma_1 \quad (1)$$

With high values of  $\tau$  and of the liquid stream speed down the pipelines, which depends on values of filling operation productivity  $W$ , in tanks there might take place a significant accumulation of electrical charges of the same polarity.

Arising in this manner the electrical field may become a source of static discharges from the fuel surface with energy, sufficient for ignition of the air-steam medium above the fuel interior of the tank. The energy saved in the tankage is characterized with electrical potential  $U$ , which permits to associate the change of  $U$  with the conditions of the ignition charges emerge. Therefore in international practice as a criterion of electrostatic danger for filling operations with liquid hydrocarbons the maximum accepted value of the electrical potential on surface of the tank fuel ( $\Gamma$ ), not exceeding  $U_{kp} = 35 \text{ kV}$  [4] is accepted, and in the domestic practice criterion  $U_{kp} = 25 \text{ kV}$  is used [5].

Then the condition of safe exploitation of the cargo system can be formulaed as follows: for each set of values  $(I_3, \tau, w, h)$ , where  $h$  is the level of filling of the tank, condition should be observed: not to exceed the maximum value of the electrical potential on the surface of the fuel of some value  $U_{kp}$ . For systems with tanks in a form of a parallelepiped it can be recorded as follows:

$$\forall (I_3, \tau, w, h) \Rightarrow \sup_{(x, y, z) \in \Gamma^*} |U| < U_{kp} \quad (2)$$

where  $x, y, z$  are cartesian coordinates.

Thus, in the real system of the tanker, for which during its functioning it is difficult to measure the electrical potential on the surface of the fuel directly, the degree of the electrostatic danger can be determined in an indirect way. In this case its definition will be carried out in two stages. At first stage the admittance principle "dangerously - safely" is used and the borders of the serviceability zones are established by way of calculations (the method is given in work [6]). At second stage on the base of results of measurements of parameter values  $(I_3, \tau, w, h)$ , for each of their sets an assessment of the stock of the system serviceability and the degree of electrostatic danger will be carried out by a method of classification of technical conditions according the key rule, which will be considered below.

**Solution of the electrostatic problem.** Let us consider the model of the rectangular tank filling, shown on fig. 1, where  $G_1$  and  $G_2$  are tank areas, filled by fuel and air-steam space accordingly;  $\Gamma$  - conducting grounded tank walls. The calculated zones of the system serviceability are developed according to the joint solution of Poisson's and Laplac's equations.

$$\begin{aligned} \Delta U_1 &= p/\epsilon_1, (x, y, z) \in G_1 \\ \Delta U_2 &= 0, (x, y, z) \in G_2 \end{aligned} \quad (3)$$

With boundary conditions

$$\begin{aligned} U_1 I_r &= U_2 I_r = 0 \\ U_1 I_{r^*} &= U_2 I_{r^*} \\ \left. \frac{\partial \sigma}{\partial t} \right|_{r^*} &= \gamma E_{m_1} I_{r^*} - \gamma_2 E_{2m_1} I_{r^*}, \\ \sigma I_{r^*} &= -\epsilon_1 E_{in} I_{r^*} + \epsilon_2 E_{2n} I_{r^*} \end{aligned} \quad (4)$$

where  $\rho$  is the density of the volumetric charge;  $\sigma$  is the density of the surface charge;  $-\gamma_2$  is the specific electrical conductivity of the air-steam area;  $E_{1n}$  and  $E_{2n}$  are normal constituents of the electrostatic field intensity in areas, filled with fuel and air-steam space accordingly;  $\epsilon_2$  is the permittivity of the air-steam space.

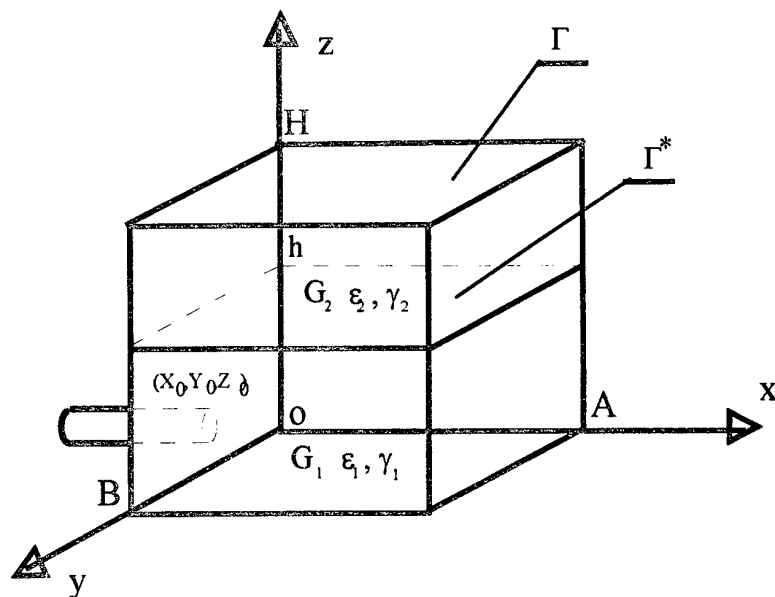


Fig.1 Calculated model of the rectangular tank filling.

With slow filling of tanks, that corresponds to the quasistatic mode of the surface charge formation, i.e.  $\partial\sigma / \partial\tau = 0$  and  $E_{1N}|_{r^*} = 0$ , the boundary conditions (4) can be replaced with the following:

$$\begin{aligned} U_1|_r &= U_2|_r = 0 \\ U_1|_{r^*} &= U_2|_{r^*}, \\ \left. \frac{\partial U_1}{\partial z} \right|_{r^*} &= 0 \end{aligned} \quad (5)$$

With the known distribution of a volumetric charge density the ways of mathematical models construction of which are shown in the paper [7], we receive a set of values of the electrical potential for various combinations of parameters ( $I_3, t, w, h$ ). The zone of the system serviceability is built as a projection of function section  $U(I_3, t, w, h)$  with the plane at level  $U_{kp}$  on the plane of parameters. The values  $I_3$  and  $t$  are examined as fixed, as their values for the particular type of fuel during filling can be practically unchanged; the principle of the zone of serviceability construction is illustrated on fig. 2.

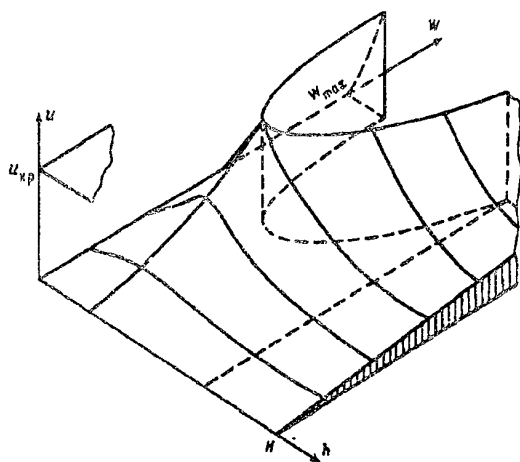


Fig.2 The principle of the system serviceability zones construction

This is the way the sets of serviceability zones for a tanker cargo system are built at various possible combinations of values of operational and electrostatic parameters. These zones can be presented both in graphic, and in tabulated forms. As an example for the physical model of the hydro-carbon fuel system of the tanker (fig.3) on fig.4 and 5 a way of construction of a zone of its serviceability is show in graphic form.

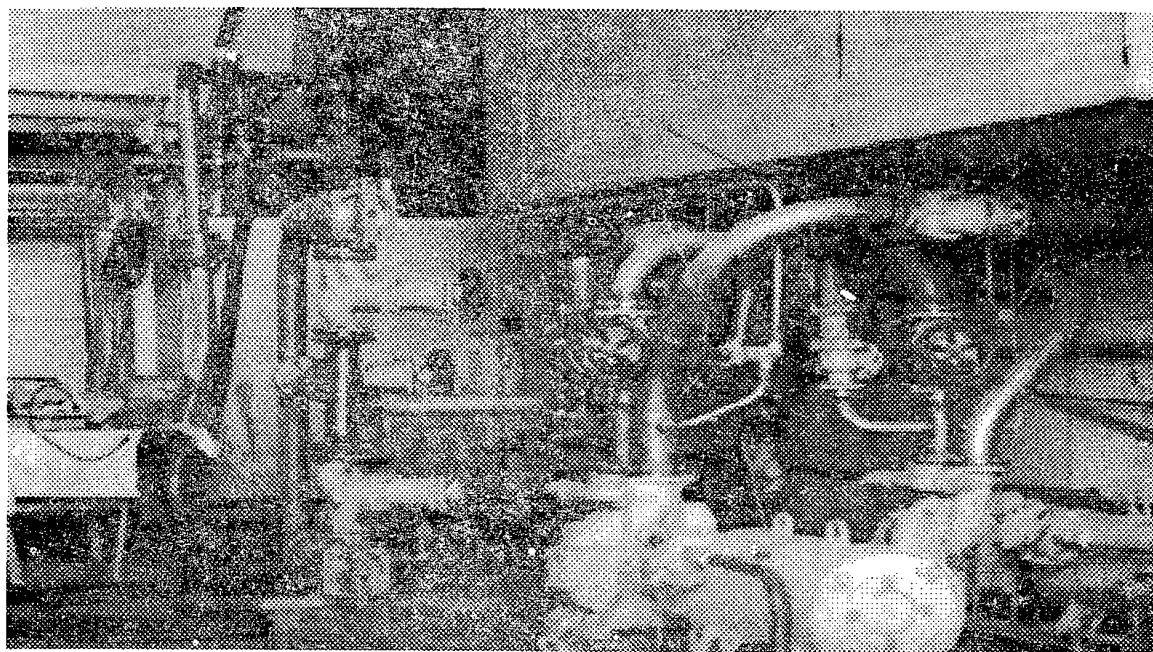


Fig.3 The physical model of the tanker cargo system

The curve  $w(h)$  differentiates the area of the system states for efficient and a non-efficient one. The estimation of the system serviceability and evaluation of electrostatic danger will include definition of a vector of the system states in relation to the function, dividing the serviceability zone into subzones.

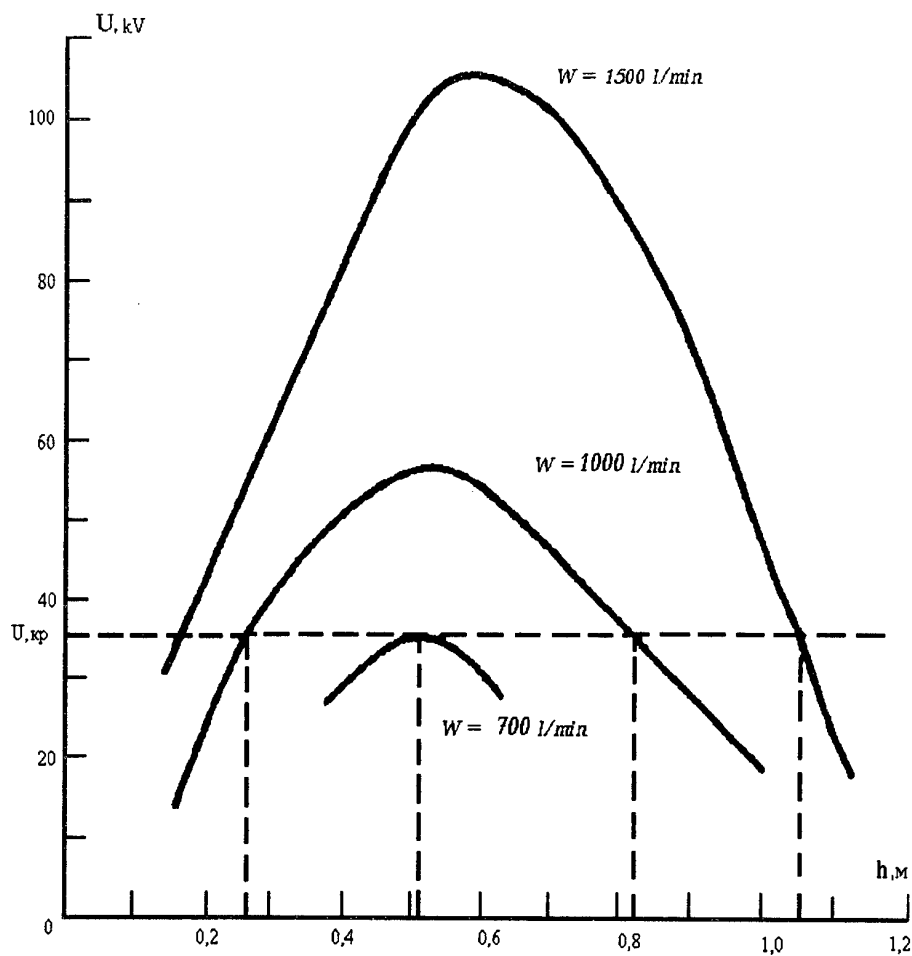


Fig. 4. Change of the electrical potential during the experiments

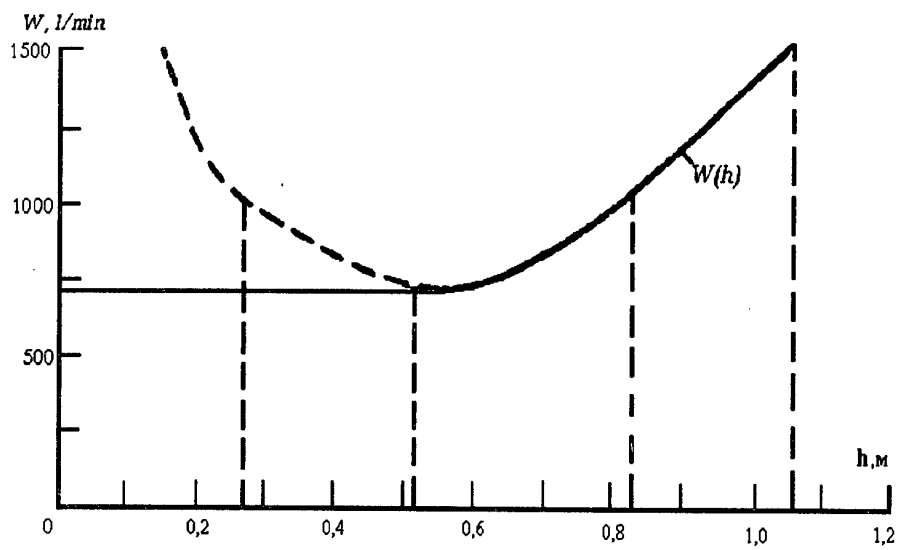


Fig. 5. Serviceability zones of the physical model

**Solution of the problem of the states classification system.** Let us introduce designations  $R_1$  and  $R_2$  as of classes according to efficient and non-efficient system states, and  $R_m$  as a vector of measured parameter values ( $I_3, \tau, W, h$ ). The key rule  $L$  determines the action, according to which  $R_m$  belongs either to class  $R_1$ , or to class  $R_2$ . As a dividing function  $g(R)$  let us name such a function, which causes acceptance of the decision, ensuring fulfillment of the following condition [7].

$$L(R_m) = R_1 \leftrightarrow g(R), \\ \forall (i, j) = 1, 2, \dots k \text{ and } R \in R_m$$

Then the admittance principle will be realized by a following ratio:

$$L(R_m) = \begin{cases} R_1, & \text{if } g(R) > 0 \\ R_2, & \text{if } g(R) < 0 \end{cases} \quad (6)$$

The special case  $g(R) = 0$  corresponds to the equation of hypersurface, dividing the considered regions of the parameter values, therefore the decision of belonging of any point of this surface can be adopted arbitrarily (either to the area, having a smaller index, or to that with the greater one).

Let us consider the calculated serviceability zone of any system with fixed values of  $I_3$  and  $\tau$  and a varying set of values ( $w_1, h_j$ ). On the plane of parameters ( $W_1, h_j$ ) let us construct a grid so that the equivalent indexes  $i$  and  $j$  were crossed on curve  $w(h)$  (fig.6). To such grid there will correspond a square matrix

$$A_k [R_{ij}] = [w_1, h_j], \forall (i, j) = 0, 1, 2, \dots, m, \quad (7)$$

or

$$A_k = \begin{vmatrix} R_{00} & R_{10} & R_{20} & \dots & R_{m0} \\ R_{01} & R_{11} & R_{21} & \dots & R_{m1} \\ R_{02} & R_{12} & R_{22} & \dots & R_{m2} \\ \dots & \dots & \dots & \dots & \dots \\ R_{0(m-1)} & R_{1(m-1)} & R_{2(m-1)} & \dots & R_{m(m-1)} \\ R_{0m} & R_{1m} & R_{2m} & \dots & R_{mm} \end{vmatrix} = \begin{vmatrix} w_0, h_0 & w_1, h_0 & \dots & w_m, h_0 \\ w_0, h_1 & w_1, h_1 & \dots & w_m, h_1 \\ w_0, h_2 & w_1, h_2 & \dots & w_m, h_2 \\ \dots & \dots & \dots & \dots \\ w_0, h_{m-1} & w_1, h_{m-1} & \dots & w_m, h_{m-1} \\ w_0, h_m & w_1, h_m & \dots & w_m, h_m \end{vmatrix}$$

From the matrix follows, that the dividing function  $g(R)$  can be determined as

$$g(R) = j - i, \forall (i, j) = 0, 1, 2, \dots, m \quad (8)$$

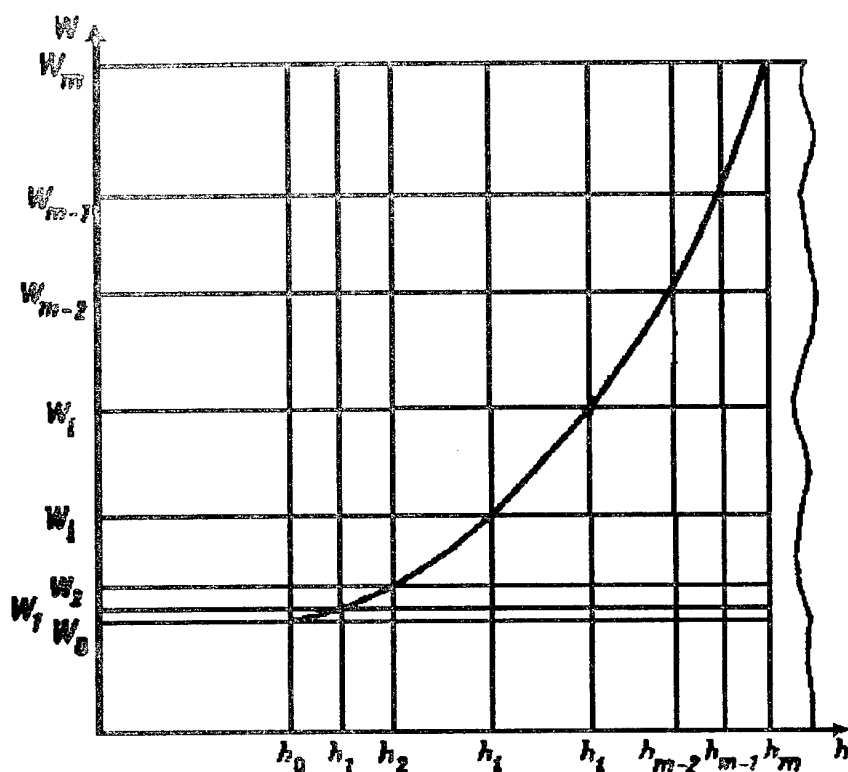


Fig.6. Approximation of the dependence  $W = f(h)$

According to (8) in matrix  $A_k$  to the right of the main diagonal the function  $g(R)$  accepts negative values, and to the left - the positive ones. Then according to the mark of function  $g(R)$  it is possible to judge whether the state of the system is efficient. It is obvious, that the greater the value  $g(R)$ , the less is the probability that the system will proceed into a non-efficient state. The removal of vector  $R_m$  from the main diagonal increases serviceability.

Let us introduce the quantitative characteristic of the system stock of serviceability  $\delta R_m$  accepting values

$$\delta R_m = \begin{cases} 1 & \text{with } g(R) = m \\ 0 & \text{with } g(R) = 0 \\ -1 & \text{with } g(R) = -m \end{cases} \quad (9)$$

In each next diagonal of matrix  $A_k$ , parallel to the main, the values  $g(R)$  differ per unit, therefore one should classify a state of the system by diagonals of a square matrix. Then, being moved from value  $R_{om}$  up to value  $R_{m0}$  each following step of reduction of index  $i$  (or increase of  $i$ ) results in a unity discrete accretion of value  $\delta R_m$ . The size of one accretion with the uniform grid will make  $1/m$ .

Thus, the accretion of one of the indexes  $i \pm 1$  or  $j \pm 1$  assumes the transfer of value  $\delta R_m$  in the next subzone  $R_{ij}$ . In case of the simultaneous accretion of two indexes for 1 or one of them for  $2\delta R_m$  acquires value with accretion  $2/m$  and etc. The number of steps,

characterizing the amount of subzones, through which move values  $\delta R_m$  from any of its values up to  $\delta R_m = 0$ , which is determined by  $g(R) = 1 - i$ , with the values  $\delta R_m$  for any pair  $(w_i, h_i)$  in each diagonal are the same.

Then the stock of the system serviceability is calculated according the formula

$$\delta R_m = g(R) / m \quad (10)$$

As an example let us construct the table of values of a stock of the system serviceability with grid  $m = 5$ . The value of one accretion for given example is equal 0.2

0	$R_{00}$					
0.2	$R_{01}$	$R_{11}$				
0.4	$R_{02}$	$R_{12}$	$R_{22}$			
0.6	$R_{03}$	$R_{13}$	$R_{23}$	$R_{33}$		
0.8	$R_{04}$	$R_{14}$	$R_{24}$	$R_{34}$	$R_{44}$	
1.0	$R_{05}$	$R_{15}$	$R_{25}$	$R_{35}$	$R_{45}$	$R_{55}$
$\delta R_m =$	1.0	0.8	0.6	0.4	0.2	0

The table shows, that the values of parameters on the main diagonal correspond to a zero store of the system serviceability and this stock is increased in accordance with removal of values to the left from this diagonal.

Let us introduce the quantitative characteristic of electrostatic danger degree, determined as

$$\delta S_m = 1 - \delta R_m \quad (11)$$

Then its values can be calculated according to the formula

$$\delta S_m = [m - g(R)] / m \quad (12)$$

**Designing of means.** Parities (10) and (12) permit to develop algorithm for designing means of diagnosing and regulation of the system work in safe modes of its operation (fig. 7).

In this case the following sequence of operations is observed.

1. There are introduced into the memory of the board computer:  
a calculated matrix of the system technical states concerning the set of parameter values  $\{I_{3k}, \tau_p, w_i, h_i\}$  and numerical values of corresponding to them indexes  $\{k, p, i, j\}$ , where  $K$  is the index of the tank filled with fuel;  $p$  is the index of the kind of fuel;  
the key rules for classification and calculations of a stock of serviceability, determined by parities (8), (10).
2. During the filling operation the control of parameters  $\{I_{3k}, \tau_p, w_i, h_i\}$  is carried out and their actual values enter the computer.
3. In the computer.

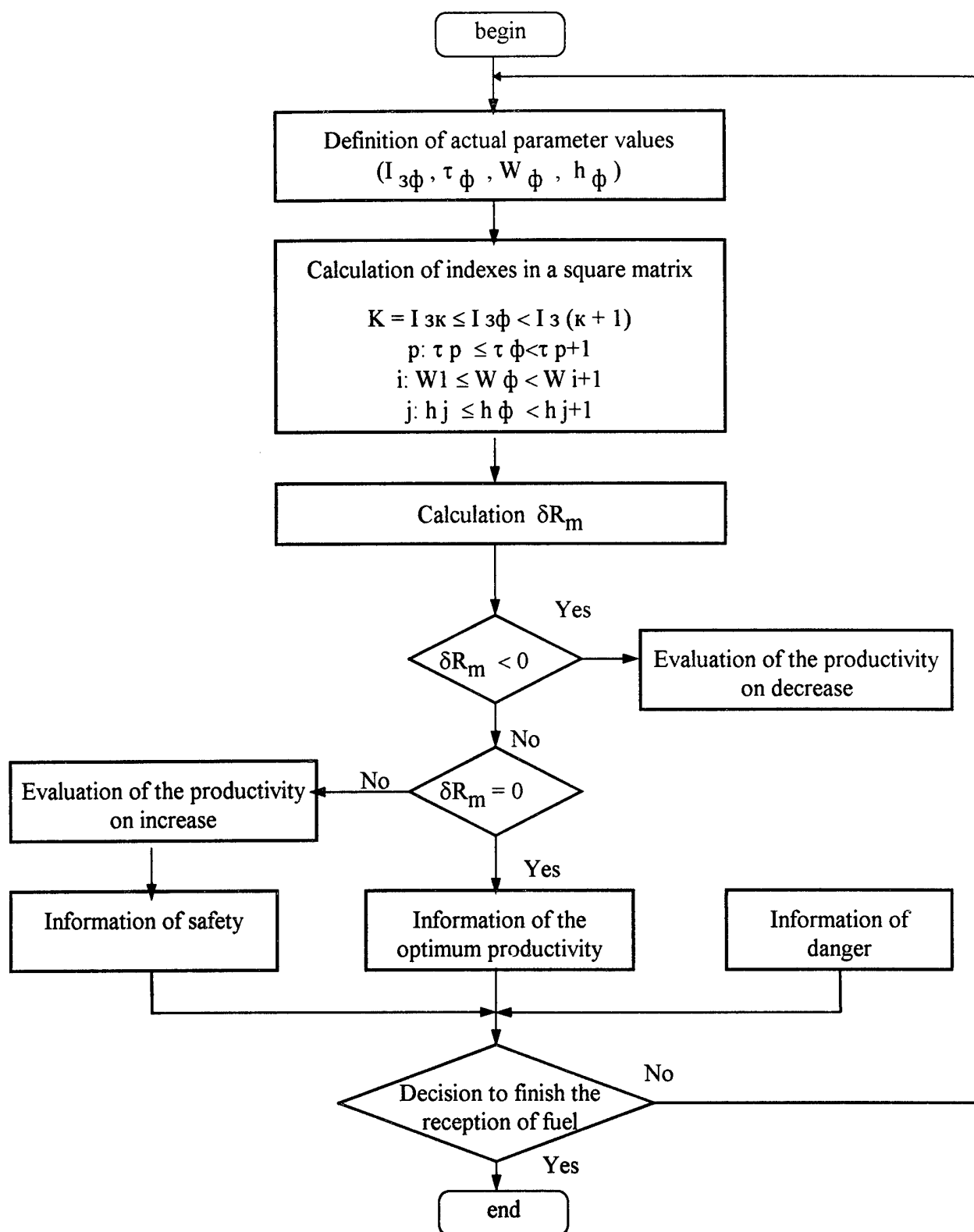


Fig.7 Algorithm of diagnostic and regulation of the tanker cargo system operation

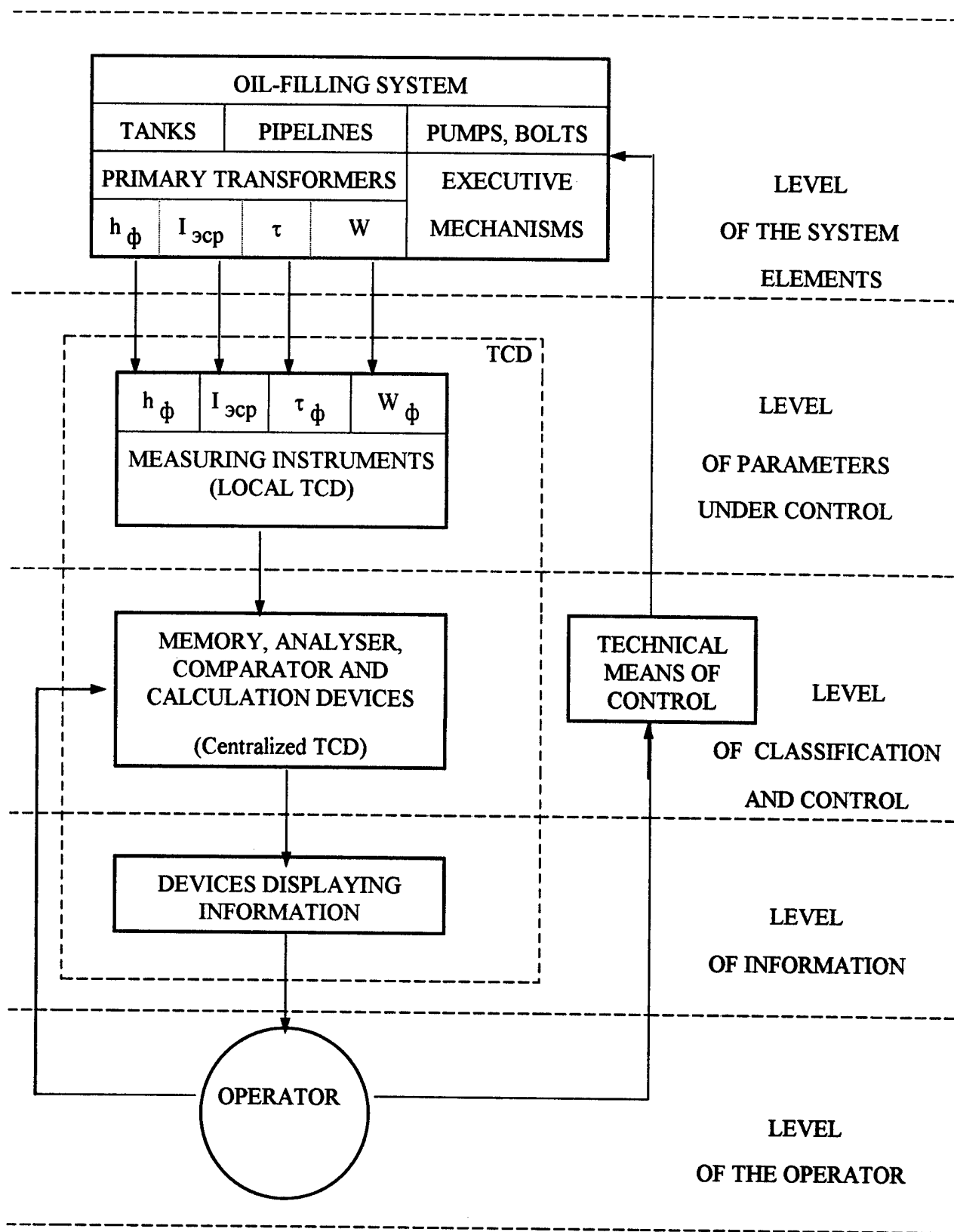


Fig.8 Structure of the diagnostic system

The search of similar (or close to them) parameter values is carried out, indexes are defined with these values and dividing function  $g(R)$  is calculated,

The mark and value of the system stock of serviceability are defined. When

$\delta R_m < 0$  the operator gets the information of danger. When  $\delta R_m > 0$  the information is given that the filling operation is carried out with stock of safety and its productivity can be increased. When  $\delta R_m = 0$  the information on optimum productivity of filling the particular tank is presented.

4. When the information on values of the system stock of serviceability is received the operator adjusts the filling process of the tanker, operating the productivity of fuel submission. The stated sequence can be fulfilled by the system of diagnosing, which includes: the tanker system of hydrocarbon fuel, means of diagnosing and operator. The basic elements of such system can be presented by a multilevel structure, shown on fig. 8. According to the structure the means of diagnosing contain:

Measuring instruments of actual values) of four parameters: fuel charge current in the pipeline ( $I_\phi$ ), time constant of charge relaxation in fuel ( $\tau_\phi$ ); level of filling the tank with fuel ( $h_\phi$ ) and productivity of filling the tank.

The constant storing device (CSD) for keeping the information on calculated zones of the hydrocarbon fuel system serviceability during filling each tank with a different kind of fuel.

$$\{I_{3k}, \tau_p, w_i, h_i\};$$

The device for analysis and choice of serviceability from CSD for combinations of parameter values

$$\{I, I_{3\phi}, \tau_p, \tau_\phi\};$$

The device for search of identical parameter values or the similar ones  $\{w_i, w_\phi, h_i, h_\phi\}$  with the purpose of indexes identification  $\{ij\}$

The device for system serviceability values calculation  $\delta R_m$  and its mark depending on the efficient (or non-efficient) zone the vector of the system state

$$(\delta R_m > 0 \text{ or } \delta R_m < 0) \text{ is located.}$$

The device for calculation of the electrostatic danger degree -  $\delta S_m$

The device for calculation of productivity value accretions for its increase or reduction;

The display unit.

### Conclusions

1. A technique of the quantitative estimation of electrostatic danger during filling operations on tankers is offered, the values of which should be accepted for regulation of filling processes with the choice optimum of filling productivity.

2. The paper shows structural realization of this technique by diagnosing means, the purpose of which are electrostatic safety and rational efficiency of filling operations.

3. The offered way of electrostatic danger evaluation can also be used on other objects of the national economy, concerning transportation and oil-products storage.

### *References*

1. Mills G.S., Oldham R.C. (1983) Evaluation and prevention of electrostatic hazards associated with oil tanker operations // Trans.9. Mar. E(TM).. V. 95. Paper. 44. № 2, p. 18
2. Галка В.Л., Щигловский К.Б. (1993) Вопросы электростатической защиты танкеров Судостроение, № 10. С. 9 - 11
3. Леб Л. (1983) Статическая электризация. М.; Л.: Госэнергоатомиздат, стр. 408
4. Butterworth G.J., Brown K.P. (1982) Assessment of electrostatic ignition hazards during tank filling with the aid of computer modeling // J. Electrostatic. № 13/ P/ 9-27
5. Максимов Б.К., Обух А.А., Тихонов А.В. (1989) Электростатическая безопасность при заполнении резервуаров нефтепродуктами. М.; Энергоатомиздат, стр.152
6. Галка В.Л. (1990) Метод построения зон работоспособности грузовой системы нефтеналивного судна. Судостроительная промышленность. Судовая электротехника и связь. Вып.11. Стр.38-48
7. Галка В.Л., Щигловский К.Б.(1994) Модель распределения электрических зарядов при заполнении резервуаров слабопроводящими жидкостями и расчет средств электростатической защиты. Электричество. № 8. Стр.54-58
8. Фор А.. (1989) Восприятие и распознавание образов. М.: Машиностроение. Стр.272

# THE PROBLEM OF THE SUSTAINABLE DEVELOPMENT

Mikhail Ignatiev

S.Petersburg State Aerospace Instruments Academy and Independent Institute of Sustainable Development , 67, Bolshaja Morskaja ul. St.-Petersburg, 190000, RUSSIA,

Tel. (812)-210-7044, Fax. (812) - 315-7778, E-mail: kvm@cit.spb.su

**Abstract.** The complex developing systems are being considered with point of view of the adaptational maximum phenomenon. It is maintained that the sustainable development can take place only in the zone of adaptational maximum on the example of the region.

## **Keywords**

Model of development, uncertainty, combinatorial analysis, decision support making.

## **Introduction**

In this time a lot of scientists and different organizations are studying the problems of the sustainable future according to United Nations Organization decisions and national programs of the sustainable development. For example, President of Russian Federation in February 1994 had affirmed the strategy of the sustainable development of Russia, which proclaimed the noospherical approach. Part of this strategy is the program of Baltic Sea region, the program of Black Sea region, the program of Caspian Sea region, the program of Far East Sea region and the program of Arctic Ocean region. Today each region has the important meaning for economic and social development, but each region has a lot of difficult problems such as transportation, energy, ecology, conflict etc. The global simulation can help in decision making process on different levels and can decrease some hazards . The monitoring had shown many hazards in these regions and the creation of global simulation system is actual, but it is necessary to solve several problems:

1. The determination of main hypothesis, which will be the base of model.
2. The determination of model structure.
3. The determination of organizational and technical possibilities for realization of global simulation.

In our paper we will try to answer on this questions using our experience in the global simulation system of Baltic Sea region. Main purpose of global simulation is determination of conditions for sustainable development and safety increasing.

## COMBINATORICAL SIMULATION

We have a lot of different models [7], but their general defect is the absence of combinatorial approach in evident form. When we have the complex system, we can observe a lot of appearances, but we cannot observe the essences. We propose to consider the groups of the appearances and to introduce the new variables - the essences of our appearances. For example, when we have sentence

word1+word2+word3

( 1 )

our words are the appearances and we can introduce the sense, which ordinary implies but not designates:

$$(\text{word1}) \cdot (\text{sense1}) + (\text{word2}) \cdot (\text{sense2}) + (\text{word3}) \cdot (\text{sense3}) = 0 \quad (2)$$

When we have the equation  $F(x_1, x_2, x_3) = 0$ , we can turn to such form by means of differentiation

$$\frac{\partial F}{\partial x_1} \cdot \frac{dx_1}{dt} + \frac{\partial F}{\partial x_2} \cdot \frac{dx_2}{dt} + \frac{\partial F}{\partial x_3} \cdot \frac{dx_3}{dt} = 0 \quad (3)$$

where the partial derivatives are the appearances and the derivatives with respect to time are the essences.

In general we can write the equations (2) and (3) in form

$$A_1 \cdot E_1 + A_2 \cdot E_2 + A_3 \cdot E_3 = 0 \quad (4)$$

This model is the algebraic ring and we can resolve this equation relatively the appearances  $A_i$  or the essences  $E_i$  [1,2,3]:

$$\begin{aligned} A_1 &= u_1 \cdot E_2 + u_2 \cdot E_3 \\ A_2 &= -u_1 \cdot E_1 + u_3 \cdot E_2 \\ A_3 &= -u_2 \cdot E_1 - u_3 \cdot E_2 \end{aligned} \quad (5)$$

or

$$\begin{aligned} E_1 &= u_1 \cdot A_2 + u_2 \cdot A_3 \\ E_2 &= -u_1 \cdot A_1 + u_3 \cdot A_2 \\ E_3 &= -u_2 \cdot A_1 + u_3 \cdot A_2 \end{aligned} \quad (6)$$

where  $u_1, u_2, u_3$  - the arbitrary coefficients, which can use for decisions of the different tasks on the initial manifold (2) or (3). For example, if we would like to reach the maximum of  $x_3$ , we can assign the arbitrary coefficients

$$u_2 = -b \cdot A_1, u_3 = -b \cdot A_2$$

and we receive

$$\begin{aligned} \frac{dx_1}{dt} &= u_1 \cdot A_2 - b \cdot A_1 \cdot A_3 \\ \frac{dx_2}{dt} &= -u_1 \cdot A_1 - b \cdot A_2 \cdot A_3 \\ \frac{dx_3}{dt} &= b \cdot (A_1 \cdot A_1 + A_2 \cdot A_2) \end{aligned}$$

and if  $b > 0$ , then variable  $x_3$  strives to maximum on stable.

In general if we have  $n$  variables and  $m$  initial manifolds, then the number of arbitrary coefficients will be

$$s = C_n^{m+1} \quad (7)$$

For example, if we have  $n = 7$ ,  $m = 1$ , then  $s = 21$ .

It is important, that we have full sum of combinations and have the all variants of decisions. The combinatorical simulation is the useful heuristic approach for investigation of complex systems.

## ADAPTATIONAL MAXIMUM PHENOMENON

We will describe a system and its interaction with the environment according to the scheme adjusted below - Fig.1. The result of interaction influences both the system and the environment. The system control unit generates control signals to decrease the difference between the system coordinates  $X_1...X_k$  and the environmental ones  $Y_1...Y_k$ .

It is supposed that the behavior of system with  $n$  variables is given to an approximation of  $m$  manifolds,  $n > m$ .

In case the system is considered as a multidimensional generator where at least a part of system variables interacts with environment variables, and the objective of system is to decrease the functional of discoordination between them, the system control unit has two instruments of influence on system. First, this is the tuning - the change of undeterministic coefficients in the differential equations of system taking into account that the more these coefficients are the more accurate the responses of the system to the change of environment will be (channel **a** on Fig.1).

Second, this is the learning - the imposition of new restrictions on the system behavior (channel **b** on Fig.1). The amount of arbitrary coefficients, which describes the behavior by the system of equation, is changing in the process of learning, of consecutive imposition of new and new restrictions on the system behavior. In the systems with the number of variables more than six the amount of arbitrary coefficients increases first and then going through the maximum begins to decrease according to the formula (7).

The Table 1 reflects the number of arbitrary coefficient in structure of equation as a function of number of superimposed restrictions.

This phenomenon permits to explain the processes of growth, complication and death of a system. The existence of adaptational maximum is proved by numerous biological and economical systems. Under the analyze of complex cybernetic systems the author discovered the adaptational maximum phenomenon (Ignatiev 1963) and the following statement is placed in the basement of the concept of sustainable development:

**SUSTAINABLE DEVELOPMENT CAN TAKE PLACE ONLY IN ZONE OF ADAPTATIONAL MAXIMUM.**

This statement determines the plan of activity of strategically, tactical and operational levels: living cycles of the systems should be investigated, the adaptational maximum should be defined and the activity should be planned to retain the system within adaptational maximum zone in spite of environment changes.

If we enunciate the word "system", we select some structure out of the whole world and this structure becomes opposite of all the remaining world, which we consider as an environment. The system interacts with the environment, which is some more than the system. Thus the first operation is to confine the system from the environment.

The second operation is the orientation of system. For this purpose the equivalent equations must be resolved for the derivatives of the appointed variables.

The third operation is definition of the structure of undetermined, arbitrary coefficients.

In the capacity of the example we are considering the region and the imitative model of the region for decision making support. The aim of this simulation is the improvement of living conditions in the region areas with inadequate socio-economic and cultural infrastructure.

## SIMULATION OF REGION

Each scientific group, governmental organization and private company has its own model of sustainable development, but if we would like to reach the mutual understanding and mutual coordinational decisions, we must work out the unificate model. It is difficult task, we will study another participants model with pleasure and propose our model. General model of development consist of seven main blocks:

1. block of population with attributes of health, education, employment,
2. block of passionarity with reflect the intentions of groups of population,
3. block of territory which includes the geoinformational systems,
4. block of ecology with attributes of quality of air, water, forest etc,
5. block of production which includes industry, agriculture and science,
6. block of finance and finance flows,
7. block of external relations of region which includes the entering and outgoing flows of energy, substance, information and peoples.

In the frame of combinatorical model we will have the equation

$$\sum_{i=1}^7 A_i \cdot E_i = 0 \quad (8)$$

and after to resolve relatively  $E_i$  we will have

$$\begin{aligned} \frac{dA_1}{dt} &= u_1 A_2 + u_2 A_3 + u_3 A_4 + u_4 A_5 + u_5 A_6 + u_6 A_7 \\ \frac{dA_2}{dt} &= -u_1 A_1 + u_7 A_3 + u_8 A_4 + u_9 A_5 + u_{10} A_6 + u_{11} A_7 \\ \frac{dA_3}{dt} &= -u_2 A_1 - u_7 A_2 + u_{12} A_4 + u_{13} A_5 + u_{14} A_6 + u_{15} A_7 \\ \frac{dA_4}{dt} &= -u_3 A_1 - u_8 A_2 - u_{12} A_3 + u_{16} A_5 + u_{17} A_6 + u_{18} A_7 \\ \frac{dA_5}{dt} &= -u_4 A_1 - u_9 A_2 - u_{13} A_3 - u_{16} A_4 + u_{19} A_6 + u_{20} A_7 \\ \frac{dA_6}{dt} &= -u_5 A_1 - u_{10} A_2 - u_{14} A_3 - u_{17} A_4 - u_{19} A_5 + u_{20} A_7 \\ \frac{dA_7}{dt} &= -u_6 A_1 - u_{11} A_2 - u_{15} A_3 - u_{18} A_4 - u_{20} A_5 - u_{21} A_6 \end{aligned} \quad (9)$$

where  $u_1, u_2, \dots, u_{21}$  - the arbitrary coefficients which we can use for control of our system.

In computer model instead of variables  $A_1, A_2, \dots, A_7$  it is possible to use the relevant files of data.

In the case of cooperation between countries we must examine the next systems :

$$\sum_{i=1}^7 A_i^j \cdot E_i^j = 0; j = 1, 2. \quad (10)$$

Similarly to (9) we will have:

$$\begin{aligned} \frac{dA_1}{dt} &= v_1 \cdot D_{23} + v_2 \cdot D_{24} + v_3 \cdot D_{25} + v_4 \cdot D_{26} + v_5 \cdot D_{27} + \\ &+ v_6 \cdot D_{34} + v_7 \cdot D_{35} + v_8 \cdot D_{36} + v_9 \cdot D_{37} + v_{10} \cdot D_{45} + \\ &+ v_{11} \cdot D_{46} + v_{12} \cdot D_{47} + v_{13} \cdot D_{56} + v_{14} \cdot D_{57} + v_{15} \cdot D_{67} \\ \frac{dA_2}{dt} &= -v_1 \cdot D_{13} - v_2 \cdot D_{14} - v_3 \cdot D_{15} - v_4 \cdot D_{16} - v_5 \cdot D_{17} + \\ &+ v_{16} \cdot D_{34} + v_{17} \cdot D_{35} + v_{18} \cdot D_{36} + v_{19} \cdot D_{37} + v_{20} \cdot D_{45} + \\ &+ v_{21} \cdot D_{46} + v_{22} \cdot D_{47} + v_{23} \cdot D_{56} + v_{24} \cdot D_{57} + v_{25} \cdot D_{67} \\ \frac{dA_3}{dt} &= -v_1 \cdot D_{12} - v_6 \cdot D_{14} - v_7 \cdot D_{15} - v_8 \cdot D_{16} - v_9 \cdot D_{17} - \\ &- v_{16} \cdot D_{24} - v_{17} \cdot D_{25} - v_{18} \cdot D_{26} - v_{19} \cdot D_{27} + v_{26} \cdot D_{45} + \\ &+ v_{27} \cdot D_{46} + v_{28} \cdot D_{47} + v_{29} \cdot D_{56} + v_{30} \cdot D_{57} + v_{31} \cdot D_{67} \\ \frac{dA_4}{dt} &= -v_2 \cdot D_{12} - v_6 \cdot D_{13} - v_{10} \cdot D_{15} - v_{11} \cdot D_{16} - v_{12} \cdot D_{17} - \\ &- v_{16} \cdot D_{23} - v_{20} \cdot D_{25} - v_{21} \cdot D_{26} - v_{22} \cdot D_{27} - v_{26} \cdot D_{35} - \\ &- v_{27} \cdot D_{36} - v_{28} \cdot D_{37} + v_{32} \cdot D_{56} + v_{33} \cdot D_{57} + v_{34} \cdot D_{67} \\ \frac{dA_5}{dt} &= -v_3 \cdot D_{12} - v_7 \cdot D_{13} - v_{10} \cdot D_{14} - v_{13} \cdot D_{16} - v_{14} \cdot D_{17} - \\ &- v_{17} \cdot D_{23} - v_{20} \cdot D_{24} - v_{23} \cdot D_{26} - v_{24} \cdot D_{27} - v_{26} \cdot D_{34} - \\ &- v_{29} \cdot D_{36} - v_{30} \cdot D_{37} - v_{32} \cdot D_{46} - v_{33} \cdot D_{47} + v_{35} \cdot D_{67} \\ \frac{dA_6}{dt} &= -v_4 \cdot D_{12} - v_8 \cdot D_{13} - v_{11} \cdot D_{14} - v_{13} \cdot D_{15} - v_{15} \cdot D_{17} - \\ &- v_{18} \cdot D_{23} - v_{21} \cdot D_{24} - v_{23} \cdot D_{25} - v_{25} \cdot D_{27} - v_{27} \cdot D_{34} - \\ &- v_{29} \cdot D_{35} - v_{31} \cdot D_{37} - v_{32} \cdot D_{45} - v_{34} \cdot D_{47} - v_{35} \cdot D_{57} \\ \frac{dA_7}{dt} &= -v_5 \cdot D_{12} - v_9 \cdot D_{13} - v_{12} \cdot D_{14} - v_{14} \cdot D_{15} - v_{15} \cdot D_{16} - \\ &- v_{19} \cdot D_{23} - v_{22} \cdot D_{24} - v_{24} \cdot D_{25} - v_{25} \cdot D_{26} - v_{28} \cdot D_{34} - \\ &- v_{30} \cdot D_{35} - v_{31} \cdot D_{36} - v_{33} \cdot D_{45} - v_{34} \cdot D_{46} - v_{35} \cdot D_{56} \end{aligned} \quad (11)$$

where  $D_{ij} = \det \begin{vmatrix} A_i^1 & A_j^1 \\ A_i^2 & A_j^2 \end{vmatrix}$ , and  $v_1, v_2, \dots, v_{35}$  - the arbitrary coefficients which we can use for control of our systems.

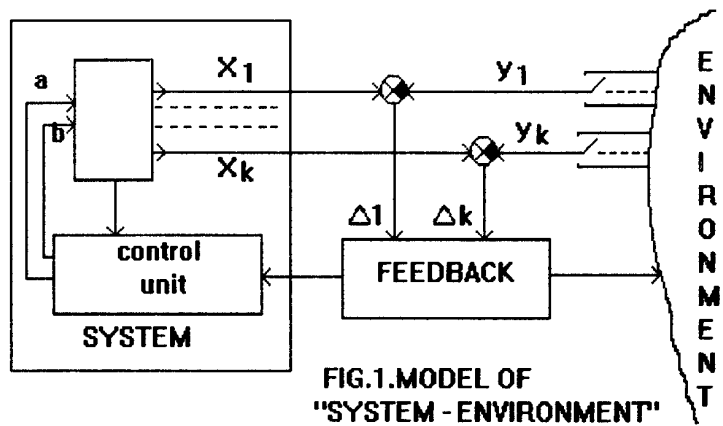
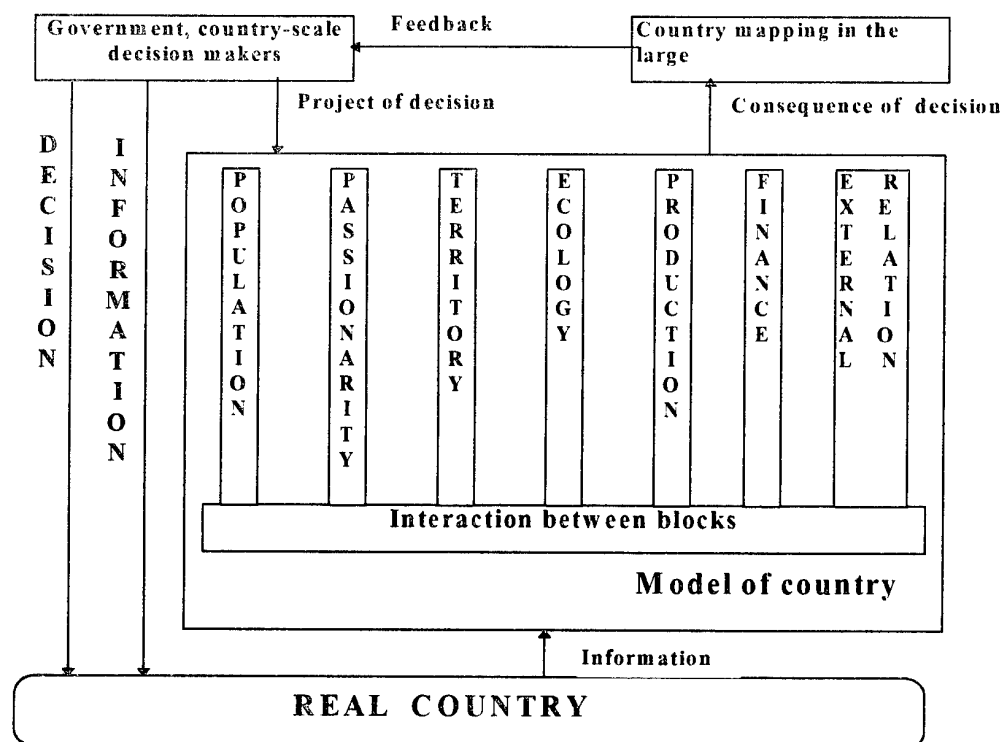


TABLE 1

m	1	2	3	4	5	6	7	8
n\								
2	1							
3	3	1						
4	6	4	1					
5	10	10	5	1				
6	15	20	15	6	1			
7	21	35	35	21	7	1		
8	28	56	70	56	28	8	1	
9	36	84	126	126	84	36	9	1



**Fig.2 Simulation of country for decision making support.**

We can compare system (9) and system (11) and can see that after cooperation the number of the arbitrary coefficients increase and increase the adaptational possibilities. For example, when we would like to define the strategy of development without of territory changes, we must suppose:

$$\frac{dA_3}{dt} = 0, \text{ whence :}$$

$$-u_2 \cdot A_1 - u_7 \cdot A_2 - u_{12} \cdot A_4 + u_{13} \cdot A_5 + u_{14} \cdot A_6 + u_{15} \cdot A_7 = 0 \quad \text{from (9)}$$

$$-v_1 \cdot D_{12} - v_6 \cdot D_{14} - v_7 \cdot D_{15} - v_8 \cdot D_{16} - v_9 \cdot D_{17} - v_{16} \cdot D_{24} - v_{17} \cdot D_{25} - v_{18} \cdot D_{26} - v_{19} \cdot D_{27} + v_{26} \cdot D_{35} + v_{27} \cdot D_{46} + v_{28} \cdot D_{47} + v_{29} \cdot D_{56} + v_{30} \cdot D_{57} + v_{31} \cdot D_{67} = 0 \quad \text{from (11)}$$

We can use the arbitrary coefficients for investigation of the different strategies of development for the decision making support relatively population, passionarity, ecology, production etc.

The discovered regularity of complex systems development permits to create the imitation model. Fig.2 is the block-scheme of simulation of country for governmental decision making support.

As the invariant kernel of model for the countries of region it is possible to use above-mentioned model in which the particularity each of country must be considered in the shell of that model.

Each country of region can have the same model for mutual coordinated decisions.

## CONCLUSION

The global simulation of region is very complex and important problem, but for realization of this problem it is necessary to prepare:

- the unification model of region and each country,
- the unification understanding of sustainable development,
- the unification data bases about each block of model.

## REFERENCES

1. Ignatiev M.B. "Holonomical automatic systems" Izd. Nauka, Moscow-Leningrad, 1963.
2. Ignatiev M.B. "Global projects and sustainable development" in proceedings of the 1995 Western Multiconference, Las Vegas, USA, January 1995.
3. Ignatiev M.B. "The problem of the sustainable development of Europe", in proceedings of the 7th European Simulation Symposium, Erlangen, Germany, Oct.1995
4. Chernenco V.I. and Ignatiev M.B. "Multimodal transportation on North-West Russia for sustainable development", in proceedings of Sustainable Inter-Regional Transport in Europe Conference, Kouvola, Finland, June 10-12, 1996
5. Ignatiev M.B. "The program of sustainable development for Baltic sea region" Proceedings of International Conference of World Futures Study Federation, Pushkin-St-Petersburg, July 7-10, 1996
6. Cycles and chaos in economics. 1992. J.Berhabib, ed, Princeton univ. press, New Jersey
7. "Global model matrix" in SIMULATION, vol.66, number 5, May 1996, 321-330 pages.
8. Ignatiev M., Nikitin A." The problem of Global Simulation of Mediterranean Region" Proceedings of the 8<sup>th</sup> European Simulation Symposium, October 1996, Genoa, Italy.
9. The 1993 Bankruptcy yearbook & almanac. Ed. Christofer M. McHugh, Boston, NGR, 1993.

## BIOGRAPHICAL NOTE

**Professor Mikhail B. Ignatiev**, Doctor of technical science, Honorable scientist of Russia, Laureate of the State Prize of USSR, Member of Natural Science Academy, Member of Metrological Academy is the Head of Computer Systems Department of St-Petersburg State Aerospace Instruments Academy and Director of Independent Institute of Sustainable Development. He published more than 300 scientific articles, inventions and books including "Holonomical automatic systems", 1963; "Robot-manipulators control algorithms", 1972, translated to English - USA Virginia Press, 1973; "Computing process control", 1973; "Underwater robots", 1977; "Simulation of machine systems", 1986; "Model and systems for complex experimental investigations", 1986; "Active method of reliability guarantee of algorithms and programs", 1992, etc.

He was the pioneer in robot field, in 1990 he organized the sustainable development group and now he is at the head of scientific program "Sustainable development of Russian regions". He is working in the complex systems simulation field.

# **COMPLEX METHODOLOGY OF SAFETY REGULATION - SYSTEM OF DYNAMIC BARRIERS (SOB) PREVENTING THE DEVELOPMENT OF EMERGENCY TRANSIENTS AT NUCLEAR POWER PLANTS**

A.S. Stebenev, A.P. Eperin, M.M. Khoudiakov,  
Leningrad NPP, Leningrad Region, Sosnovy Bor, 188537, Russia  
Fax: +7(81269)62-518

I.D. Rakitin  
RRC Kurchatov Institute, 1, Kurchatov Sq., Moscow, 123182 Russia  
E-mail: sdm@atis.kiae.su

**Abstract.** Work justification: a need in an unambiguous and illustrative methodology of Nuclear Power Plant (NPP) safety control for practical use by professionals. Work aims and objectives: development of an unified professional language for all the NPPs to identify power unit status, the NPP operation violations and corrective actions, as well as NPP emergency status control algorithms optimization (or symptom oriented emergency actions – SOEA). Results to be reached: optimization of all the areas of the NPP safety related activities.

In accordance with the proposed System of Dynamic Barriers (SDB) all the activities aimed at the NPP safe and reliable operation are integrated into a single structure. It is supposed that at this NPP, in accordance with the Quality Assurance Program, the required level of the equipment, personnel and procedure quality has been reached, and the appropriate surveillance for revealing and eliminating latent weaknesses is executed. To perfect the normative, other NPPs' good practice and their own operation experience are used.

All initial events (failures) are initiated by the loss of "NORMAL" function by any quality component. This moment is considered to be the beginning of the transient progress, and the transient itself is looked upon as a consequence of the seven main types of the power unit conditions (states): from "normal" to "severe accident".

**Keywords.** power unit conditions (states), barriers

**Introduction.** The main reason to work out the SDB methodology was the necessity to obtain complex approach to safety regulation of NPP which could become common as for all types of NPP as well as for practical use by the specialists.

That's why, the proposed methodology uses generally accepted terms, principles and approaches that guarantee reliability and safety of NPP<sup>1,2</sup>. Also unambiguous and simple gradation of types of power generating units (PU) states (conditions) is done. Methodology is complex, since all types of possible PU states, all kinds of safety activity and all components guaranteeing quality are considered in it.

The purposes of the methodology elaboration are:

- to create a language for specialists, common for all types of NPP, which provide identification of PU states and disturbances in NPP work, troubleshooting activity,
- to optimize system of dynamic barriers at NPP.

In accordance with the proposed System of Dynamic Barriers (SDB) all the activities aimed at the NPP safe and reliable operation are integrated into a single structure. It is supposed that at this NPP, in accordance with the Quality Assurance Program, the required level of the equipment, personnel and procedure quality has been reached, and the appropriate surveillance for revealing and eliminating latent weaknesses is executed. To perfect the normative, other NPPs' good practice and their own operation experience are used.

The key terms of the methodology are "types of states" (or "states") and "dynamic barriers" (or "barriers"). Remark. The proposed "dynamic barriers" not to be mixed up with common used "physical barriers - protective radio-activity limits".

SDB is presented as a scheme in Figure 1. The concepts and their internal correlation are muted in natural way in the scheme. Progress of accident is shown as a PU sequential transition from one state to another. As a base of a state classification the designed transient of PU, from small deviation down to severe accident, was chosen. The states of PU were chosen from the point of view of safe control of NPP.

It can be said, that dynamic model of progress of accident schemed in Figure 1 presents transitional regime of a PU as a consecutive change of unit's states. It is worth remembering that "small" problems in NPP exploitation are potential accidents. Taking into account probability of their further progress, the problems can reach the area of foreseen or not foreseen by the design project accidents as a result of failure of quality guaranteeing components:

- equipment,
- staff (personnel),
- man-machine auxiliary interface devises and implements.

According to the research<sup>2</sup>, "the probability of an operator error during the first few minutes of an incident is equal to 1, but after 40 minutes it is equal to  $10^{-4}$ ". And let us recall that common practice of socalled Symptom Oriented Emergency Actions (SOEA) or operating procedures is being engaged just only after the 5-th barrier is crossed<sup>3</sup>.

Thus, it is necessary to have a complex system of barriers that should block a progress in an incident or accident transient from the very beginning and with the main goal to increase reliability of human factor.

Naturally, there is a direct connection between the number of such accidents at NPP and the reliability of equipment, the qualification of staff and the adequacy of man-machine interface implements. In other words, the number of such accidents at NPP is a measure of reliability and safety level of NPP. The final goal of any significant for safety activity is to make a concrete contribution to make stronger appropriate dynamic barrier.

Therefore, SDB presents a high-level structure with an unambiguously defined niche ("place, time and space point") for every kind of reliability and safety related control and regulation at NPP. The main consideration in the paper is focused on:

- functions of operators when there is a progress in the incident/accident transient;
- support of operators in order to organize a safe control of NPP,
- analysis and identification of disturbances in NPP work, troubleshooting activity.

**SDB description and examples of application.** All kinds of activity guaranteeing safe and reliable operation of NPP are muted in common structure as shown in Fig. 1. It is supposed that the "Quality Assurance Program" is being carried out at given NPP, so that needed quality level of equipment, staff and auxiliary technique of a man-machine interface is guaranteed<sup>4,5</sup>.

Note: The proposed by us term "Man-machine auxiliary communication interface means and implements", in our opinion, is more comprehensive than the term "Procedures", as it includes **all** the means necessary and sufficient for the operator's reliable and safe operation of the power units (from the information display system, support system, diagrams, instructions, procedures, communications, hardware algorithms and up to the language of communication). This term or about the same term "auxiliary technique of human-machine interface" better describes the gist of problem as a technique which allows for an operator to carry out safe and reliable control of PU. It is advisable to use shorter and more adequate term - implement" while analyzing the operator activity.

Internal and external expert examination of activity as well as a supervision of an operation should be carried out in order to reveal "latent" shortcomings of components that guarantee its quality, namely, (a) equipment, (b) personnel, and (c) implements.

As a result of internal and external expert examination of NPP's activity certain corrections should be made and quality standards for equipment, staff and implements should be improved. To improve standards the "good practice" of other NPP can also be used.

The start of a safety violating transitional process in terms of SDB is initialized by a failure of a quality component (when the 1-st barrier is crossed). The moment of its initialization is regarded to be the beginning of a transitional process and the transitional process itself is considered to be the sequence of state (condition) changes of PU.

The basis of classification of PU state types is a sequence of states during the progress of the negative safety violating transient from some failure of a quality component down to severe accident. The numbers of states of PU are:

- # 1 - no failure;
- # 2 - failure (incident) which doesn't lead to power decrease;
- # 3 - failure (incident) leading to power decrease;
- # 4 - failure (incident) leading to engaging the safety protection systems;
- # 5 - failure with breaching of critical safety criterion (accident state);
- # 6 - accident with partly damaged core;
- # 7 - severe accident with serious damage of core and environment impact.

"Intermediate safe" states that correspond to states listed above are supplied with index "S" (refer to Figure 1). According to this Figure the initial incident events that are not foreseen by the design project, are transferred right away to "Accident state", type "5", because of defence safety systems were not counted on preventing them (as by definition).

The Figure 1 is divided into three vertical parts that include the following information about:

- "negative" PU states (## 2-7);
- "intermediate safe" types of states (states, to which PU should be transferred when corresponding negative ones are appeared);

- achieving goals, ## 2.S-7.S;
- ways of achieving goals (technical and organizational matters).

As we said before, the key terms of SDB are: types of PU states ("states") and dynamic barriers ("barriers").

Term "state" means minimum quantity of information concerning deviation of limits and conditions of normal (or safe) operation which guarantees a definite identification of the current PU state as one of the seven state types.

Term "barriers" means the conditional borders between the PU states which prevent a transitional process from its undesirable development by means of a purposeful conversion of negative PU states into corresponding safe ones.

Concretely, the term "barrier" represents the complex of technical and organizational actions with optimum contribution in barrier reliability by each of the three quality components, namely: "equipment" (E), "personnel" (P), and "auxiliary technique of man-machine interface implements" (I).

Every barrier has the number (1-7) corresponding to the state which should be transferred to a safe one. If any negative process begins, automatic and/or manual actions that are provided by corresponding barrier should be undertaken. The crossing of any barrier represents an incident or significant accident at NPP which requires the correcting steps as well as elucidation of direct and root causes of the accident.

If any barrier is crossed (corresponding failure of a quality component) the analysis of its causes should be undertaken together with investigation of not used (missed) possibilities of quality components for the purpose of making the crossed barriers stronger.

To guarantee the safe control of the NPP, operator must timely identify the PU state and undertake adequate steps in order to transfer the state of the PU to a safe one, (so called "control by the state").

Particular (but important) case of "control by the state" is the use of SOEA procedures or system to identify one of the cases "4.S" or "5" after the sanctioned engaging of protective safety systems (so called "scram").<sup>2,5</sup>

So, the operator's support system should be focused in the first place on information support of operator in order to help him to identify all PU states mostly in safety violating transitional regimes of the unit. Other words, the scheme shown at Figure 1 appears to be high-level generated videograph of an operator support system. This videograph allows to watch and control the progress of negative transients in a form of "dynamics of safety important states" by the administrative staff, operators and specialists of Crisis Centers.

The formal notation and classification of correcting actions and failures of quality components maintains two indexes: one is the barrier number to be strengthened (or the one which was crossed) and the other is the component number which makes the crossed barrier stronger (or the one which failure led to crossing the barrier).

Example 1. The correcting action "5.P" means that the purpose of the action is the strengthening of the 5-th barrier by means of training the personnel how to act when the 5-th state happens (accident state).

Example 2: the installation at the Leningrad NPP Units (RBMK-1000 type) of the Emergency Reactor Sub cooling System (ERSS) is identified by two symbols – 4.E, as the 4-th barrier was made stronger in "equipment" component.

Example 3: the crossed barrier coding "3.P" means the crossing of the 3-d barrier because of the failure of "personnel" quality component. Formal identification of incidents at NPP consists of sequence of crossed barriers including failure components and final state type of PU.

Example 4. The failure of level regulator in drum-separator at the RBMK-type reactor caused the deviation of level above the appropriate "level set-point" and led to engaging the preventive automatics control for power decrease from normal 100% down to 60%. *Analysis and identification:* the code of a regulator failure in our sense is "1.I" because the 1-st barrier was crossed in "I" component ("auxiliary technique of interface implements"), not in "equipment" component as it were be under a common IAEA identification<sup>2,4</sup>. According to the proposed SDB methodology all processes are classified from the safe control of NPP point of view. So it is the failure of "implements for operator in a technological process control". Unsatisfactory attempts of the operators to compensate the incident disturbance in the drum-separator level led to crossing of the 2-nd barrier – failure "2.P". The 3-d barrier fulfilled its safety function duty and transferred the state # 3 to the state "3.S" by decreasing the power level down to 60%. So, the incident code description is represented as follows: 100% 1.I- 2.P -60% 3.S, where 100% – the initial power level before the incident, "3.S" – PU state after the incident, 60% – the final power level.

Complex databases of NPP (RBMK-type) incidents and barrier crosses are compiled now on the described above basis at the Leningrad NPP in order to undertake comparative analysis of dynamic barriers and correcting actions. It could be done for NPPs of different kinds.

The top priority steps at NPP are those that exclude (lessen the probability of) the failures that lead to the crossing of several barriers at the same time when there is no possibility to strengthen the crossed barriers with the help of other quality components.

The SDB as a complex system must guarantee reliable and safe control of PU so the SDB and NPP should be designed at the same time. SDB must have reasonable barriers and optimum contribution of each quality component in each barrier. Databases of incidents at NPP of all kinds are used for those purposes. Thus, the optimum structure of dynamic barriers and high reliability of staff activity can be guaranteed during the designing process.

SDB is supposed to be further developed continuously because each kind of activity represents the root of more detailed steps in a personnel activity.

**Conclusion.** The SDB proposed and proved at the Leningrad NPP methodology of organization and regulation of important for NPP safety activity can be used:

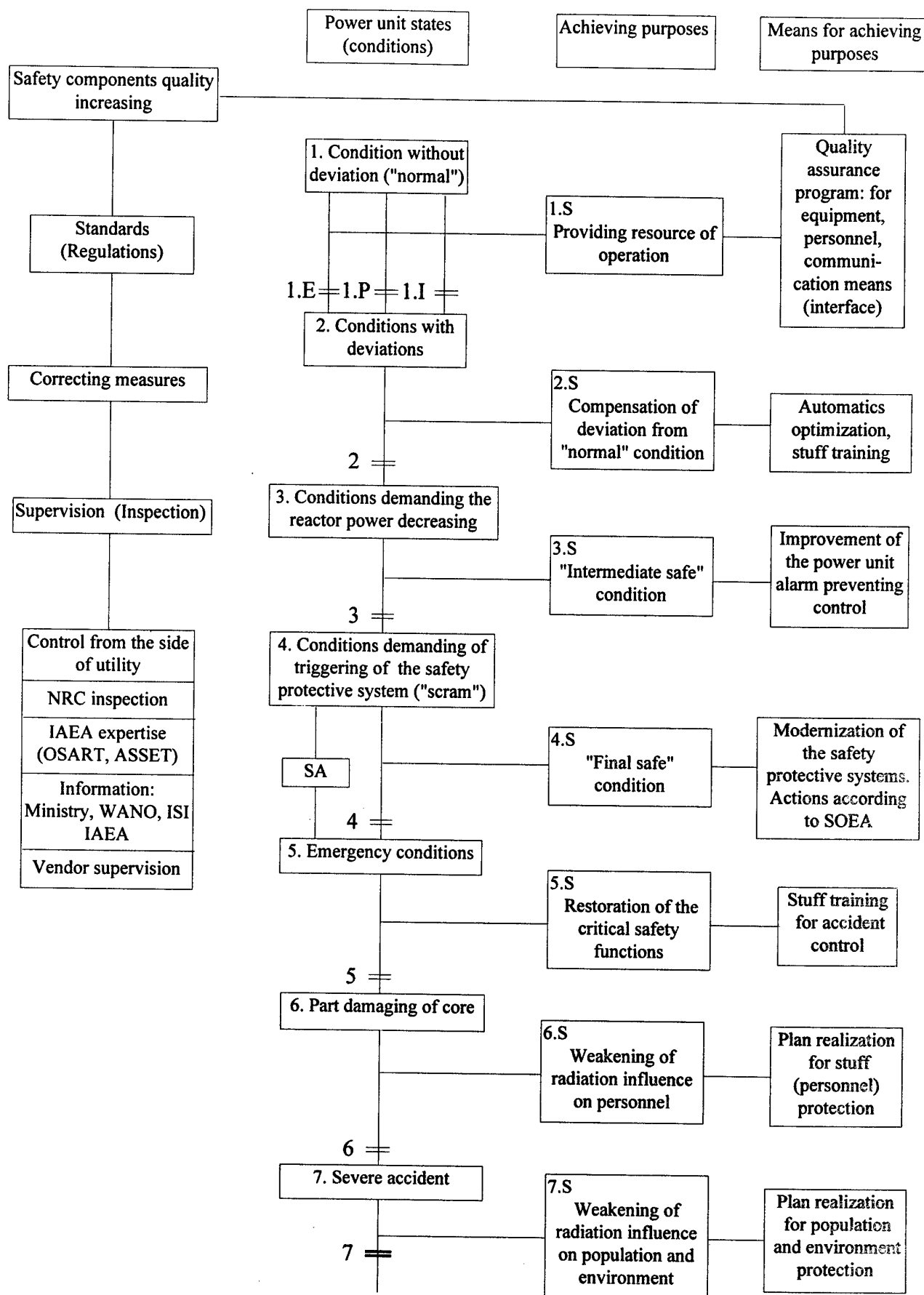


Fig. 1. System of dynamic barriers (SDB) preventing propagation of emergency transients on NPPs

- for improvement of algorithms realizing preventive control and emergency regulation of the main technological process;
- as a common algorithmic language which can help specialists to elaborate certain kinds of activity to improve safety and reliability of NPP;
- for analysis and formal estimation of incidents at NPP and for compilation of databases consisting of crossed barriers cases and quality components failures';
- for organization of universal operator support systems;
- for comparative analysis of certain SDB at reactors of a different kinds;
- for working out simulator training scenarios with various combinations of incidents and failures of different quality components;
- to acquaint the personnel of a NPP with the concept of "control by the state" as well as to teach it to understand the problem of safe and reliable operation of NPP.

---

## References .

- <sup>1</sup> *Basic Safety Principles for Nuclear Power Plants – A Report by the International Nuclear Safety Advisory Group*. IAEA Report # 75-INSAG-3. Vienna (1988).
- <sup>2</sup> *Safety Culture – A Report by the International Nuclear Safety Advisory Group*. IAEA Report # 75-INSAG-4. Vienna (1991).
- <sup>3</sup> *Accident Management Programmes in Nuclear Power Plants - A Guidebook* IAEA Report # 368. Vienna (1994).
- <sup>4</sup> *ASSET: Review of Safety Performance of Operational Nuclear Power Plants –USER's MANUAL*. IAEA Report (Draft of 3 May 1995). Vienna (1995).
- <sup>5</sup> *General Regulatory Guide for Safety Assurance of Nuclear Power Plants (OPB-88)*. ATOMIZDAT, Moscow (1988). (In Russian).

# KNOWLEDGE REPRESENTATION IN DIAGNOSING TASKS OF THE EXPERTISE AREAS WITH STRUCTURED SUBSPACE

Galina Miasnikova

Institute of Problems of Mechanical Engineering

Russian Academy of Sciences

Tel.: (812) 217-81-85, E-mail: sol@sapr.ipme.ru

**Abstract.** Scientific approach and technique for the decision of a construction problem of diagnosing objects graph-models is stated. The approach is based on revealing structured space from the whole expertise area and its transformation to parameters space. The given offers can be used at the decision of a problem of search of diagnostic parameters effective set for some expertise areas (technical, organizational and etc.) at construction of diagnosing systems including neural network principles.

**Keywords.** Diagnostics, knowledge representation, structured subspace, graph-model, automated method.

**Introduction.** The offers, stated in the given article are formulated for any expertise areas (EA), in which it is possible to allocate a structured subspace. Technical, organizational and other problems, in particular problems of diagnostics on these areas, deal with such areas. For organizational systems the diagnostic problems make sense revealing faulty functioning elements of system in result of unintentional or deliberate wrong actions. Therefore the conclusions, made in the article are distributed to all these areas, though sometimes the terminology is used from area of diagnostics of means.

One of the first problems on the way of diagnosing system engineering (DS) independent on chosen principles and means of their realization (including the use of neural networks) is a problem of revealing and representation of knowledge about diagnosing object (DO), that is construction of DO model. Subject area, with which experts in diagnostics deal, is knowledge about DO - complex technical (organizational etc.) object, including elements of a various physical nature. Therefore the development of DO models is connected to processing by the expert of large volume of the information about object, including its structure, units, parameters, describing its functioning in various modes, quantitative assessments of DO works, analytical dependence between parameters, the most probable malfunctions, possible defects and their displays etc.[1].

The first difficult problem, soluble on model, is a problem of determination of set of diagnostic parameters, necessary for revealing of malfunctions of an object. In practical aspect it is a problem of multicriterial optimization, as at its decision it is necessary besides diagnostic possibilities of parameters themselves to take into account the presence of means of control, having the necessary qualitative characteristics (reliability, speed, accuracy, acceptable dimensions and cost etc.). This problem is still urgent for actively developing at the moment neural network approach [2], used for realization of diagnostic algorithms. Development the neural network includes a problem of determination of entrance parameters, which until now

has no theoretical solution. At the same time the sizes and complexity of the neural network, and its diagnostic abilities depend on its solution [3].

For example, such fact, that at an estimation of the non-return risk of the bank credits the accuracy of eleven applied methods lays in limits 18%-38 % [4], speaks about actuality of theoretical development of this problem. The analysis shows, that this border can be objectively caused by unadequacy of used model of expertise area.

In this connection at the neural network approach to the solution of diagnostic problems it is recommended to use modeling subject area as graph - model [1]. Such model allows to present and to connect all knowledge about DO, required for the solution of diagnosing problems. The theoretical and practical apparatus of the solution of the problem of the diagnostic parameters effective set determination, construction of diagnostic algorithms and other problems is developed [5].

The mathematical model of such AE is represented as set of functional parities, logic conditions and restrictions, which describe functioning of idealized object, connect outputs of the object with inputs, internal parameters of the object, entry conditions, external initials and internal noise in time, forming thus sign space, suitable for processing by mathematical methods.

As the mathematical apparatus of modeling is used the apparatus of the graph theory with consideration of the relations on set of DO functioning properties, on set of its parameters or on set of functional elements.

We shall present the basic concepts of such graph - model, and also we shall describe the theoretical approach and method of its construction for some subject areas (in which structured space can be found).

The DO graph - model will be formed by assignment to elements (tops and ribs) of abstract graph  $G(X, U, T, I)$  (where  $X$  - a set of the tops,  $U$  - a set of the ribs,  $T$  - a set of diagnostic parameters efficiency indicators,  $I$  - a set of the weights ribs) pithy sense of researched DO. The abstract set  $X$  elements acquire sense of particular properties or parameters, essential from the point of view of the object normal functioning, and topology - of particular cause-consequent connections between these properties, presented in the matter part as a kind of either functional dependency or other forms, or as a verbal description.

In a common case the set of the graph tops can be association of subsets of the various classes tops:

$$X = \bigcup_{i=1}^n N_i, \quad (1)$$

Where a  $N_i$ -subset of  $i$  class.

For example, in technical sphere the set of the graph tops can be determined by the parity [5]:

$$X = KUYURUFUVUE, \quad (2)$$

where:

$K \{k_1, k_2, \dots, k_b\}$  - input parameters (parameters of other objects, influencing on DO work);

$Y \{y_1, y_2, \dots, y_r\}$  - output parameters (a set of DO output parameters);

$R \{r_1, r_2, \dots, r_l\}$  - "characteristics" (main quantitative assessments of DO work);

$F \{f_1, f_2, \dots, f_g\}$  - functioning process parameters of (characteristic of processes, making the main process of DO functioning);

$V \{v1, v2, \dots Vm\}$  - auxiliary parameters (parameters of accompanying processes, which do not participate in the main process realization);

$E \{e1, e2, \dots En\}$  - structural parameters (physical, chemical, electrical, geometrical and other properties of object elements, for example, details, units, and etc.);

$D \{d1, d2, \dots Dp\}$  - defects (discrepancy of DO structural parameters significance to installed norms).

In organizational sphere it is possible to find analogues of these classes. Thus the person is the «fault» carrier. In this connection such concept as Person state, executing this or that function can be a structural parameter. This state can be «good working order», if he carries out this function correctly and «faulty», if he admits inadvertent errors or deliberate wrong actions. Revealing parameters, sensitive to change of this person state is the first stage of the diagnostic problem decision in organizational sphere.

Cause-consequent connection between parameters (tops of graph-model) have the characteristic  $I_{ij}$  - weight of the rib, estimating tightness information dependence between the appropriate parameters. Thus each top also has weight  $T_i$  as characteristic, allowing to estimate the appropriate parameter from the point of view of its opportunity of satisfaction to following inconsistent requirements:

- 1) Cover of all defects;
- 2) Maximum informationity and sensitivity to defects;
- 3) Minimum set;
- 4) Maximum availability to the control;
- 5) Minimum measurement time;
- 6) Minimum the control cost;
- 7) Maximum dividing ability;
- 8) Maximum reliance of control results.

Methods of an each parameter efficiency estimation, determination of effective set of diagnostic parameters, and also development of diagnostic algorithms are presented in [5] and can be used at neural network creation for the diagnostic purposes.

The example of such graph-model fragment is presented on fig. 1.

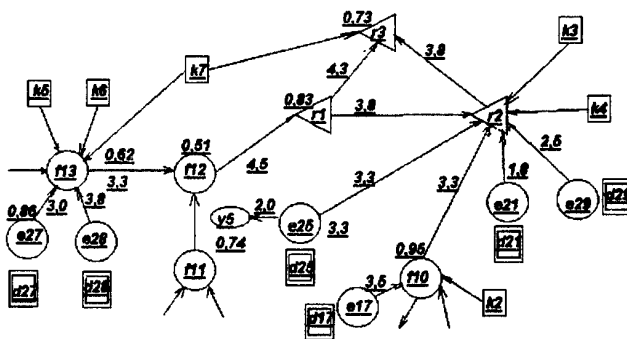


Fig. 1. The graph-model fragment.

However, for application of these methods first of all it is necessary to solve problem of graph - model creation, i.e. problem of revealing from the whole subject area the information, which corresponds (1) and includes diagnostic one.

For considered EA the approach for knowledge acquisition about object, based on revealing and description of some interesting spaces and finding of connections between them is offered [6].

There are such EA (artificial and natural), the purpose of which existence is fulfillment of some problems (functions). For fulfillment of each of these functions there is a set of interconnected subfunctions, which are described by such concept as functional structure. The ways of realization of these functions and physical condition of organs their realizing, determine properties of EA, reflecting by its parameters (1). So that to reveal a complete set of parameters, describing the given EA, it is necessary to reveal its functional structure and to find cause-consequent connection between functional bodies, process, occurring in object and a condition of its parameters.

We shall make it as follows.

We shall describe expertise area as

$$L = \langle M, R \rangle, \quad (3)$$

where: M - family of the basic sets of concepts AE, appropriate to definite its subspaces.

R - family of the relations on these sets.

$$\text{In our case: } M = W \cup Q \cup X, \quad (4)$$

where:

W - a set of functional blocks of object, inducing its properties (space of functions);

Q - a set of functioning properties of allocated blocks (space of properties);

X - a set of object parameters, included in its graph-model (space of parameters).

R unites three sort of the relations:

$$R = \{R_1, R_2, R_3\}, \quad (5)$$

where: R1 - relation «to be the cause», R2 - «to be characterized» and R3 - «to influence».

If:

$$w \in W, \quad q \in Q, \quad k \in K, \quad f \in F, \quad r \in R, \quad v \in V, \quad d \in D, \quad e \in E,$$

then:

$$w R_1 k; \quad q_i R_1 q_j; \quad q R_2 (k, f, r); \quad k R_3 (f, r); \quad e R_3 (f, r, v); \quad d R_3 e. \quad (6)$$

Thus, the first relation R1 connects elements of sets W and H (the set of functional blocks and the set of functioning properties), and also elements inside set H. Second R2 - connects elements of set H to elements of subsets K, F, R sets X. Third - R3 connects elements of subsets K, Y, R, F, V, E and D inside set X.

Thus it is known, that the space of functions is structured and its structure is described by a function scheme of an object (for technical and organizational objects this scheme is developed at their creation). The presence of such space in researched EA gives a chance, using system of the descriptions (6), to find all elements of sets H and X and their relation on graph-models. The offered approach, uses peculiarities of EA, and also of a Person, capable to be concentrated on small problems with small number of variables. It allows to include consistently in the sphere of consideration the known knowledge (elements of a function scheme), being a nucleus, around which new knowledge can be formed.

The essence of the general approach to construction of graph - model of such subject area consists of the following:

1. To describe structure of space of functions  $W$ .
2. According to the description  $W$  using consistently the relation  $R_2$  to find elements  $q$  of set  $Q$  (space of properties), thus constructing, part of a semantic network, describing researched AE.
3. To transform space  $Q$  to space  $X$  by the description with the help of the relation  $R_3$  of each its element  $q$  in the terms of the appropriate elements of space  $X$ , thus building, the other part of a semantic network. When we finish transformations we shall receive required working graph - model of object.

The graph-model construction method corresponds to the formulated approach (originally developed for technical sphere in [5]). It has extended to various types EA and has been given below.

The basis for construction of graph - model is a function scheme of object. For an example it is represented on fig. 2.1, on which figures 1-6 designate functional blocks, and symbols  $x_{01}$ ,  $x_{02}$ ,  $x_1$ - $x_6$  - connection between them.

In a common case output and input of functional elements (blocks), being functions of time  $x_i = f_i(t)$ , can be connected analytically:  $x_i = f_i(x_j)$ . For construction of graph-model quantity  $x_i$  are considered as the main functional properties of an object and are represented on a model in a kind of the graph tops (fig.2.2). Cause-consequent connections between properties of an object, following from the essence of its functioning, cause a set  $U$  of the graph ribs.

On the following stages of synthesis initial graph-model  $G_1$  in the properties space, displaying processes of DO functioning, will be transformed to graph-model  $G_2$  ", displaying the relations on a set of DO functioning parameters with additions and specifications of initial graph-model.

The peculiarity of this technique consists of such organization thinking of the expert, at which his knowledge, based on structured information of the DO function scheme, is taken fragmentarely, but consecutively. Such reception allows to the expert to focus the attention in a determined sequence on all details of a problem not losing a general direction of its research.

On fig. 2 the main stages of DO graph-model construction technique in the parameters space are resulted. Tasks, soluble on each stage are listed below. The work begins from a preliminary stage.

Preliminary stage. Discovery of parameters sets.

1. Describe pithyly DO;
2. Draw the basic outline of an object;
3. Present analytical and qualitative dependences;
4. Single out  $K$ ,  $Y$ ,  $R$ ,  $F$ ,  $V$ ,  $E$ ,  $D$  of parameters sets, describing DO functioning and condition - (2).

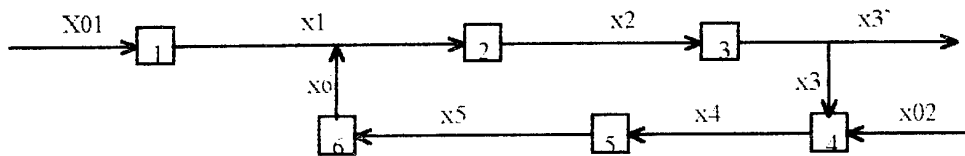
Stage 1. Discover the essential properties of DO functioning.

1. Discover essential properties of  $x_1, x_2, \dots$  objects functioning ( or properties of its main function ).
2. Identify chosen properties with functions, executed by DO components.
3. Single out the main cycle of DO functioning and construction of the simplified functional block diagram (FBD), where 1,2,...,6 - are functional blocks.

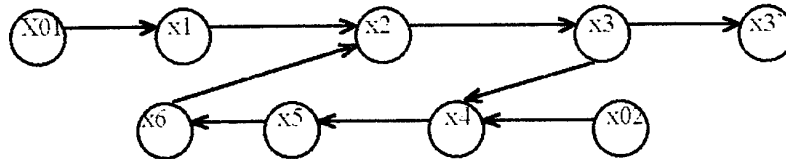
Stage 2. Build an initial G-model  $G_1$  in properties space

1. Present the properties ( functions ) by  $x_1$ - $x_6$  tops of graph-model.
2. Discover cause-effect connections between functions (the graph tops).

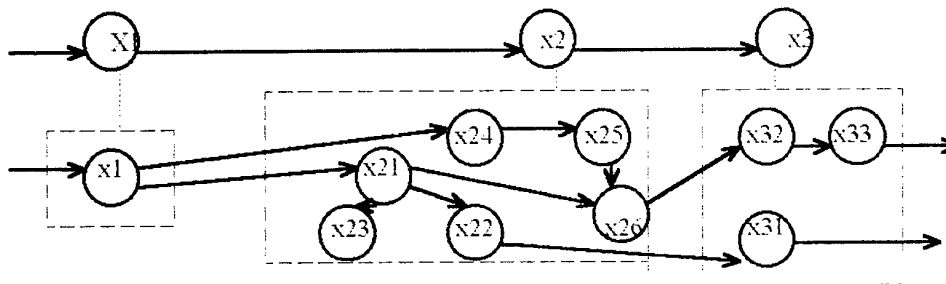
1. Discovery of essential properties of DO functioning.



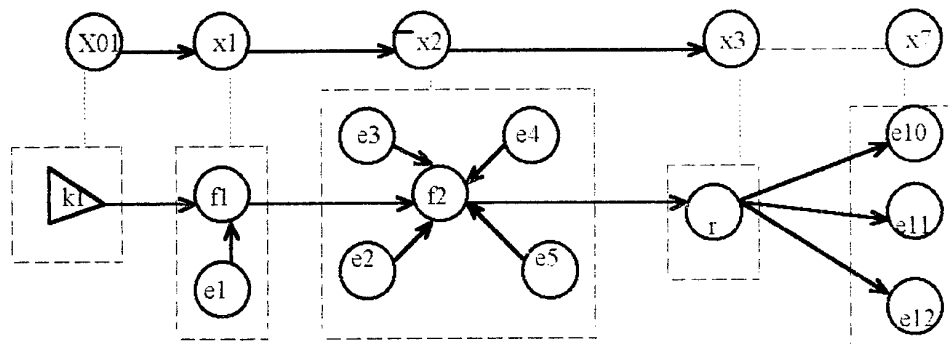
2. Building of initial graph-model into properties space - G1.



3. Supplementing and disintegrating of initial graph-model (G1-G1').



4. Graph-model presenting in the parameters space - G2.



5. The introduction of defects (infringements) into model - G2'.

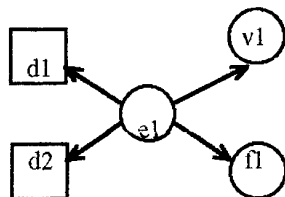


Fig.2. The main stages of OD expert knowledge extraction method

3. Connect the graph tops by the ribs in conformity with cause-effect connections. Cause-effect connections (the ribs) exist, if  $x_i$  "causes"  $x_j$ .

4. Transform FBD to oriented G-model in properties space.

Stage 3. Supplement and disintegrate the initial G-model of properties; go if necessary to a more detailed model (reception of graph-model  $G1'$ ).

1. Bring in properties of importance at functioning of an object, but not participating in a main cycle. The relation between  $x_i$  and  $x_j$  makes sense "to influence".

2. Carry out the separation of complex properties (detailing)

3. Carry out ribs in the graph with separating and added properties under condition of gomeomorphing display of graph.

Stage 4. Present  $G2$  graph-model in the parameters space.

1. Each property is presented by main process functioning parameters  $F$  and by its quantitative characteristics  $R$ .

2. The tops - parameters connect themselves in conformity with the initial  $G1$  graph-model in a properties space.

3. To the tops  $F$  and  $R$  are those structural parameters  $E$  attached, which values influence the values of parameters  $F$  or  $R$ . Execute conditions of gomeomorphing display of graphs.

Stage 5. The introduction of defects (infringements) into a model ( $G2$  "graph-model").

1. Each structural parameter is presented by possible conditions:  $e_i - \{d_1, d_2, \dots, d_n\}$ . Usually the conditions  $d_1$  - the "norm" and  $d_2$  - "not the norm" is introduced. If necessary bring in accompanying parameters  $V$ , which appear together with defects.

Thus, consistently on stages the working DO G-model is built, which then is researched and transformed with the purpose of search of effective diagnostic parameters set.

The automation of DO graph-model development on the given method has caused the creation of special technology of extraction of knowledge of the expert - developer of DO model [1]. The features of this technology consist in granting the user of an opportunity for:

1) work with DO model at all generalization levels beginning with the top one (bearing the information on its structural connections), gradually passing on all levels of detailing, finishing by the lower one which is enough for the input of all the necessary information;

2) sequential concentrating of the user's attention on separate fragments of a task;

3) input, necessary processing and keeping of all information connections between fragments of a task at all summarizing levels.

For automation of graph-model development on the given technology a specialized graphic editor and an interface are created. They enable the user to build graph-model and to conduct its research on a display screen (automatically according to its analytical task or according to fragments in conformity with instructions of a dialogue system) at simultaneous formation of data and of knowledge bases, necessary for the work with users models of different professional level, and also of special soft wear for the decision of computing tasks.

This approach was used in organization sphere, for example, for modelling of wages payment organization process at some firm. During it the system links were

revealed, in which an abuses may be, diagnostic parameters for revealing of these abuses, and ways of organization structure improvement for increasing its stability against abuses.

**Conclusions.** It is offered to use peculiarities of the expertise areas with structured subspace, for diagnostic knowledge representation about an object. The mathematical description of connected subspaces EA and appropriate automated method of its graph - model creation are represented.

The realization of the given functions in the COMPUTER gives a potential opportunity to improve (to supplement and to specify) knowledge, used at construction of DO model, in accordance with accumulated data in DO real operation conditions, realization of additional experimental and analytical researches of connections between its parameters.

### **References**

1. Dobrinin, V.Y., G.I. Miasnikova, E.D. Solozhentsev (1994) Simulation process automation of diagnosing objects. Theory and information technology of safety simulation for complex systems. Edited by I.A. Ryabinin, Preprint 104, issue 2, IMSP.
2. Wasserman, F. (1992) Neural computing engineering. Moscow: Mir.
3. Sinitcin, E.V. (1996) Neural network and finance. Bank technology. J., №1, 1996, Moscow.
4. Jurgen, S., E. Stichel. (1993), Consumer Loan Analysis Using Neural Networks. Adaptive Intelligent Systems, S.W.I.T.(ed), Elsevier Science Publishers B.V.
5. Osis, Y.Y., Gelfandbein Y.A., Markovich Z.P., Novozhilova N.V. (1991). Diagnosing on graph-models: On examples of air and automobile engineering. Moscow, Transport, -224p.
6. Miasnikova, G.I. (1996) Graph-models and neural networks in problems of diagnosing of technical and bank systems. Abstracts. The First International Conference Instrument designing in the person ecology and safety. Russia, St.-Petersburg, 30 October-2 November.

**CHAPTER XIV:**

***SIMULATION OF PROCESSES IN OCEAN***

THIS CHAPTER INCLUDES PAPERS  
PRESENTED AT THE CONFERENCE SESSION:

**SIMULATION OF PROCESSES IN OCEAN**

Organized by: *Prof. Anatoly A. Rodionov,*  
*Dr. Vasily A. Popovich.*

# OBJECT-ORIENTED MODELING THE OPERATIONS RESEARCH PROBLEMS

Dr. Vasily V. Popovich, Prof. Anatoly A. Rodionov  
Division for Applied Search Programmes  
St. Petersburg's Scientific Centre of the Russian Academy of Sciences  
University Emb., St. Petersburg, Russia,  
Tel: (812) 218 41 67, E-mail: [popovich@popovich.spb.ru](mailto:popovich@popovich.spb.ru)

**Abstract.** The aspects of setting out the problems to develop parallel architecture and algorithms of object-oriented models for operations research problems in order to create computer-aided decision making processes under real conditions have been considered.

Major consideration has been given to development of unified methodological approach in applied and theoretical research and in computer simulation based on the up-to-date paradigm - the object-oriented approach.

Theoretical aspects are demonstrated based on the example of modeling the well-known set of problems of moving object search theory.

The complex of computational models using PC in Windows 3.11 environment following RAD process of Borland Delphi 1.0 is being developed simultaneously with theoretical research. The current version of software can be forwarded as distributive for all the organizations and persons concerned.

**Keywords.** Operation research, object-oriented design, theory of search.

**Introduction.** Urgency of the article is proved by urgency of the object-oriented approaches which are being implemented in computer technology related activities as well as by the fact that so important sphere of knowledge as the operations research (O.R.) [7] with far-ranging application is not given proper consideration in literature as a subject of computer modeling.

Since O.R. is basically applied scientific trend, the necessity to model the fragments of real operations is related to the necessity to model various parallel processes, though the object-oriented approach supposes to have parallel structure and algorithms.

## Object-Oriented Models. Basic Statements, Terms, Principles

The necessity to consider and to define more exactly the term "model" is justified by extremely wide application of this term in natural sciences. Depending on application, subject matter and semantical sense, it can have various meanings.

The term "model" is conventionally interpreted as a specimen, pattern, mock-up, graphical or mathematical representation of some specific features of investigated object, fragment of objective reality or thinking.

Because of such a wide use of the term “model” it can be defined in various ways. Subject matter of the present article provides for the following definition.

Model is a description of a certain real or abstract object by available means for its further studying.

Ways to present model can be divided into two general types:

- physical;
- verbal-symbolic.

The subject of our study in this article is verbal-symbolic models based on the methods of ideal (abstract) modeling.

As a rule, one of these two modeling procedures is being used in operations research.

While studying the objective reality knowledge on the present subject matter is being accumulated and qualitatively changed. Plenty of models differentiating by different criteria related to completeness of available information, its formalization, orientation towards the end user are being applied. In all applied problems conceptual and analytical models are the most typical models depending on a degree of formalization and subject matter.

Conceptual model implies a certain verbal description reflecting the fragments of real (abstract) system with predetermined (available) completeness.

Descriptive conceptual modeling means description of models.

Prescriptive conceptual modeling implies a certain formalized (algebraic) description reflecting the fragment(s) of real (abstract) system with predetermined (available) completeness.

Analytical models have a number of features:

- their development is based on a certain theory or scientific concept;
- they provide the basis for obtaining quantitative assessments.

Analytical modeling can be divided into the mathematical and the simulation ones.

The mathematical model (M.M.) stands for an approximate description of a certain group of objective world phenomena by means of mathematical symbols.

The simulation model (S.M.) stands for direct description of a certain group of objective world phenomena using PC facilities. S.M. is a way to conduct a computer based experiment.

The mathematical models are subject of the research of a famous theory of models [4] developed as a result of merging of algebra and mathematical logic.

In the present research X model for a certain system (set) Y implies a set A, where each X element corresponds to the one in Y.

Specifying the correlation between the elements of X and Y sets is termed as mapping or function and is denoted as  $f: X \rightarrow Y$ . Such a mapping is being increasingly interpreted as “morphism”.

One can make distinctions between two basic types of morphism: homomorphism (similarity) and isomorphism (invariable similarity). Formally homomorphism conditions can be presented as follows:

$$x \in X, y \in Y \quad f: x \rightarrow y$$

Isomorphism is invariable mapping of the X set to the Y set:

$$x \in X, y \in Y \quad f: x \rightarrow y \quad f^{-1}: y \rightarrow x$$

Logic of conventional model designing can be presented as follows: (Figure 1):

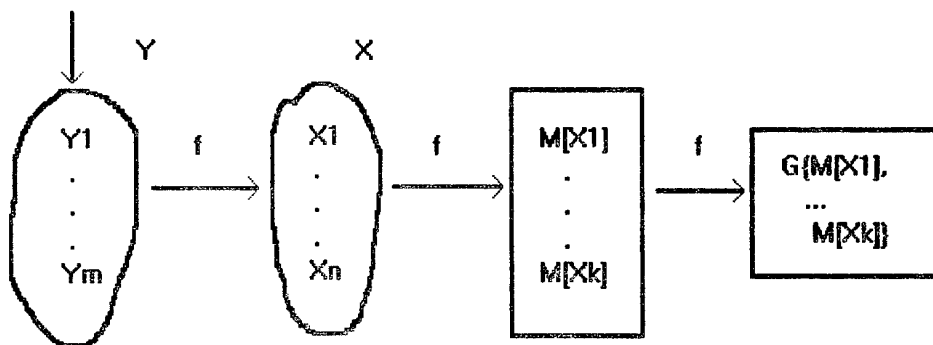


Fig. 1 Conventional design of system models.

The first stage support the development of the conceptual model for a subject matter (in the present article it is the conceptual model of operations research).

The second stage provides for development of the analytical model for the elements of Y set and (if possible) of the general analytical model for the Y set, i.e. solving the problems of analysis (obtaining the elements of the X set) and synthesis (obtaining general models for the X set).

The third stage provides for development of numerical methods for quantitative assessments of the elements of the X set.

The fourth stage provides for development of the computer model for the elements of the X set, that enables to get both separate assessment of the elements  $x \in X$  and some assessments of the emergent properties of the investigated operation (system).

As regards to the conventional modeling, each stage is quite independent in selection of policies and practices, concept and theory of research.

It is quite common and natural for development of some applied and basic research used at different stages. But from the point of view of the end user (applied researcher) of the developed model it is not always convenient, as such a design does not provide for the basic element of the studied systems - their evolution.

The idea of using unified methodology for research complicated system becomes very attractive and quite evident. Moreover, such an idea has already been used by various scientists.

However, the weak point of this approach and a number of other well-known ideas is their being proposed only "on paper", and the final, the most important stage - the stage of computer modeling does not comply with the general research concept.

At the moment, the intensive scientific and practical research aimed to remove this default is being in progress. One of the most promising direction is the object-oriented approach. At present, this methodology has been most completely implemented at the stage of analysis and synthesis of operation environment of up-to-date computer systems. In order to expand this concept to the theory of models, it is necessary to get clear about terms. The new term - object-oriented model (O.O.M.) is proposed to be introduced. Using this term as a certain invariant will enable to develop modeling design (Figure 2) within the unified methodology, with the aim to provide for the basic system element - the evolution at all the stages of research and to avoid other conventional restrictions.

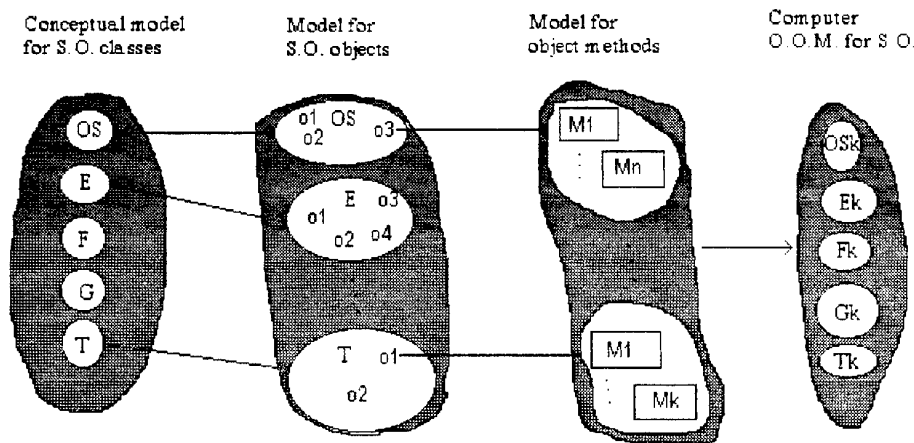


Fig.2 Design of object-oriented models for search operations.

Analysis of the two designs represented in Figures 1, 2 proves that the essential difference between the two modeling designs is that in the first case each stage provides for its own research system, and in the second case (Figure 2) the general model structure both at the first and at the last stages is isomorphic.

It must be noted that in the first modeling case the concept of function provides the basis for analysis. In the second case the object including both a certain finite set of functions (methods) and a set (aggregate) of necessary data serves as such a basis.

The basic difference of the O.O.M. from the conventional one consists in the fact that the internal structural changes in the O.O.M. does not result in the necessity to review the other stages not connected with this change.

Object-oriented model stands for a model where the investigated phenomenon or process is represented in the form of object and (or) hierarchy of objects and their interaction.

Object means a certain tangible (real or abstract) substance [1]. The term "object" is known to be in compliance with the following underlying principles:

- encapsulation (aggregate of data and methods (functions));
- inheritance (ensuring the methods and data down the whole hierarchy of objects (within a class));
- polymorphism (expansion of the methods (functions) to the whole object hierarchy (within a class)); each case (object) provides for the overlapping of methods (functions) that means their deferent realization.

As compared with the term "function", the term "object" implies a certain set of functions (methods) and a certain set of parameters attributable to a specific class or case (object). In such a view, computer based realization of the object hierarchy becomes of essential importance because of existing restrictions for processor machine cycle and for random access memory, which means emergence of the problems referring to parallelism of two levels. The first level is related to characteristics of the specific operation environment and computer platforms. The second level is related to the subject matter. As regards to the real time system it is quite evident and clear: a number of objects should function simultaneously at the same intervals but as shown below such a problem also exists for the research systems which are not related to the real time systems.

### **Object-Oriented Search Operation Model**

In the research operations the key term is "operation" which means a certain system of actions with the participation of human being aimed to achieve a specific goal set in the operation. Design of the search operation model as well as of any complex system model is related to solving a number of rather complicated problems. In terms of ideology a major problem is that we are not in a position to adequately describe any real object of nature. It is worth recalling Kant's "thing in itself". In such a view, while conducting the object-oriented analysis (O.O.A.) three basic research stages for O.O.M. description can be specified [6], such as: information model, model of states, model of processes.

Owing to the limitations of the present article let us take as an example the classical theory - theory of movable objects search [3], which is considered to be the founder of all operations research methods.

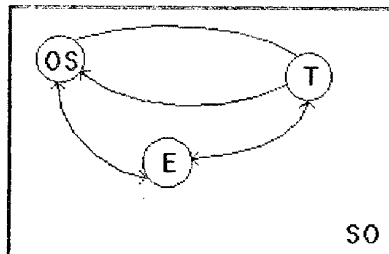
As a result of the O.O.A. for classical search problem a distinction can be made between the principal abstraction comprising the simplest search operation: observing system (O.S.), target (T.), environment (E.) and the uniting class - search operation (S.O.). Detailed consideration of the subject matter and its approximation to the real operations research results in development of the S.O. objects classification.

The principal objects of the search operation class at the conceptual level are as follows:

a) First object. Abstract search operation. superclass.

$$S.O.=[O.S.,T.,E.]$$

Objects correlation:



Target movement is the mandatory condition for the search operation.

The search operation aims at target detecting. Efficiency criteria: min T (time), max P (probability). Tasks to be undertaken:

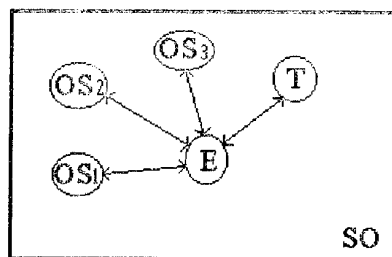
- 1) S.O. planning;
- 2) S.O. support;
- 3) analysis of conducted S.O.

b) Second object.

Finite number of observers is the basic feature.

$$S.O.=[O.S....O.S.,T.,E.]$$

Objects correlation:



Target movement is the mandatory condition for the S.O.

The S.O. aims at target detecting. Efficiency criteria: min T., max P.

Tasks to be undertaken:

- 1) S.O. planning;
- 2) S.O. support;
- 3) assessment of conducted S.O.;

Additional tasks to be undertaken within the scope of the principal ones:

- optimization of search efforts;

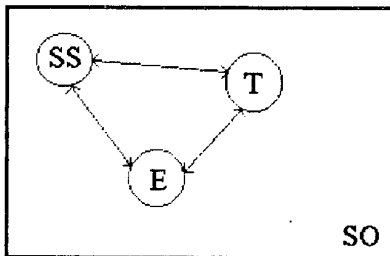
- S.O. feasibility study.

c) S.O. third object.

$S.O. = [Search\ system\ (S.S.), T., E.]$

The basic feature - existence of the S.S.  $S.S. = O.S. + C.$  (maneuvering carrier).

Objects correlation:



Target movement is the mandatory condition for the search operation. Efficiency criteria: min T., max P.

Tasks to be undertaken:

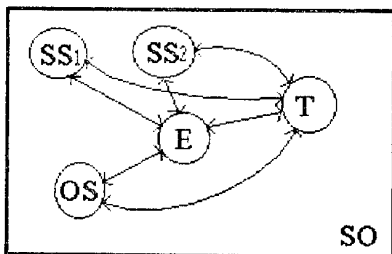
- 1) S.O. planning;
- 2) S.O. support;
- 3) analysis of conducted S.O.

d) Fourth object.

The basic feature - finite number of O.S. and S.S.

$S.O. = [O.S., O.S., S.S., S.S., T., E.]$

Objects correlation:



Movement of target and searching system is the mandatory condition for search operation.

The S.O. aims at target detecting. Efficiency criteria: min T., max P.

Tasks to be undertaken:

- 1) S.O. planning;
- 2) S.O. support;
- 3) assessment of conducted S.O.;

Additional problems subject to solution within the scope of the principal ones:

- optimization of search efforts;
- S.O. feasibility study.

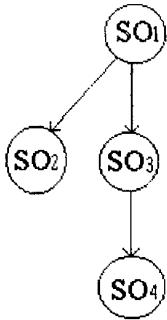
Target movement is the mandatory condition for the S.O.

The S.O. aims at target detecting. Efficiency criteria: min T., max P.

Tasks to be undertaken:

- 1) S.O. planning;
- 2) S.O. support;
- 3) assessment of conducted S.O.

The general hierarchy of S.O. classes can be presented as follows:



For practical and research purposes the S.O. class tree can be easily extended.

It can be concluded that the principal modeling purposes are as follows:

- 1) S.O. planning;
- 2) S.O. support;
- 3) analysis of conducted S.O.

The auxiliary tasks which if necessary can be included into the principal ones are as follows:

- set up the S.O. target: detection of signal, class, project, etc.;
- optimization of search efforts;
- efficiency assessment at the following levels: information, operation, engineering and economic.

Let us briefly characterize the S.O. modeling tasks.

### S.O. Planning

This stage aims to substantiate the suboptimum composition of forces and scenarios of their proceeding in time and space. To implement this stage we shagest the following sequence of steps:

- a) selection of S.O. efficiency criterion (criteria);
- b) selection (substantiation) of S.O. efficiency indices;
- c) selection of O.S. (S.S.) efficiency criterion;
- b) selection (substantiation) of O.S. (S.S.) efficiency indices;
- d) selection (substantiation) of mathematical, simulation models for S.S. efficiency indices calculation;
- e) selection (substantiation) of mathematical (simulation) models for S.O. efficiency indices calculation;
- f) Model for O. S. (S.S.) optimization; O.S. application, search direction, etc.;
- g) S.O. optimization model;
- h) selection of S.O. plan alternatives to be approved.

## S.O. Support

This stage provides for quick support for real search operation. It includes a number of steps (subtasks):

- a) substantiation of the alternatives of long-term and "step by step" planning;
- b) substantiation of necessary change in S.O. plan;
- c) substantiation of prompt interaction of S.S. (specifying coordinates, guidance, S.S. location control, etc.);
- d) Making a decision either to terminate or continue S.O.

## Analysis of conducted S.O.

The present stage is primarily aimed at revealing the factors which are not taken into account but they substantially affect the S.O. result.

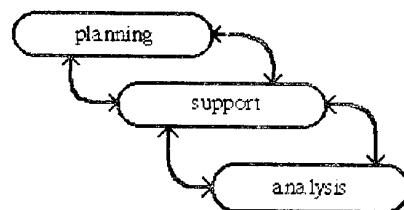
A number of steps is intended to be done, such as:

- a) comparing theoretical and actual results.

A number of specific tasks is intended to be carried out:

- substantiating rules for collection and processing statistical data;
  - validation of statistical hypotheses;
  - verifying mathematical models by calculation of correlation factors;
- b) revealing clusters in statistical samples and their interpretation;
  - c) selecting (substantiating) key factors to take them into account in further S.O. planning.

It should be noted that the process of S.O. object-oriented modeling is interactive and can be presented as follows:



Implementation of the above S.O. modeling stages results in the necessity of parallel implementation of some model fragments both within the scope of separate stages and within modeling stages. It is apparent that the support stage is typical real time process.

OODLE (Object-Oriented Design Language - language of object-oriented design of programmes, data base and module) notion is being applied for graphical presentation whatever language is used.

Synchronization of parallel process implementation is realized by two classes" timer (T.O and assessable memory controller (M.C.). In terms of technical realization, 3 levels of parallel process should be distinguished:

- 1) parallel existence and performance of the object supported by computer platform and (or) programming system;
- 2) parallel process implementation within the frameworks of given S.O. research stage, e.g. at the "S.O. support" stage;
- 3) parallel realization of the all S.O. modeling stages. This level is the most complicated one and not only regarding to computer resources, but because of complicated correlation between the stages. Such a complication is justified by a weak S.O. formalization at the levels of such general stages, as planning, support and analysis. Presently, acceptable formalization of the methods has been developed only at the level of internal realization of each stage.

**Conclusions.** Using the up-to-date programming paradigm - object-oriented approach in research operation provides a new tool in development of this promising direction of complicated systems research with the participation of human being.

Introduction of the new term O.O.M. enables to study the whole research process from conceptual model to program realization within the unified methodology and system of graphical and analytical presentation.

Using O.O.A. enables to divide the complicated stage of parallel algorithms and structures realization into three levels:

- level of language, computer platform supporting O.O.A.;
- parallel execution of applied classes at the level of parallelism;
- parallel execution at the level of domains (relatively independent subsystems). In this article, at the level of S.O. stages implementation, such as planning, support and analysis.

## **References**

1. Grady Booch. Object-Oriented Design with Applications. Benjamin/Cummings Publishing Company. Redwood City, Calif.1990.
2. J.Klir. Systems science. Automation of system problems solution. Translation from English under A.M. Gorlin's edition. M.: "Radio and Communication", 1990 (in Russia).
3. Koopman B.O. Theory of Search: 2. Target Detection. Operations Research. - 1956.- v.4, No.6.
4. G. Caisler, Ch. Chan. Theory of Models.- M: World, 1957.
5. V.Mitko, V. Popovich. . Application of object-oriented approach for sea observation systems modelling.-UNESCO,"RI-94", St.Petersburg, May 1994.
6. Sally Shlaer and Stephen J.Mellor . Object Lifecycles: Modeling the World in States, Prentice-Hall, 1992.
7. E.S. Ventzel. Operations Research. Problems, Principles, Methodology. M: World, 1977 (in Russia).

# THEORY AND PRACTICE OF AUTOMATED STRUCTURAL-LOGICAL SIMULATION OF SYSTEMS

**Mozhaev Alexander Sergeevich**

Naval Academy  
emb. Viborgskaja 73/1  
Saint-Petersburg

**Abstract.** Concept of New information technology of automated construction of mathematical models is stated. Description of foundations of theory of automated structural-logical simulation of systems (ASLS) is given. Educational - research computer-program set of ASLS is demonstrated. Accumulated experience of application in different domains and of training of specialists in theory and methodology of ASLS is offered.

**Keywords.** Models synthesis, automation, system, structure, logic, probability, computer program set, design, application, training.

Automation of constructing different mathematical models for objects and processes of complex system nature is an important modern scientific and practical problem of system analysis. Two studies [1, 2] are devoted to two important ways of solving the problem. One more perspective way of automation of simulation is connected with development of well-known logical-probabilistic methods (LPM) of system analysis [3, 4]. This issue gives summary of new scientific and practical results achieved during last years in elaboration of special theory, methodology and computer software on automation of constructing structural-logical models of complex system nature objects [5-17].

## Automated structural-logical simulation of systems theory concept

1. Initial key idea of automated structural-logical simulation (ASLS) consists in automated constructing of different mathematical models of system nature objects and processes by means of computer software. The structure of these objects and processes is to be set up by special circuits of functional integrity (CFI) [6, 7, 8].
2. Automation of constructing mathematical models is based on solving a new class of so called model tasks by means of computer software. Automatically created models consisting of mathematical calculations are the result of solving these tasks, that allows to calculate indexes of different features of examined systems.
3. Using computer software to solve model tasks requires creating special complex algorithms to provide automated computer constructing of mathematical calculations models. These algorithms must be able to construct another algorithms - models for quantitative evaluation of features of examined systems. That is why they are sometimes called A/A-algorithms (algorithms that make another algorithms) [6].
4. It is necessary to develop especial theories, methodologies and complete sets of software in order to automate constructing of mathematical models of systems. This situation is conditioned by the fact that existing now simulation theories are designated (and are able) only to "hand-made" constructing of systems models. These hand-made techniques of constructing models are slightly (not enough) formalised and can't be directly used for automated constructing. Examples of automated simulation theories development are stated in [1, 2]. General structure of offered theory of automated structural-logical simulation of systems (ASLS theory) is given on fig.1.

<b>ASLS theory METHOD</b>			
<b>General logic-probabilistic method for simulation of systems</b>			
Primary simulation	Secondary simulation		Application of models
Stage I	Stage II	Stage III	Stage IV
Formalized structural logical task statement systems simulation.	Constructing of logical model of system's functioning.	Constructing of different types of mathematical calculations based models, such as: analytic, Markov's, statistics and network models.	Calculation values of variables (indexes), analysis, selections, optimization, synthesis, working out management plans.
Executed by hand	Automated execution under special computer program run		

<b>ASLS theory SUBJECT MATTER</b>	
Theoretical object	Applied object
<p>Working out methods and means for development of general logic-probabilistic method of ASLS theory, such as:</p> <ul style="list-style-type: none"> <li>- initial formalized task statement for automated simulation;</li> <li>- basic methods, algorithms and computer programs for automated constructing of different mathematical calculations based models;</li> <li>- computer's interpretation forms of models and methods for calculation values of variables (indexes), analysis, selection, optimization, synthesis of systems, working out and grounding on management plans.</li> </ul>	<p>Working out applied parts and directions of ASLS theory, such as:</p> <ul style="list-style-type: none"> <li>- constructing models of reliability, survivability and safety;</li> <li>- simulation and calculation of real efficiency of complex systems;</li> <li>- simulation of functioning of complex systems including organizational and technical components;</li> <li>- probabilistic and deterministic systems' simulation in real time.</li> </ul>

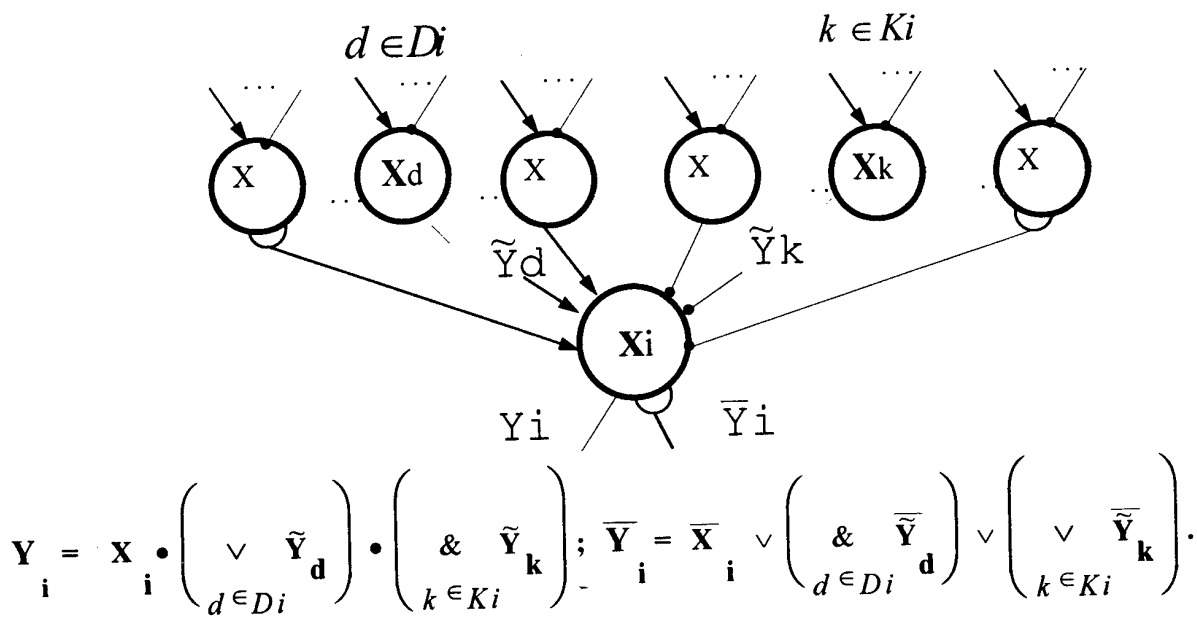
<b>ASLS theory METHODOLOGY</b>	
Methodological foundations of methods, algorithms and computer programs of automated structural-logical simulation for complex systems of different type, class and designation.	Methodological foundations of theory and computer programs applications for automated structural-logical simulation in different branches of science research, projecting and use of complex systems, in training of personnel and managing.

**Fig.1. Automated structural-logical simulation of systems theory subject matter**

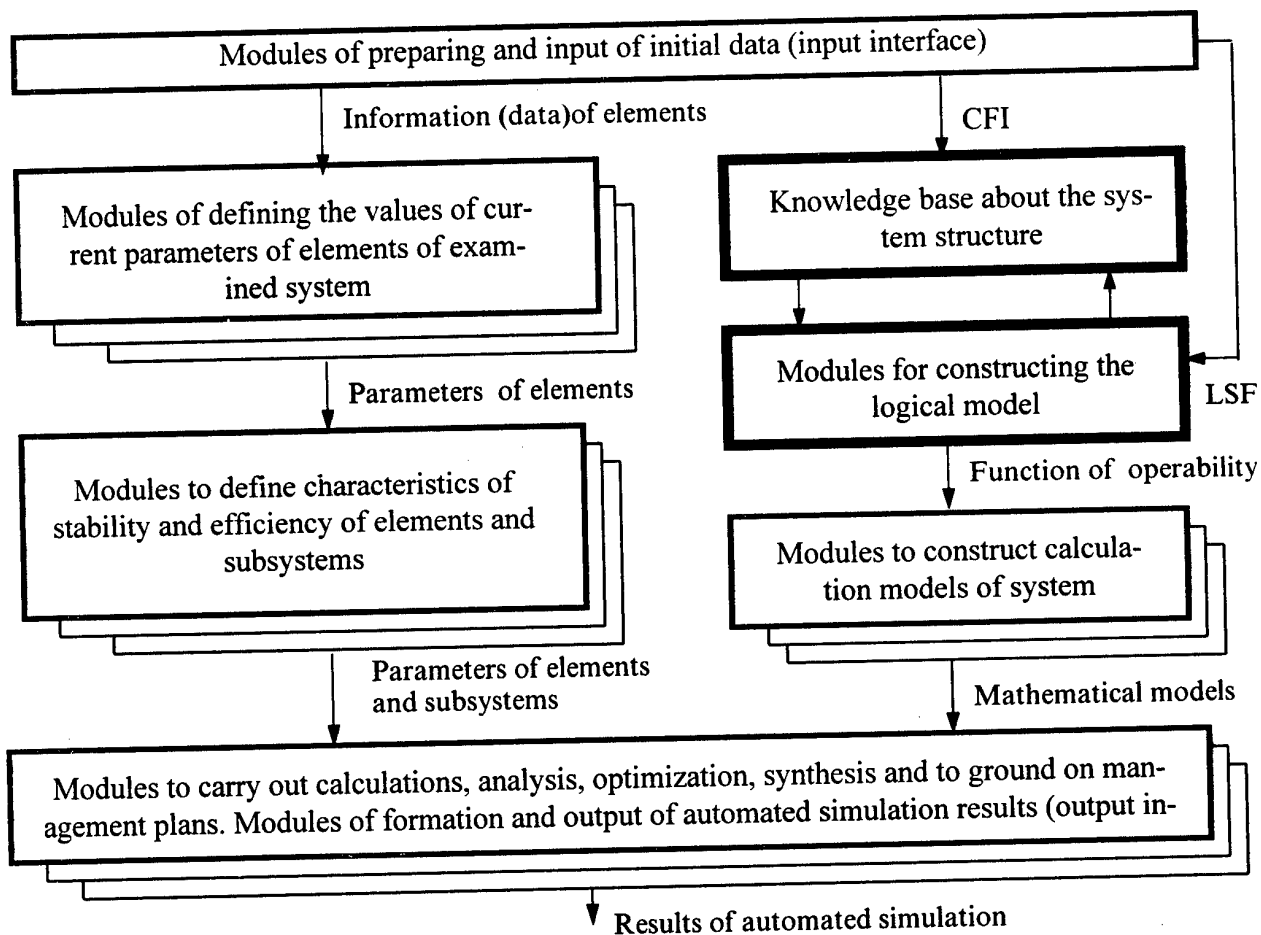
## General logic-probabilistic method of ASLS theory

So called general logic-probabilistic method (GLPM) of system analysis [5-8] serves as method of ASLS theory. GLPM represents further development of classic logic-probabilistic methods [3, 4] in following basic directions.

1. To describe the structure of systems, classic of logic-probabilistic methods (LPM) use trees of events or connectivity graphs, which realise only two logic operations "AND" and "OR". This set of operations is not functionally complete. It permits to construct only so called monotonous logic models of systems functioning in the form either of shortest ways of successful functioning or of minimal sections of failures. Functionally-complete set of logic operations "AND", "OR" and "NOT" was used to fulfil structural and logical simulation for the first time in GLPM [7, 8]. Especial graphic technique based on circuits of functional integrity (CFI) was developed in GLPM to describe systems structure. Basic representation means of CFI are given on fig.2. Universal graph-analytic method and algorithm for automated constructing of any kind of monotonous and non-monotonous logic functions of operability of examined systems were developed to provide logic description of models. It allowed to use in GLPM all the possibilities of classic algebra of logic and to automate constructing technique both for any kind of known monotonous models and for very important new class of non-monotonous models of functioning of complex system naturel objects. By means of non-monotonous models you can, for instance, take into account multifunctionality and qualitative complexity of systems, influence of damage factors (wreck, accidents, defeat, etc.), make probabilistic analysis of safety, estimate risk of functioning, find and pick out faults in projecting and many other important system peculiarities of real objects.
2. Subject matter of simulation has moved out of the borders of classic algebra of logic for the first time in GLPM. Elaboration was made of special laws of algebra of logic for groups of antithetic events (GAE) and for formal rules of calculation of corresponding probabilities [5, 8]. It allowed to use in GLPM not only binary elements (having 2 states), but also elements with any number of their own states. Special methods were developed to take account of stochastic dependencies between the elements, of their multifunctionality, of failures because of general cause, and also of different sequences of random events [8, 9].
3. New logic models, titled as logic functions of transitions (LFT), are applied in GLPM side by side with traditional logic functions of operability (LFO) [5, 14]. It has allowed to construct automatically two basic classes of models:
  - traditional models for LPM on the basis of logically connected states of separate elements;
  - new models for LPM on the basis of transitions between states of examined system as a whole.
4. In GLPM inside the borders of above mentioned classes the processes of constructing of following four types mathematical models of systems are automated;
  - analytic models, being represented in the form of polynomials of probabilistic functions;
  - statistical models, being represented in the form of rules to carry out computer statistical testing;
  - Markov models, being represented in the form of discrete and continuous chains with due regard to presence of dependent events and restorations;
  - network models, determined on a few classes of search tasks on a multitude of solutions, models of calculation of random sequences of events, models of network planning and control.



**Fig.2. Representation means of circuits of functional integrity of general logic-probabilistic method of ASLS theory**



**Fig.3. Typical circuit-diagram of computer program set of automated structural-logical simulation**

5. Complex automation is made in GLPM for all basic stages of mathematical simulation (see fig.1). Typical circuit-diagram of computer-program set of automated structural-logical simulation is represented on fig.3. New technology of automated structural-logical simulation of systems has been practically made using this basis. During the stage I it provides "hand-made" constructing by the user and putting into the computer program a circuit of functional integrity (CFI) of an examined system. Unlike the hand-made technology all the next stages of constructing of logical and mathematical calculations models of system (analytic, statistical, Markov and network models) and of calculating the system characteristics are being carried out completely in automated manner. Under it a purpose redistribution of functions between a human being and computer is taking place. Creative component of simulation remains at human being disposal, and it is being turned to reality in forms of constructing CFI, setting logic criteria of functioning, defining parameters of elements, plan and regime of automated research. Computer software carries all the rest cumbersome and hard, although formalised, technique of constructing mathematical calculations models and helps the user to make calculations in order to solve problems of analysis, optimisation, synthesis, to work out and ground on management plans. Technology of automated simulation is many times superior to top possibilities of hand-made simulation at quickness, exactness, dimension and number of variants.

### **Subjects matter of ASLS theory**

Two relatively independent objects - theoretic object and applied object - represent the subject matter of ASLS theory. Theoretic object includes all stated above problems of development and automation of general logic-probabilistic method of ASLS theory. Applied object includes the set of sections and directions of practical use of ASLS theory. The most characteristic of them are the following:

1. Many real tasks of ASLS are distinguished by such large dimension, that they can't be solved by means of modern and even perspective computer technique. The cause of it with all objectivity is in very high double power of exponential complexity of stages of logical and probabilistic simulation. Research has shown that there is no general solution of dimension problem. By now certain results have been obtained in ASLS theory in three particular directions of solving dimension problem.
  - a) the most efficient algorithms and computer programs for automated constructing of logical and probabilistic functions are developed on the basis of combining (composing) a number of particular logic-probabilistic methods into general procedures. Some of them are given below [7, 8]:
    - universal graph-analytic method of constructing monotonous and non-monotonous logic functions of operability;
    - combined and gradient methods of transition from logic functions of operability to polynomials of probabilistic functions.
  - b) Different method of decomposition of processes of constructing systems models [7, 10] are developed and installed in GLPM:
    - method of mono-coherent structural decomposition;
    - method of mini-coherent structural decomposition;
    - method of logical decomposition;
    - method of decomposition in antithetic hypotheses.
  - c) A number of efficient methods of constructing high-dimensional models of systems are developed and installed in computer programs on the basis of automated statistical simulation [5] and approximate analytic simulation [13]. The most perspective thing is adaptation of methods, algorithms and computer programs of ASLS to concrete subject matter domains and classes of tasks to be solved.

2. The following independent sections of ASLS theory are connected with analysis of features of systems stability-reliability, survivability and safety of different systems. Principle approaches to solution of tasks were developed inside the borders of classic LPM [3,4, etc.]. In ASLS theory these approaches have been completely performed and have got further development in following directions.
  - a) The following new possibilities have been performed in reliability domain side by side with classic tasks of simulation and calculation both the probability of no-failure operation of non-restorable systems and in-commission rate of restorable systems [6,13]:
    - automated constructing of monotonous and non-monotonous models of reliability of systems which are not being represented by trees of events or connectivity graphs;
    - taking into account the influence of sets of antithetic events in models as well as failures on general cause and multiplicable states of elements;
    - calculation of time-probabilistic characteristics of reliability of systems (mean-time-to-first-failure, mean time between failures and mean time of restoration);
    - constructing of approximate models for calculating the probability of failure (or probability of no-failure operation) of restorable system of arbitrary structure;
    - method of simulation and evaluation of reliability of mixed systems, consisting of restorable and non-restorable elements, by means of special new index - probability of availability of mixed system [6] - is developed, this index occupies intermediate place between traditional models of non-restorable and non-constrained restorable systems.
  - b) In survivability analysis domain the ASLS theory allows to put into effect without any constraints all known approaches to monotonous logical-probabilistic simulation and to calculate characteristics of conditional laws of survivability of systems by means of known table methods [11]. Principle new and perspective are the methods of automated Markov's simulation of survivability [11]. They allow, in particular:
    - to construct automatically monotonous and non-monotonous dynamic mathematical models of survivability in the form of Markov's chains and calculate characteristics of conditional law of survivability (CLS) for all typical models of defeat;
    - to construct dynamic models and calculate characteristics of CLS for any current initial state of a system;
    - to take into account in created models the processes of restoring elements and different kinds of dependencies.
  - c) In safety analysis domain the ASLS theory allows to put into effect completely all basic theses of known concept of logical-probabilistic theory of complex systems safety (scenario approach) [4]. At the same time a number of principle new possibilities is put into effect in ASLS theory for further perfection and development of mentioned concept on the basis of the methods of non-monotonous simulation of accident progressing processes, of accounting sets of antithetic events, of multipliable states of elements, of different initial states of system-nature objects and their qualitative complexity, operative accounting of variable (changeable) conditions of systems functioning, causes of origin and progressing of accidents, defeatable influence, etc. New problems of simulation and calculation of systems safety have been also solved [12]:
    - basic theses of probabilistic analysis procedure of nuclear power stations safety have been developed, answering the typical requirements of IAEA, that confirms the opportunity of creation of appropriate home-made computer software on the ASLS basis;

- general technique of automated structural-logical simulation and estimation the risk of complex systems functioning has been developed;
  - approaches have been offered to automated deterministic structural-logical simulation for purposes of monitoring of safety functioning of complex systems.
3. Three principle new and perspective applied directions have been formed under development of ASLS theory subject matter.
- a) Automated simulation and calculation of indexes of real efficiency of qualitative complex systems functioning. Such systems are able to fulfil their functions in different submultitudes of their states with different efficiency. That is why analysis of these systems requires correct combining of stability models (reliability models, survivability models, safety models) and conditional efficiency of their functioning. Such combining in GLPM can be carried out on the basis of constructing non-monotonous models in every submultitude of qualitative different states of system and then putting them together in desirable general model of real efficiency [5, 13].
  - b) Development of GLPM allowed to widen the domain of practical use of ASLS theory having including not only classes of technical objects but also classes of complex organisational and organisational-technical systems of different kinds, types and purpose. At the same time both general and also reciprocally contrary purposes and tasks of functioning of separate elements and subsystems can be taken into account.
  - c) Complex (combined) automation of mathematical models constructing technique allows to put into effect new technology of operative structural-logical simulation for check and control of complex systems in real-time functioning. Such real-time simulation will allow to find operative answers, grounded from the scientific point of view, to all current changes of purposes, tasks, operation regimes and functioning conditions of complex system-nature objects and processes.

### **ASLS theory methodological foundations**

Methodological foundations of ASLS theory are connected with solution of tasks of development and practical use of methods, algorithms and computer programs of ASLS (see fig.1). They constitute two relatively independent methodologic directions.

1. Methodic problems of elaborating of methods, algorithms and computer programs of automated simulation, developed inside the theory of ASLS, include following basic theses [8, 14]:
  - generalised structure of typical computer program set for automated simulation, diagram of which is represented on fig.3;
  - basic principles and general procedure of elaboration of methods and algorithms of ASLS for systems.

In our institution (Naval Academy) a number of computer program sets of automated simulation are developed and successfully used in educational process and science research. Educational branches are stated and scheduled training of professionals is carried out in elaboration and use of ASLS computer program sets [15]. A number of methodical textbooks on theory and methodology of ASLS are ready to be published.

2. Methodical problems of practical use of ASLS theory now are represented by special procedures of using computer program sets of automated simulation in different domains of systems research and control [16, 17], and also by generalisation of already accumulated experience of use these procedures in many institutions.

Educational-research example of computer program set for automated structural-logical simulation "CPS ASLS version 5.0", developed by chair of automated control systems, Naval Academy, has got the most circulation in science and educational in-

stitutions of St-Petersburg. Methods of analytic and statistical automated simulation are being put into effect in this example on the basis of GLPM, and also there is a training regime and textbook on practical use. CPS ASLS is represented in this conference show now and may be handed if desired to everybody free of charge.

CPS ASLS being applied a great amount of science research has been carried out in many institutions. Six doctoral dissertations and more than twenty candidate of science dissertations have been worked out and successfully stood up for by using CPS ASLS. Studies [18, 19] where all the tasks of simulation are solved by means of CPS ASLS version 5.0 may serve as typical example of practical use of ASLS theory, although without any references to the authorship.

**Conclusions.** Automated structural-logical simulation is perspective new scientific direction of systems analysis. Experience accumulated in theoretical elaboration and practical use allows now to carry out concrete studies in adaptation of theory and computer program sets of ASLS to different applications of research, protecting, maintenance and practical use.

### *References*

1. Ivanishev V.V. Automation of simulation of data flow systems. Leningrad, Science, 1986, 139 p.
2. Buslenko V.N. Automation of imitative simulation of complex systems. Moscow, Science, 1977, 240 p.
3. Ryabinin I.A., Cherkesov G.N. Logical-probabilistic methods of research of complex-in-structure systems reliability. Moscow, Radio and communications, 1981, 264 p.
4. Ryabinin I.A. Scientific school of logical-probabilistic method, and the concept of logical-probabilistic theory of complex systems safety. Pre-print 101. Theory and information technology of complex systems safety simulation. Issue 1. Edited by Ryabinin I.A. St.Petersburg: IPM RAS, 1994, p.p. 3-22.
5. Mozhaev A.S. Modern state and few directions of development of logical-probabilistic methods of systems analysis. Part 1. Pre-print 101. Theory and information technology of complex systems safety simulation. Issue 1. Edited by Ryabinin I.A. St.Petersburg: IPM RAS, 1994, p.p. 23-53.
6. Mozhaev A.S., Alekseev A.O. Automated structural-logical simulation and probabilistic analysis of complex systems. Pre-print 104. Theory and information technology of complex systems safety simulation. Issue 2. Edited by Ryabinin I.A. St.Petersburg: IPM RAS, 1994, p.p. 17-42.
7. Mozhaev A.S. General logical-probabilistic method of analysis of complex systems reliability. Educational text-book. Leningrad, Naval Academy, 1988, 68 p.
8. Cherkesov G.N., Mozhaev A.S. Logical-probabilistic methods of complex-in-structure systems reliability calculations. In "Reliability and quality of industrial goods". Moscow, Knowledge, 1991, p.p. 3-65.
9. Mozhaev A.S. Tasking the account of time sequences of elements failures in logical-probabilistic models of systems reliability. In "Reliability of energetics systems", Inter-high school collection. Novocherkassk, NPI, 1990, p.p. 94-103.
10. Mozhaev A.S., Travin E.V. Methods of decomposition of logical-probabilistic of complex technical systems reliability. Theses of reports on science-technical conference: Ways of raising the technical level and reliability of machines", Minsk, IP NDM AN BSSR, 1986, p.p. 27-28.
11. Ivanov M.V., Mozhaev A.S., Ryabinin I.A. Logical-probabilistic methods of calculation of survivability of ships' automated electrical-energetic systems. Problems of shipbuilding, series 'Ship's automation', 1984, Issue 30, p.p. 3-16.

12. Mozhaev A.S. Automated structural-logical simulation in solving the tasks of probabilistic analysis of safety. Pre-print 110. Theory and information technology of simulation of complex systems safety. Issue 4. Edited by Ryabinin I.A. St.Petersburg: IPM RAS, 1994, p.p. 16-38.
13. Mozhaev A.S. Educational-methodical textbook on automated structural-logical simulation and calculation of indexes of reliability, survivability and safety of systems by means of personal computer. St.Petersburg, Naval Academy, 1992, 98 p.
14. Mozhaev A.S. Simulation method in the theory of automation of control. Educational textbook. St.Petersburg, Naval Academy, 1995, 39 p.
15. Mozhaev A.S. Methodical problems of development and application of computer programs of automated structural-logical simulation and systems analysis. In coll. "Important problems of perfection of educational-training process in Academy". Issue 5. St.Petersburg, Naval Academy, 1992, p.p. 87-99.
16. Mozhaev A.S. Theoretical foundations and experience of structural-logical automated simulation and systems analysis. In "Methodical problems of large energetics systems reliability research". Issue 41. Computational models of electrical energetics systems reliability research. Comparative analysis. Irkutsk, SEI SO AN SSSR, 1991, p.p.91-105.
17. Komin V.Y., Mozhaev A.S., Alekseev A.O. Automation of constructing reliability models of navigational equipment system. Report on IX Science-Technical conference: Navigation's role in providing human activities at sea. Part III. Gdinya, Naval Academy named after Heroes of Westerplatte (AMW Wewen. 907/94), 1994, p.p. 80-92.
18. Solozhentsev E.D., Solozhentsev V.E. Logical-probabilistic simulation and risk analysis in bank and organisational systems. Pre-print 110. Theory and information technology of complex systems safety simulation. Issue 4. Edited by Ryabinin I.A. and Solozhentsev E.D. St.Petersburg, IPM RAS, 1994, p.p. 68-88.
19. Solozhentsev E.D. Science-research study: Elaboration of graph-models logical-probabilistic models for numerical estimation and analysis of risk of credit unreturn. Carrying out research on examples. Under agreement with joint-stock company "Bank St.Petersburg". St.Petersburg, IPM RAS, 1995, 86p.

# COMPUTER TECHNOLOGY OF SEISMIC DATA PROCESSING WITH THE PURPOSE OF DETECTION OF OBJECTS WALKING AND MOVING ALONG THE EARTH SURFACE

Alexander A. Pahomov  
Jupiter-Z  
Science and Technology Centre,  
Saint Petersburg

**Abstract.** Some problems of building of digital seismic stations using for detection of «walking» and other objects, moving along the earth surface, are being solved. This station unites computer technology and algorithms of data processing and decision making typical of a man-interpreter. The station utilizes a detailed multi-criteria rule of making decision on detection and flexible multi-parameter system of adaptation to certain conditions of functioning and an object character, that is important for a user. The using of this station in real conditions shows its high characteristics.

**Keywords.** Seismic and acoustic data processing, observation systems, detection of objects of predetermined classes, multi-criteria rule of making decision, multi-parameter adaptation.

A problem of detection of objects moving along the earth surface is characterized by numerous applied aspects and a wide sphere of application. The so-called «walking» objects are of particular importance among all moving objects. They include a walking man, a running man, a pack animal walking or running with a man on its back, etc. Firstly, these objects are notable for their ability to move along both a broken ground covered by a forest or bushes and difficult-of-access terrain lying far from roads. Secondly, their moving can be connected with various illegal types of activity and crossing different frontiers, boundaries, zones, etc. Besides detection and location of moving tracked and wheeled vehicles is of considerable interest.

Stations based on computer technology of processing of seismic data and used for detection and location of the above-mentioned objects appeared in various countries of the world (the USA, Great Britain, Germany, Russia). However, their characteristics could not be regarded as characteristics satisfying demands of customers. That is why there was an attempt to design a station which would unite computer technology of data processing and algorithms of data processing and decision making typical of a man-interpreter. (More detailed discussion of some problems of use of such algorithms in digital information systems is given in another article of the author published in the present collection).

The station is passive and detects objects by their primary seismic and acoustic fields. It conditions its ecological safety. All elements of the system which receive and transmit signals are buried and cannot be seen by a detached observer. They are characterized by high secrecy and, as a result, good «vitality».

In comparison with other known analogous seismic devices the proposed system ensures a two-threefold increase of a detection range, higher reliability and a lower number of false alarm signals. It is achieved by use of absolutely new methods of processing of seismic information which allow to detect objects of only a predetermined class. Thus, it excludes traditional two-stage (detection-classification) processing which inevitably leads to complexity and higher cost of a system and sometimes begins to annoy a user by detection of either «no one knows what» or «everything».

The efficacy of the proposed system is explained by use of algorithms on which an activity of an experienced interpreter of visual seismic information is usually based. Besides the station utilizes a detailed multi-criteria rule of making decision on detection and a flexible multi-parameter system of adaptation to certain conditions of functioning and an object character. It is important for a user. The simplest modifications of algorithms of data processing used in the system include 25 variables which provide the corresponding adaptation.

The station consists of a part for outside use and a digital complex for processing, making archives, storage and printing of information. A part for outside use is responsible for receiving seismic signals, their preamplifying and transmission to a digital complex. It includes seismic sensors, preamplifiers, a power supply block and main cables.

A digital complex is meant for processing of received signals, isolation of a useful signal, making decision on detection of targets of predetermined classes, almost on-line presenting information on a screen of a colour monitor, making archives and printing obtained results in the form of a document.

The station versions envisage use of a digital system of information transmission which makes it possible to convert signal to a digital form, to regulate their amplification coefficients and a bandpass of sensors, to provide sensors with remote power supply and to transmit information along a single main coaxial cable.

The station can be used with other observation systems. In particular, when an object of a predetermined class is detected it can switch on secret videocameras, microphones, audiosystems which allow to hear all conversations and sounds within an area of its action, etc. Certain information (video- and audiosignals, etc.) can be transmitted via the above-mentioned coaxial cable of the station using channelling. The same cable can be used for provision of telephone and fax-modem communication between observation posts which receive information on transgressors and trespassers.

The proposed station allows to make various systems for detection of moving objects. Such systems can be of:

- a zone (area) type when sensors are placed evenly and cover some area;
- an object (local) type when sensors are placed in immediate proximity to a protected object;
- a boundary (linear) type when sensors are placed along some line running across the ground.

Each sensor detects a «walking» man or a pack animal on an area with a radius of not less than 70-80 m. This result is attainable on various soils, in a different character of a covering layer and woods. It is irrespective of a movement direction, a character of steps and weather conditions. The station provides reliable detection of

objects within the range of this area even in case of seismic noise due to remote earthquakes. Moving vehicles can be detected at a distance which is 3-5 times longer. As for a detection line sensors are placed with an interval of 100 m. It can be considered that every sensor controls a zone which has a form of a square with a side of 100 m. Thus, the station makes it possible to form a detection line with a width (depth) of 100 m. Its length can vary from some kilometers to several dozens and even more. If a detection line is too long or if a number of sensors used in detection systems of some other types is too great a digital complex of processing, making archives and printing is equipped with several monitors presenting information on a general situation in the region under protection and a situation typical of those places where moving objects have been detected. A digital complex configuration is determined by a type and size of a detection system, its location and a character of problems solved by a user.

If there is no predetermined object of detection in a zone of sensors' activity a monitor display presents information on location of sensors, zones of responsibility of each sensor (for a detection line systems - squares with a side of 100 m), an ordinal number of every sensor of a system, a distance scale, a current date and time. This image can be superimposed on a map of the country where the system is installed.

If an object of detection approaches and penetrates into a zone of a sensor's activity one can hear a sonic alarm signal and the following information appears on a monitor screen: an ordinal number of an activated sensor, a zone of its responsibility, time of the first episode of an object detection, its class (from a list of detectable classes chosen by a user beforehand), information on bearings. The proposed station allows to take bearings at a distance making 60-70% of the maximum range of detection that is why one can see such words as «insufficiency of information» instead of bearings when an object is detected at maximum distances.

Interrogation of sensors is carried out every 5-8 sec, a new image appears on a screen after the same period of time. Information on a total period of time spent by a trespasser in a zone of each sensor and all determined bearings are presented. All information on a detected object can be erased by an operator using a panel or it is done automatically after some period of time fixed by a user. Archives are made automatically; the information can be printed if necessary.

The cost of the station depends on a size and configuration of a formed system of detection, a number of classes of objects which should be detected, necessity of taking bearings, a character of sources of natural and artificial noise in the area of a system location (adaptation of the station to environmental conditions is carried out by a company-manufacturer, i.e. Jupiter-Z Science and Technology Centre, Saint Petersburg, fax (812)-234-47-12).

# DIGITAL INFORMATION SYSTEMS AND SOME PROBLEMS OF APPLICATION OF ALGORITHMS OF DATA PROCESSING AND DECISION MAKING CHARACTERISTIC OF A MAN-INTERPRETER

Alexander A. Pahomov  
Jupiter-Z  
Science and Technology Centre,  
Saint Petersburg

**Abstract.** Some problems of realization in the digital information systems of the algorithms of the data processing and taking decision, characterizing to a man-interpreter, are being solved. We consider some possibilities and peculiarities of visual perception of an interpreter which can be used in digital information systems. We analyze the main difficulties of using such algorithms.

**Keywords.** Data processing, visual perception, information sign, space-time area, decision made, efficacy.

Use of computers (personal computers) with the purpose of solving some applied problems including processing of large amounts of data of physical observations has become a routine event. Replacement of one generation of computers by another which took place during 30 years resulted as a rule in some qualitative leap in significance of obtained results. At the very beginning it was connected with the fact that new and more efficient computers allowed to use though more complicated but still known formal solutions of problems (with a greater amount of data, higher complexity of problems, taking into account a greater number of correlation links, etc.). A physical essence of a problem and a creative thought of a physicist-investigator were often of secondary importance.

The situation was aggravated by a quick growth of a used park of computers and its frequent renewal which demanded training of a big number of specialists in this field. In the long run such a park became a reality and it was renewed continuously. But instead of movement from a physical essence of urgent practical problems towards use of computers which was more fruitful for development of applied physics there appeared more active movement in the opposite direction, i.e. from a powerful computer to solution of those problems which "came to one's hand" and were suitable for application of a known method of information processing.

As a result, solution of some problems connected with processing the results of physical observations lost any sense. As for search for new solutions it reached a deadlock. Developers of many information systems, in particular those intended for detection, classification and determination of parameters of various objects on the basis of their physical fields, do not see the possible ways of qualitative improving the results of solution of their problems. At the same time such possibilities exist. They are connected with use of just those algorithms of data processing and decision making in digital information systems which are typical of work of a man-interpreter.

This trend of application of computers came into existence in the 60s-70s. But being represented by a scanty number of physicists who used computers for interpretation of results of physical observations it was lost in the ocean of works carried out mostly by specialists in the fields of calculations and applied mathematics.

As it has already been mentioned, a pure mathematical and calculating approach exhausts itself. Thus, a trend connected with use of algorithms typical of work of a man-interpretor in information systems becomes important again and even can acquire priority.

What peculiarities of information perception and decision making are prone to a man-interpretor and can be used in digital information systems? A man usually perceives about 90% of all information with the help of his eyes. This figure is higher as far as an interpretor is concerned. An auxiliary material in the form of some graphic images (for example, amplitude or phase spectra of signals) is presented by some up-to-date digital information systems to an operator for decision making. However, it is far from use of all possibilities of visual perception.

Let us dwell only on some possibilities and peculiarities of visual perception of an interpretor which can be used in digital information systems. He is able:

- to observe large amounts of material presented in a space-time area moving his eyes to and fro in any direction;
- to fix his eyes on peculiarities of the material which, for example, has been subjected to preliminary processing and is presented in a space-time area;
- to compare properties of various parts of visual representation of data under processing in a space-time area;
- to study a structure of presented material in various parts of a space-time area;
- to analyse back-, middle- and foregrounds of processed data, i.e. to look at this material as if from various distances;
- to study peculiarities of distribution of information signs proper in a space-time area as data concerning these signs cannot be accumulated with the help of such mathematical operations as summing or integrating;
- to estimate parameters of distribution of information signs in a space-time area from a quantitative point of view;
- to carry out detailed analysis of a structure of properties of information signs proper which can be repeated in various places of representation of data in a space-time area.

It will allow an interpretor both to detect signs of a signal from an objects of predetermined classes in processed data and to estimate their characteristics (coordinates, movement velocity, etc.) with the help of additional measurements and processing procedures.

Sometimes capabilities of an operator's acoustic analyzer are used in existing information systems intended for detection of objects and determination of their parameters on the basis of acoustic and seismic fields. Of course, this increases efficacy of application of such information systems though capabilities of an acoustic analyzer are considerably smaller than those of a visual analyzer. The crucial point is that almost every operator can be taught *to see* peculiarities (even very delicate ones) of distribution of processed data in a space-time area. At the same time only a talented person with an ear for music can be taught *to hear* shades of an acoustic signal. Besides, many peculiarities which are seen on a signal record cannot be heard and distinguished by ear even by an experienced operator with a good ear for music.

It is quite natural that use of discussed capabilities and peculiarities of visual perception of an operator in digital information systems is accompanied by some difficulties the main of which are as follows.

Analysing data presented in a space-time area an interpretor makes several consecutive decisions. A chain of decisions is rather long and branchy. Every decision made by an interpretor is entailed with (accompanied by) verification of these or those

conditions. Use of an algorithm of interpreter's work in a digital information system demands performing a great number of conditional operations and, thus, more time for realization of this algorithm. At the same time a practical experience shows that speed of operation of modern computers allows to carry out on-line processing and decision making based on these algorithms.

Every decision of an interpreter in a long chain of decisions is entailed with a solution (or comes to a solution) of bialternative problems in which these or those characteristics of processed material are compared with threshold values. These threshold values reflect interpreter's conceptions of such characteristics of processed material which allow to consider that there is that very peculiarity which accompanies a physical phenomenon of interest. A lot of decisions are made and there can be many threshold values (two and more dozens in solution of real problems). Determination of these values in adjustment of a computer algorithm is a very complex and creative task which demands much time and joint work with experienced interpreters. It is very difficult to point out optimum meanings for threshold values as many of them are connected with each other and a change of any of them in this or that direction has both a positive and negative effect on a final or important intermediate result of solution of the main problem.

An interpreter can easily turn from one algorithm to another. Sometimes he acts by intuition and does not bring a version of reasoning which he selected primarily and then discarded to its logic end. This allows an interpreter to save time. If an algorithm is formal and lacks accurate quantitative criteria such transition from one version of processing to another will be impossible. As a rule these criteria can be formulated only as applied to a final result of processing. That is why use of computer algorithms demands additional time (in comparison with a man-interpreter) which is spent on bringing to an end every out of all possible versions of processing.

An interpreter's reaction to a nonstandard situation (i.e. a situation which is not envisaged in a problem under solution) very quickly and can stop processing or use additional data and algorithms. Realization of it in a computer algorithm can cause serious difficulties in its development and use and, as a result, increase expenditures.

One more difficulty connected with use of algorithms typical of a man-interpreter's performance lies in extreme complexity of theoretical, a priori investigation of their efficacy. In other words, it is difficult to obtain estimates of the so-called potential efficacy or maximum efficacy which have become widespread. As for algorithms under discussion it is possible to get estimates only of actual efficacy and to use them with this or that experimental material in controlled conditions with their subsequent transfer to another situations which are identical from the point of view of conditions.

Use of algorithms typical of a man-interpreter's performance in specific digital information systems intended for detection and location of objects of predetermined classes on the basis of their seismic fields has shown that characteristics of these systems are 3-5 times better than those of the best systems based on traditional mathematical approaches to information processing.

# COMPUTER SIMULATION OF MULTIPLICATION SEARCH TASKS

Dr. Vasily V. Popovich, Alexey L. Tretyakov.

Division for Applied Search Programs

St. Petersburg Scientific Centre of the Russian Academy of Sciences

University Emb., St. Petersburg, Russia,

Tel: (812) 218 41 67, E-mail: popovich@popovich.spb.ru

**Abstract.** Computer simulation of the new class of moving objects search tasks (multiplication tasks) is examined. Two variants of the model notion are suggested. An approximate model and its generalization.

**Keywords.** Theory of search moving objects, additive and multiplication search tasks, computer simulation.

**Introduction.** In [1, 2, 3, 4] the statement of new class tasks of search theory was examined. In these tasks the condition of additive search potentials of detection means is not true.

In this article will suggest a computer model of appraisal efficient by system of different means of detection are situated on single carrier.

The calculations executed by the following equations :

$$K_{cmin} = \sin^{-1}(b) - \sin^{-1}(a), \quad K_{cmax} = \pi - \sin^{-1}(b) - \sin^{-1}(a).$$

where

$K_{cmin}$  - minimal-possible value of target course;

$K_{cmax}$  - maximal-possible value of target course;

$$b = \left( \frac{D_n}{D_s} \right), \quad a = \left( \frac{V_c D_n}{V_n D_s} \right);$$

$$S = D_s^2 \Delta K - S', \quad S' = \int_{K_{cmin}}^{K_{cmax}} \frac{D_n V_\rho}{V_n \sin K} dK.$$

$S$  - general square of area of possible target location (OWMC).

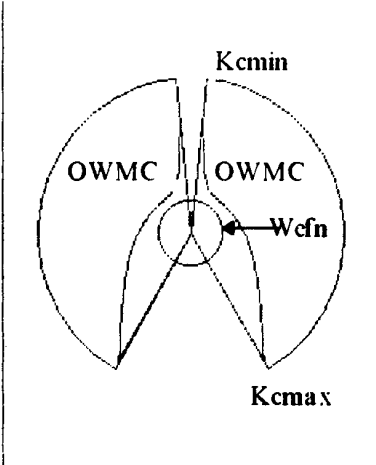
Figure 1 shows the general view on the model window.

Characteristic and purpose of window elements of the task.

The panel "**Original data**" - intend for operational correction original data for calculations.

Original data correction can be made by the next method :

- use "mouse" or button "Tab" for choose parameter that you want to correct (after it its background colour will change into blue); on bottom line of the window will appear short name and measure of current parameter;
- after correction you must press button "Run" to begin execution of decision process.



The diagram on the left shows a circular search area divided into two segments labeled 'OWMC'. A central point is labeled 'Wefn'. Two radial lines from the center to the outer edge are labeled 'Kcmin' (top) and 'Kcmax' (bottom). Below the diagram is the text 'Скорость наблюдателя (узл)'.

The form on the right is titled 'Исходные данные:' and contains the following input fields:

Vn =	6
Vc =	8
Wefn =	5
Tj =	3

Below this is the section 'Результаты:' with the following output fields:

Sowmc =	932.1
Kcmax =	151.8
Kcmin =	4.1

At the bottom of the form are three buttons: 'Run', 'Print', and 'Save'. Below the 'Save' button is the text 'Справка F1'.

Fig. 1 View on the modal window.

Button **"Run"** intend to start execution of decision process. This button must be pressed after each correction.

On the panel **"Results"** is shown results of decision after each pressing **"Run"** button.

Button **"Print"** intend to print original data and results of task decision.

Button **"Save"** intend to save window of results on a hard or floppy disk into the file that have **"BMP"**-format. So results may be included into different reports and documents by using text editors, for example Microsoft Word 6.0 and later versions.

Button **"F1"** is used to receive context help, where intend of the task and work organization on PC are described.

The panel **"Original data"** includes following parameters :

Vn - *observer speed (knot);*

Vc - *target speed (knot);*

Wefn - *effective wide of the search line of observer (miles);*

Tj - *track registration time of target (hours);*

The panel **"Results"** includes following parameters :

Sowmc - *square of possible target location area (miles);*

Kcmax - *maximal value of target course (grad);*

Kcmin - *minimal value of target course (grad).*

This task of appraisal of search efficient will belong to multiplication tasks of search.

The model that was described before is fair for majority of practical applications. But for general case it needs to be made some more precise. So if we take into account real



Now we can define the time, which target passage the calculation distance  $S_p$ .

$$t_p = \frac{S_p}{V_p}.$$

For calculation the time of delay of devices for track registration, examine Figure 3.

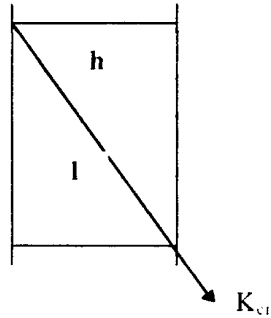


Fig. 3 Calculation the registration time.

From this figure we can calculate the distance, which observer passage in track up to exit to background and (after this) the registration time, assume, what devices work in real time scale. In contrary case time of delay must be increase on time of decision adoption  $t_k$ .

$$l = \frac{h}{\sin K_{ci}} \Rightarrow t_n = \frac{h}{V_n \sin K_{ci}}.$$

Condition of target presence on line of current course may be write in the next form :

$$t_p > t_n + t_k.$$

Sometime more comfortable decide converse task (for graphical show).

Determine the point on target course (on line  $K_{ci}$ ), beginning with which may presence (in increase side) the target, track of which was intersected by observer.

Write:  $S_p = t_n V_p \Rightarrow \frac{D_n}{\sin \alpha} \sin(\alpha + \xi) = t_n V_p \Rightarrow$

$$\sin(\alpha + \xi) = \frac{t_n V_p}{D_n} \sin \alpha \Rightarrow \alpha + \xi = \sin^{-1} \left( \frac{t_n V_p}{D_n} \sin \alpha \right) \Rightarrow$$

$$\xi = \sin^{-1} \left( \frac{t_n V_p}{D_n} \sin \alpha \right) - \sin^{-1} \left( \frac{V_c}{V_p} \sin K_{ci} \right) \Rightarrow$$

$$\sin^{-1} \left( \frac{D_x}{D_n} \sin \alpha \right) = \sin^{-1} \left( \frac{t_n V_p}{D_n} \sin \alpha \right) - \sin^{-1} \left( \frac{V_c}{V_p} \sin K_{ci} \right);$$

Mark in:

$$a = \left( \frac{t_n V_p}{D_n} \sin \alpha \right); \quad b = \left( \frac{V_c}{V_p} \sin K_{ci} \right). \quad D_x - \text{segment OA (Figure 2)}.$$

Then:

$$\frac{D_x}{D_n} \sin \alpha = \sin(\sin^{-1} a - \sin^{-1} b) = a \cos(\sin^{-1} b) - b \cos(\sin^{-1} a).$$

It is known, that  $\cos(\sin^{-1} b) = \sqrt{1 - b^2}$ ; then  $\frac{D_x}{D_n} \sin \alpha = a \sqrt{1 - b^2} - b \sqrt{1 - a^2}$ ;

$$\begin{aligned}
\text{Or : } \frac{D_x}{D_n} \sin \alpha &= \frac{t_n V_\rho}{D_n} \sin \alpha \sqrt{1 - \frac{V_c^2}{V_\rho^2} \sin^2 K_{ci}} - \frac{V_c}{V_\rho} \sin K_{ci} \sqrt{1 - \frac{t_n^2 V_\rho^2}{D_n^2} \sin^2 \alpha} = \\
&= \frac{t_n V_\rho}{D_n} \sin \alpha \frac{\sqrt{V_\rho^2 - V_c^2 \sin^2 K_{ci}}}{V_\rho} - \frac{V_c}{V_\rho} \sin K_{ci} \frac{\sqrt{D_n^2 - t_n^2 V_\rho^2 \sin^2 \alpha}}{D_n} \Rightarrow \\
D_x &= t_n \sqrt{V_\rho^2 - V_c^2 \sin^2 K_{ci}} - \frac{V_c \sin K_{ci}}{V_\rho \sin \alpha} \sqrt{D_n^2 - t_n^2 V_\rho^2 \sin^2 \alpha};
\end{aligned}$$

Comment:

$$\sin \alpha = \frac{V_c}{V_\rho} \sin K_{ci}, \text{ then } D_x = t_n \sqrt{V_\rho^2 - V_c^2 \sin^2 K_{ci}} - \sqrt{D_n^2 - t_n^2 V_\rho^2 \sin^2 \alpha};$$

Interval of target presence on target course line must satisfy by the next condition :  $D_x \leq D_s$ .

So we have making more precise original model [1] for general case. Quantitatively it expressing in insignificant increase square OWMC and approach  $K_{cmin}$  to nought. It is typically for pursuit situation.

**Conclusions.** Results of simulation for received analytic models allowing make next inferences:

- ◆ by decision multiplication tasks of search, for considerable rang of applied tasks will be correct using original model;
- ◆ for general case received analytic equations, making original model more precise;
- ◆ using making more precise model worth while for cases, when the speed of observer search more greater that target speed ( $V_n \gg V_c$ ) and the time of detection system reaction is insignificant.

## References

1. Попович В.В., Шпак В.Ф. Оценка эффективности комплексного применения средств обнаружения.- М.: "Красная звезда", "Морской сборник" N2, 1995г.
2. Попович В.В., Шпак В.Ф. Модель оценки эффективности комплексного применения средств обнаружения.- М.: "Красная звезда", "Морской сборник" N6, 1995г.
3. Попович В.В. Расчет суммарного поискового потенциала средств обнаружения.- М.: "Красная звезда", "Морской сборник" N6, 1995г.
4. Попович В.В., Левчин И.М. Введение в новый класс задач теории поиска подвижных объектов. UNESCO. Пятая Санкт-Петербургская международная конференция "РИ-96". Тез. докл., ч.2, стр.358.

# MODELING FORMATION PROCESSES OF INTERMITTENT TURBULENCE IN THE OCEAN

Anatoly Rodionov and Herman Papenko

St.Petersburg Scientific Centre of the Russian Academy of Sciences

Division for Applied Pilot Programms

5, University Emb.

St.Petersburg, 199034, Russia

Tel: (812) 218 26 12, E-mail: rodionov@wplus.net

**Abstract.** Theoretical-empirical (half-empirical) approach for modeling formation processes of intermittent turbulence in the ocean is proposed. Process of intermittent turbulence formation based upon developed approach is simulated.

**Keywords.** Intermittent turbulence, formation processes, hydrodynamic instability, wave disturbances, half-empirical approach.

**Introduction.** The ocean occupies more than 70 % of the Earth surface. From ancient times a man tries to master water. At the same time in hydrosphere there is significant number of "white spots".

One of effective ways of scientific and practical development of the ocean is modeling of processes in liquid environment of a planet.

- Modeling on the basis of the numerical or analytical solution of the equations of the mechanics of continuous environment;
- Modeling under laboratory conditions at observance of similarity criteria;
- Statistical modeling on the basis of known (from experiment or theoretical preconditions) distribution laws.

Specified methods, as a rule, give correct results only for the description of separate processes or phenomena, covering local spatial-temporary areas and proceeding without real interaction of components.

For the decision of many practically important problems it is necessary to receive information on complex processes and phenomena. Theoretical-empirical (half-empirical) approach for their modeling it is offered on the basis of the system analysis with allocation of basic elements, delimitation of free evolution, spatial - temporary areas of interaction and general cause-effects of relations. For the description of elements, laws of their free evolution and local acts of interaction methods of laboratory or numerical modeling can be effectively used. To describe of complex process as a whole methods of modeling on the basis of statistical or determined laws, characteristic for nature of conditions can be applied.

Process of intermittent turbulence formation on the basis of the developed approach in the work is simulated.

Intermittent turbulence, as a rule, forms in depth of the ocean under a seasonal layer of density jump and represents statistical set turbulence of areas or spots, between which environment is in a laminar state. In some of practical applications it is

necessary to have estimations of probabilities of formation and registration of turbulence anomalies. Let us construct mathematical models of these processes.

We shall consider ocean as a system of layers, each characterized by parameters critical to turbulence formation. Such parameters are Reynolds  $Re$  and Richardson  $Ri$  numbers, and also the thickness of layers. Let number  $Ri$  to be constant in limits of each of considered layer. We shall record in ocean some region with the characteristic size  $L_p$ . Restriction on the region size  $L_p$  is a condition of invariance of vertical structure of fields.

The basic mechanism of turbulence formation under a layer of jump is the mechanism of hydrodynamic instability of currents under action "unstable" wave disturbances. Let  $n$  to be average meaning of a stationary flow of wave disturbances, influencing on a considered layer, and  $n^*$  is "unstable" part of a flow. Let occurrence of unstable indignation results in turbulence formation and average time of its life equally  $t_*$ . Then the modeling of whole process is reduced to modeling a flow of wave disturbances and its "unstable part", mechanism of turbulence formation spots and their evolution with current of time. Schematically the process is shown on Figure 1.

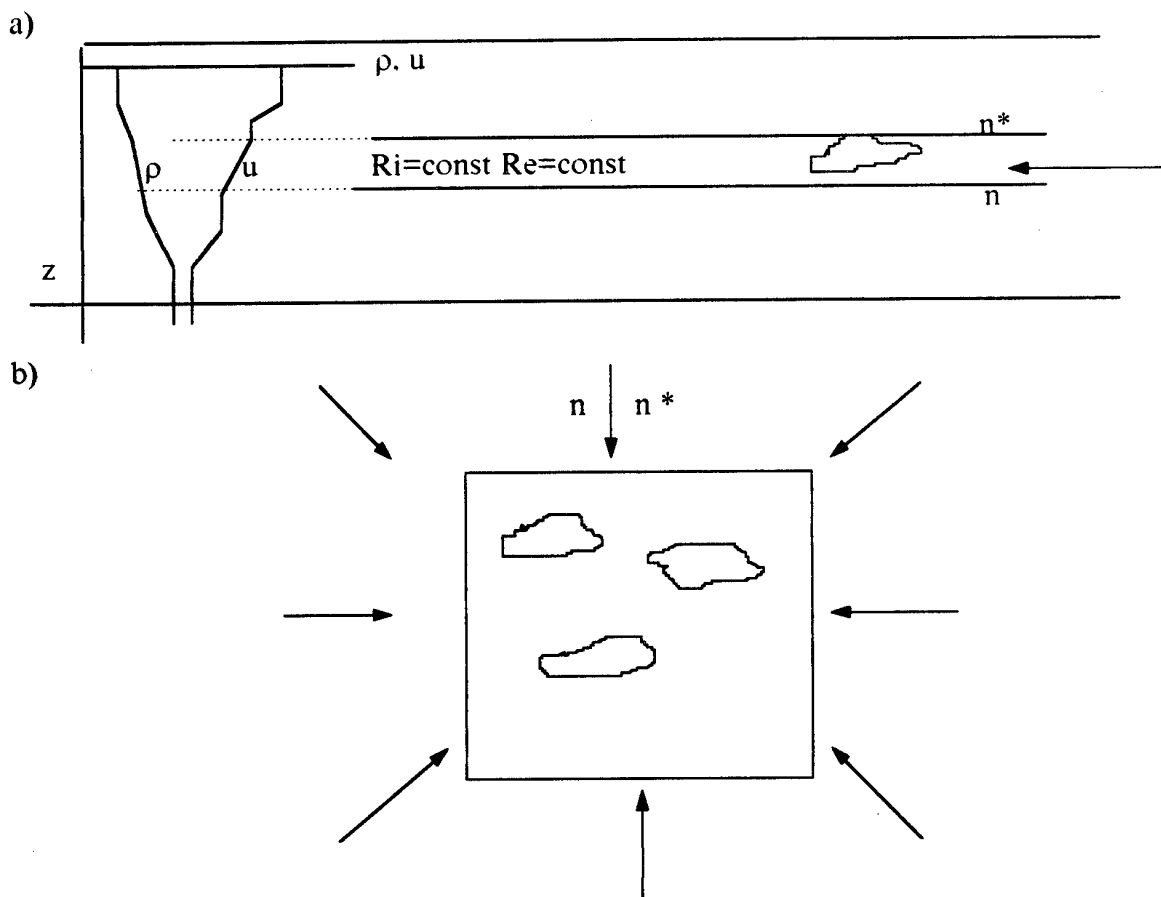


Fig. 1 The mechanism of formation intermittent turbulence under action of a flow of wave disturbances: a - circuit in vertical; b - circuit in horizontal for considered layers ( $\rho, u, z$  - density, current rate of environment and vertical coordinate).

The probability of turbulence anomalies in system in any moment with a point flow of disturbances and each "unstable" disturbance results in turbulence formation described by expression

$$P_T = \frac{n^*}{n^* + (t_k)^{-1}} \quad (1)$$

For Eq. (1) was supposed the time of life  $t_k$  on exponential law or evenly distributed in a range  $(0 \div 2t_k)$ . Eq. (1) according to results of the mass service theory is received. To find of estimation of probability of turbulence registration  $P_P$  it is necessary to relate  $P_T$  to a trajectory of movement of a measuring means (sensitive elements of gauges) through exited areas. Let the anomalies in considered region to be scattered by a casual image with average density  $\chi_T$  (average quantity of anomalies per unit of the area), and the typical cross size of anomaly has size  $l_T$ , then probability of crossing is equal  $m$  anomalies on the way  $L_P$

$$P_L = \frac{\exp\{-\gamma L_P\} (\gamma L_P)^m}{m!} \quad (2)$$

Where  $\gamma = \chi_T l_T$  - intensity of a flow of crossings of anomalies. From Eq. (2) probability of crossings though one anomaly on the way  $L_P$  follows, that

$$P_L = 1 - \exp\{-\chi_T l_T L_P\} \quad (3)$$

According to the theorem additinal of probabilities from Eq. (1), (2) for probability of anomalies registration we get

$$P_P = \frac{n^*}{n^* + (t_k)^{-1}} (1 - \exp\{-\chi_T l_T L_P\}) \quad (4)$$

We shall estimate size of a flow of "unstable" wave disturbances in considered region. It is known the overwhelming majority of disturbances of a wave nature in the ocean relates to superficial and internal waves (SW and IW). The superficial and internal waves, as a rule, are distributed as sections and have the determined vertical structure, fascinating rather extended in a vertical direction thickness of sea environment. Therefore the size  $n$  is determined by a flow of sections SW and IW.

Let  $\psi(k)$  to be a density of probability of supervision disturbances with wave number  $k$  from a range  $[k_i, k_{II}]$  ( $k_i, k_{II}$  - boundary meanings of a range of wave numbers observable in region SW and IW). For a case of a neutral stability curve of a parabolic kind we have

$$\begin{aligned}
 n^* &= \begin{cases} n \int_{k_1}^{k_2} \psi(k) dk & , \quad Ri \leq Ri_{kp} ; \\ 0 & , \quad Ri > Ri_{kp} ; \end{cases} \\
 k_1 &= \begin{cases} k_I & , \quad k_I > k_1 ; \\ \frac{1-2\sqrt{(Ri_{kp} - Ri)}}{L} & , \quad k_I \leq k_1 ; \end{cases} \\
 k_2 &= \begin{cases} k_{II} & , \quad k_{II} < k_2 ; \\ \frac{1+2\sqrt{(Ri_{kp} - Ri)}}{L} & , \quad k_{II} \geq k_2 ; \end{cases}
 \end{aligned} \tag{5}$$

Where  $k_1, k_2$  - border of a range of "unstable" wave numbers;  
 $L$  - thickness of a considered layer;  
 $Ri_{kp}$  - critical meaning  $Ri$ .

The question on a kind of function  $\psi(k)$  in depth of a considered layer is open at the moment. As a rule, the specific region of the ocean can be characterized only by a range of possible numbers  $k$ . Therefore let's assume, that  $\psi(k)$  - density of probability of normally distributed casual in a range of meanings  $[k_I, k_{II}]$ :

$$\psi(k) = \begin{cases} \frac{1}{k_I - k_{II}} & , \quad k \in [k_I, k_{II}] \\ 0 & , \quad k \notin [k_I, k_{II}] \end{cases} \tag{6}$$

In this case

$$\begin{aligned}
 n^* &= \begin{cases} \frac{4n\sqrt{(Ri_{kp} - Ri)}}{Lk_{II}} & , \quad \frac{k_I L}{2} \rightarrow 0 \text{ и } \frac{k_{II} L}{2} \geq 1 ; \\ \frac{4n\sqrt{(Ri_{kp} - Ri)}}{L(k_{II} - k_I)} & , \quad Ri \geq Ri_{max} ; \\ n \cdot \frac{k_{II} - (1-2\sqrt{(Ri_{kp} - Ri)})/L}{k_{II} - k_I} & , \quad Ri_{kI} < Ri < Ri_{kII} ; \\ n \cdot \frac{-k_{II} + (1+2\sqrt{(Ri_{kp} - Ri)})/L}{k_{II} - k_I} & , \quad Ri_{kII} < Ri < Ri_{kI} ; \\ n & , \quad Ri \leq Ri_{min} ; \\ 0 & , \quad Ri \geq Ri_{kp} ; \end{cases}
 \end{aligned} \tag{7}$$

$$\begin{aligned}
Ri_{\max} &= \max \{ Ri_{kI}, Ri_{kII} \} ; \\
Ri_{\min} &= \min \{ Ri_{kI}, Ri_{kII} \} ; \\
Ri_{ki} &= 0,5 k_i L (1 - 0,5 k_i L) ; i = I, II.
\end{aligned}$$

**Conclusions.** Calculations show, that the offered model satisfactorily describes the phenomena observable in the open ocean. It can be applied to estimations of intermittent turbulence parameters depending on the hydrophysical factors in a more general case with the account anisostrops of fields on the basis of modern computer technologies.

### **References**

1. Океанология. Физика океана. Т.2. Гидрофизика океана. - М.: Наука, 1978.- 483с.
2. Тернер Дж. Эффекты плавучести в жидкости. - М.: Мир, 1977.-432с.

# AUTOSYSTEM MARKETING RESEARCH: FORMATION AND UTILIZATION

Azarenko S.A.  
Naval Academy  
73/1, Vyborgskaya nab.  
St-Petersburg, Russia, 197045

Habibullin R.K., Chetvertakov M.M.  
1 st Naval Institute  
30, ul. Chapayeva  
St-Petersburg, Russia, 197061

**Abstract.** Main principles of building, structure and directions of usage of combined autosystem of marketing researches are being considered on an example of marketing of shipbuilding production.

**Keywords.** Autosystem, market, marketing researches.

**Introduction.** Experience of creation, development and utilization of multifunctional autosystems(1,2, etc.) testify that general architecture of any of them, even with different level of adequacy, reflects the object's structure or the object field of research. The success of creation and usefulness of such systems' utilization depends on realization of ideas and tasks of automatization objecta, process and methodological and practical regulation of ways of their achievement. The more authentic and definite are the laws working in this or that object system, the more evident will be the effect of creation indoctrination of problem oriented autosystems. Regarding this, we may say that marketing in Russia as an independent field of research and practical activity is on the stage of going. Scientific potential and foreign experience is mainly being used in this sphere. However, one may suggest that native theories and practical experience of creation autosystems to use in scientific and technical fields, together with the world experience of marketing research permit to formulate a row of methodological and applied principles of building autosystems of marketing research (ASMR).

Any autosystem is a reflection of our ideas about laws of existanse and functioning of subjects or objects of research in definite or changing due to known laws external conditions. Otherwise, such system is an informative technic-program model of objects, processes and fields of knowledge to research. One of the system-technical principles of formation such model is the principle of keeping the adequacy of its structure to processes, objects and their qualities to model. Therefore, it is necessary to have definite and clear idea about the objects of modeling, their qualities, environment and laws of functioning when forming ASMR. The second important principle is informative sufficiency of model and parametrical descriptions. The necessity of actualization only essentially important for concrete aims and tasks information is meant this, information must be sufficient to permit the consumer to analyse and get practical conclusions and recommendations.

Besides these general principles one may also name two more particular principles, which may be used in marketing research in the interests of firms - manufactures and which are carried out through methodology of system approach to creation of technical systems. Firstly, this is the principle of combination, meaning the combined analisation of consumers' features and production quality, technology of creation and securing of resources. Secondly, that is the principle of analyzing of production functioning during its full living cycle from appearing the need of it to utilization or liquidation. These two principles are mutually connected and dependent. Otherwise, it is necessary to analyse quantitative indexes of the quality of a production's item, technology of production or securing of utilization and necessary resourches (finansial, time, material, etc.) on all stages of the cycle.

Besides the principles mentioned above, the most important one is the principle of permanency of marketing research. In other words, collection, analyzing and generalization of information is a constant process and the effectiveness of it will in general define the firm's living capability in variable marketing conditions.

Considering these requirements and principles we will lean on the technological apparates and general thesis of a well-known monography of F.Kotlyar/3/, understanding marketing as a kind of human activity, aimed at satisfaction of needs by means of exchange. By marketing research we will consider the constant activity on collection,generalization and analysis of information about marketing situation with a goal to work out and to optimize the management decisions on formation of marketing strategy of a firm. Therefore, when forming the structure of ASMR one should foresee the blocks and modulus which permit to model the proper production being made or services, their outer environment both to them and to the consumer, the dynamics of cooperation between production and the product-maker and the external sphere, the aspects of creation and utilization of production directly with accounting of multimeanings of items of quality, necessary resource expenses.

The blocks and modulus of the system mentioned above are the informative surrounding of the functional system's core, which is the logical model of market. In a general case market is a combination of two multitudes: multitude of market participants and multitude of consuming functions. The last one consists of submultitude of consuming goods' qualities and multitude of required services. Participants of market are physical and juridical persons who can be manufactures, consumers or neither of them with respect to a concrete good or service.

This attitudes are being worked out in a form of tables of binar relations between these objects of three types:"consumer functions - goods (services)", "goods (services) - manufactures" and "goods(services) - consumers". For the realization of the first-type table for a definite limited market a matrix of relations between consumer functions and their substitutes on the market-goods or services provided is being built. Each column corresponds to an elementary consumer quality, and each line corresponds to a definite kind of a good(service). Moreover, each good(service) can fulfil more than one consumer function, but in the column of consumer functions there may be not any its fulfilling good(service). The two following types of the table are fulfilled like binar matrixes, in which the names of the participants (market-manufacturers) are written in the lines in one case, and of the consumers of

goods(services) in another. Elementary goods(services) are written in columns. If there is some kind of connection between a participant of the market and definite good(service), then into the proper element of matrix the Boolean value "true" is being put, and in case of this absence "false" is being put. The level and character of fulfillment of the tables can serve as a base for characteristics of definite markets and informative-logical base for the next analysis. Therefore ASMR must represent strong enough, but, at the same time accessible interface to work with these matrixes the logical core of the system. Such structure of the core may be fulfilled on the base of any program of shareable data base.

Working with logical tables of combination of goods, services and participants of the market is a preliminary macroanalyse to discover unemployed "places" of the market or to forecast the strategical directions of marketing policy. ASMR must represent a way for more detailed microanalyse of market situations to correct the current policy. Having this goal two blocks of informative-analytical support must come to life in the system: the block of data bases and System Control Data Bases and the block of models, including the block of statistics analysis and the block of models of analytical securing of taking the decisions.

The data block secures storing and updating of essential information on the participants of market. Within that the opportunity of their detailed structuring on criteria of their relations and intersection of their interests with their own firm (for example, suppliers, mediators, competitors, contact audiences, wholesale consumers, etc.) A list of structural information with quantitative and qualitative items of characteristics for every of the participants of the market may be associated with each of the participants of the market. The same descriptions are formed in the data block and for goods and services.

In the block of statistics analysis' models the mathematical methods of regression, correlation, factor analysis and if needed other methods are formed. The models of calculation of techno-economical items of production and services, depending on inner and outer conditions, are formed in the block of analytical securing of taking the decision. Here also are formed the models quantitative assessments of the level of realisation of consumers' functions and qualities of goods, with taking into account the opportunity of formation the criteria of assessments on several points. The group of models and methodics for calculation and foreseeing the prices, planning the advertising budget and other indexes, determined by particularities of definite goods and services is also formed in this block.

The structure and functions of the blocks of ASMR, mentioned above, is just a invariant base for any ASMR. Some definite structures of such systems will be depending on the specific spheres of systems, measures and particularities of wholesale markets (local, regional, international, global, etc.) As an example one can regard the particularities and additional subsystems and functions of ASMR in the sphere of international market of the shipbuilding production. Automation support must come to life by the system the spheres of sale of items already made, working out the aspect of perspective ones, forming the variety of goods, stabilize the advertising activity. According to these functions the following subsystems must be part of the structure of ASMR.

1. The subsystem of development of sale market which has to guarantee the process of marketing research in all stages, including analysis, observation and foreseeing the dynamics of the market. Modern mathematical methods which can form process of subjects of shipbuilding functioning with assessments of techno-economical effectiveness can serve as a base of special program ensuring. The results of researches must ensure to assess the general situation and perspectives of development of the economy of shipbuilding and countries-potential consumers and competitors, to foresee the development of adjacent and the need of potential consumers.

2. The subsystem of working out a basing of perspective goods of shipbuilding, which ensures the forming of requirements to the look of perspective export goods of shipbuilding and its parts (counter agent supplies). The requirements must correspond to the foreseeing results of marketing research. The subsystem is formed on the basis of methodical apparatus of research building of ships. permitting to hold various researches with assessment of techno- economical characteristics and usage of solving-optimization tasks methods. The results of researches must give way to indoctrination of the new production and and to update and modernize in time the production already being made.

3. The subsystem of forming the assortment which encures the selection of goods for a manufacturer to make and offer at the market. The methods of program and net planning may serve as a basis for the methodical apparates for they permit to form uncontradictory variants of programs on creation the samples on shipbuilding production, accounting state of the market and the perspectives of counter-agents' enterprizes. The results of research must permit forming the structure of assortment of goods being made and programs of their production for definite terms of time.

4. The subsystem of working out the advertising materials, which ensures the automatic working out of advertisement information on the products being made or to be made in the future in shipbuilding. Modern technologies for making videocomputer film with demonstration of goods' look, their mechanisms and ways of exploitation must form the base of program ensuring.

This variant of general conception and usage of autosystem of marketing research may come to life on the base of already existing and perspective program-technical ensuring. But the main problems will arise mainly on the stage of formation of a general structure and functions with their components. Together with this, sticking to the principles stated in this article will permit to decrease the risk of making strategic mistakes when creating ASMR, mistakes that can lead to the necessity of reobserving the conception system in the process of its creation. The consequences may be higher expenses and really lost time.

### **References**

1. Koriachko V.P., Kureychik V.M., Norenkov I.P. Theoretical bases of SAD. Moscow, Energoatomizdat 1987
2. Norenkov I.P., Manichev V.B. Bases of theory and design of SAD. Moscow, Vysshaya shkola 1990
3. F.Kotlyar. Bases of marketing. Moscow, Progress 1990

# APPLICATION OF MARINE HYDROGRAPHIC INFORMATION SYSTEMS FOR RESOURCE AND ECOLOGICAL PROBLEM'S SOLUTION

Arseny Mit'ko, Valery Mit'ko and Alexey Hvostov  
Electrotechnical University of St.Petersburg  
5, Professor Popov street  
St.Petersburg, 197376, Russia  
tel. (812) 239-94-89 fax. (812) 234-15-43  
e-mail: kvm @ respcs.spb.su.

**Abstract.** There are considering application's possibilities of Marine hydrographic information System for Detection and Identification of small dimensions objects and heterogeneities. It has importance for resource and ecological problem's solution.

**Keywords.** Information Systems, hydrography, convertional technologies, data bases.

**Introduction.** Marine Hydrographic Information Systems (MHIS) are used for military and economic problem's solution in accordance with its dual and convertional application's nature. Today it is become evidently in connection with changing of Navy's tasks and human interests growth to ocean resources [8,9]. The latter is conditional by proclamation XXI century as period of exploring and assimilation of world ocean for humanity.

But large-scale information possibilities of MHIS are used insufficiently at present. It can be explained by different reasons from Russia's economical difficulties to poor notification with regard to proposed investors for new research as far as the problems of conversion are concerned. Some illustrations are given below.

## Problems for solution with help of MHIS.

**Echological monitoring.** Underwater monitoring for solution of echological problems can include many directions from Detection and Identification of Buried Unexploded Ordnance (including mines), small dimensions objects to measurement of water medium parameters to obtaine data about it contamination [1,2]. Such problems can be solved by using special ships with its equipment, its information systems and information systems of the Administration of Navigation and Oceanography, The State Research Navigation-Hydrographic Institute, other organizations of St.Petersburg, where are concentrated main forces of Russian science - technical thought in this field.

**Guaranteeing for search and extraction of ocean's resources.** The problem includes many directions such as Detection and Localization of water and bottom soil ingomogenities, detection of ocean's biological masses, working out the methods of estimating the effectiveness and optimuzation of the systems of oceanographic aids for different users [4,6]. New possibilities are opening for such regions, that before have supposed by hard of accessible, as for example Arctic basin floor and Arctic ocean.

**Control for the state of underwater constructions and communication systems.** The problem includes methods of remote observation and testing for the state shuch construction as dikes, dam, underwater platforms and installation for the ocean bottom's drilling, underwater buildings for living and tourism, pipe-lines, bottom's

cables and other. Such problem takes place as example for St.Petersburg protective dam in the gulf of Finland.

***Guaranteeing for fundamental research of ocean.*** The problem includes methods of remote observation and testing for water, surface and bottom parameters, that determine many regularity of climate forming, extraordinary situations beginning, many global processes understanding. Largescale ocean's researches practically are impossible for alone State, to determine perspectives of International cooperation.

***Possibilities of MHIS for ocean problem's solution.*** Ship's information hydrographical systems, as instrument for researching the world ocean are the unique objects, that have large scale opportunities. It has on board many laboratories (up to 30) of different type, including hydrographical, hydrophysical, meteosinoptical, hydrological, biological and otherones, all that allow to produce precise complex measurement of water medium and it boundary parameteres, phisical fields depending on nature and human activity. Ship's technical base provides complete information cycle from data obtaining to processing, renresentation and documentation in suitable form for user.

The ships are equiped with sufficient number of arrangements, that allows it to work with different outboard arrangements on significant depths, including surface and bottom, peacing and selection of autonomous buoyancy stations for uninterrupted parameteres water medium registration.

The topical problems, that equipment of the ships permitted to decide last time were measuring of sea depth, collection and processing hydrological, gravimetrical and other information producing different works for limited amount of customers, though requirements for such type of works rapidly grow.

#### **Short description of MHIS sensor equipment.**

***Hydroacoustic systems.*** There are some types of sonar, as a MHIS sensor sets. First of all it is side scan sonar, based on shadow classification. The main parameters for sonar estimation are beamwidth, range resolution and signal-to-noise ratio. In addition to its sizes, weight and value have importance for different purposes of sea-bed topography, sea-bed classification. There are different systems that can be used for this purpose, based on multibeam and interferometric methods.

Theoretical and experimental investigations show advantages of complex signals application and sinthetic apperture method to solve problems of sea-bed mapping. Such application promises rang extending, signal-to-noise ratio and sonar range resolution increasing for complicated relieve sea-bed.

There were developed algorithmas for acoustical representation, measuring of sea-bed relieve and characteristics.

These algorithmas were realised with help of digital side scan sonar-hydrographic complex for sea-bed square filming (AHSAS-200, -1500, -5000). Construction of complexes provides automatization of measuring and permits express-processing and archivating of information with magnetic base [4]. Exploitation of complexes on research vessels showed usefulness of measuring data for sea-bed mapping.

The automated hydrographic complex for area surveying (ANSAS-200) is designed at base of the side scan sonar with a phase channel to look for objects on the sea bottom and also can make a bottom map in depth up to 200 m. The complex may do depth data at the real time and acoustical imaging simultaneously.

General components of the complex are:

- acoustical antennas on ship board or also on the towed fish;
- digital synthesizer of transmitted LFM pulses or tone signals;
- multichannel receiving system with digital echo-signal converter;
- computer system for data acquisition and processing IBM PC/AT type.

Basic characteristics are the following: swath width - up to 1200 m, range resolution- 0,1...0,75 m, antenna pattern - 1,5 x 60 Deg, measuring accuracy- 1 percent of depth, tow fish depth - up to 30 m, weight - 80 kg.

Complexes ANSAS-1500, -5000 possess similar characteristics for depth up to 1500 m and 5000 m with swath up to 9000 m and 20000 m, range resolution about 3 m.

Multibeam echosounder with attached set "Soilgraph" provides in common with digital map to determine stationary under water objects with high accuracy. There is bottom's characteristic can be determined if it is necessary. Information about bottom's soil can be obtained up to basalt.

There is sequentially scanning parametric sonar and synthetic aperture processing, which improves signal to reverberation level and spatial resolution.

***Magnetic Environmental Measurements.*** Non-acoustic methods to obtain underwater information include magnetic detection techniques. The objective of the magnetic measurements is the quantitative characterization of the magnetic environmental variability in selected shallow water zones. Investigations into the spatial and temporal variability of natural and man-made magnetic Ultra Low Frequency variations in coastal and deep waters permit observe perturbation of the magnetic field by a passing vessels and other anomalies.

***Gravity Environmental Measurements.*** Non-acoustic methods to obtain underwater information include gravity detection techniques. The objective of the gravity measurements is the quantitative characterization of the gravity environmental variability in selected shallow or deep water zones. Applications of gravity measurements are similar to magnetic ones.

***Hydrophysical Measurements.*** Such measurements include evaluation of chemical qualities, electrical conductivity, temperature, oxygen concentration, pH-concentration, radioactivity and many other parameters, determining ecological and resources conditions [1,3]. Advantages of MHIS consist in defining of ship's place position with high accuracy due the special navigation systems.

#### **Main ways and new possibilities of MHIS's using.**

There is the idea of organization of "Baltic international ecological and environmental center". This center supposes creation of general monitoring system, prognosing, prevention, liquidation of extraordinary situation's consequences of natural and technical birth. Training-testing, insurance centers, banks must be included into this system. Environmental monitoring system, as element of more high system's level, may be optimized so that other elements in common system. It permits to realise possibilities of its modular construction, gradual development and common presentation on regional and international market.

As one example of such system is the structure of the space-distributed environmental monitoring system, that was created on the base of Extraordinary Situations Prognosing Institute at St.Petersburg Electrotechnical University. It includes own sets, sets of dual and conventional application of Ministry defence,

interacting forces and means, as for example, boat "Echopatrol", building by shipbuilding company "Almaz" in St.Petersburg.

Structure includes:

- sensors or other checking devices of different physical fields and characteristics;
- devices of data readout and their transfer by communication and information exchange channels;
- informational and analytical centers for data acquisition and processing;
- interregional electronic mail-based data-based;
- mobile laboratories, working on regional and interregional levels;
- measuring stationary grounds, located in different regions.

Degree of cultivation for different information channels has different level and includes radiolocation sets, underwater acoustic sets, mobile laboratories, contact sensors devices, optical and infrared devices.

Information collection and processing center is created on base of St.Petersburg Electrotechnical University. There are acting structure of center, interaction of subsystems which is builded on base of information-situation matrix under different physical field for revelation such extraordinary situations as oil-product overflow, atmosphere soiling, radiation soiling, fires, floods, earthquakes, tsunamies, transport crashes. United Information System is first condition for successful action such subsystem, as MHIS.

Second condition is concluding in creation of data bank and ecological attestation for more important regions [5,10].

Successful action of united Information System must be scientifically guaranteed by State Research Navigation-Hydrographic Institute, that is subordinate to the chief Administration of Navigation and Oceanography of the MoD. The main subdivisions of the Institute provide main directions its activities:

- research and working out of new physical principles and know-how for creating the new perspective navigation-hydrographic aids, as well as the means of exploring the World Ocean;
- substantiation of proposals and development of projects concerning aids to navigation for the sea and oceanic water areas;
- working out the methods of estimating the effectiveness and optimization of the systems of meteorological and oceanographic aids for different users;
- coordination and scientific-technical substantiation of Research and Development (R & D) in the field of marine navigation, cartography, hydrography, oceanography and geophysics, writing the methodologic textbooks, guides and manuals, rules and instructions;
- substantiation and working out the proposals concerning the ecologic monitoring of water areas;
- substantiation and working out the proposals intended for national programs concerning the problems of exploring the World ocean;
- drawing up the proposals and programs of international cooperation in the field of navigation and hydrography.

Close working contacts are maintained with corresponding national and foreign services and departments [7,8]. Among them there are the International Marine Organization, the International Hydrographic Organization, International Association of Lighthouse Authorities, Inter-Governmental Oceanographic Commission of the UNESCO, the Hydrographic Society of St.Petersburg, the St.Petersburg department of the Russian Social Institute of Navigation.

The State Research Navigation-Hydrographic Institute is ready to discuss any proposals aimed at mutually beneficial cooperation in the field of R & D directed at development and creation of the new means of exploring the World ocean. Participation in all-Russian and International programs that correspond to the profile of the activity of the Institute and related problems is welcomed. Orders for information service are taken. Specialists interested in the above mentioned problems are invited to contribute to the magazine "Navigation and Hydrography".

**Acknowledgments.** Authors wish to thank the following persons who contributed to this article: research fellows and leaders of State Research Navigation-Hydrographic Institute, Administration of Navigation and Oceanography, Extraordinary Situations Prognosing Institute at St.Petersburg Electrotechnical University.

## References

1. Baljasnikov S.B., Yu.N. Zukov (1995) Metodology of obtaining and processing of information about water masses dynamic for admixture propagation modelling in sea medium. Thesis of Reports "Regional informatics-95", St.Petersburg, pp. 272-273.
2. Baljasnikov S.B., O.N.Turovcev (1994) Guaranteeing of sea echological monitoring. Thesis of Reports "Regional informatics-94", St.Petersburg, pp. 186-187.
3. Hvostov A.A., A.V. Mit'ko (1996) Information possibilities of ship "Adjaria" for underwater monitoring problems solution. Thesis of Reports "Conversional technology of hydroacoustics-96", St.Petersburg, pp. 26-27.
4. Kaevicer V.I. (1994) Application of complex signals for acoustical mapping of sea bottom. Thesis of Reports "Conversional technology of hydroacoustics-94", St.Petersburg, pp. 52-53.
5. Kiselev Yu.G., V.B.Mitk'o, V.V.Popovich (1994) SeismoArcticBank and its significance for practical hydroacoustical tasks in the Arctic ocean. Thesis of Reports "Regional informatics-94", St.Petersburg, pp. 169-170.
6. Kiselev Yu.G., V.B. Mitk'o (1994) The geological model of the Arctic basin floor in the Arctic ocean and main ways of its complication. In: Problems of Development of Marine Geotechnologies, information Science and Geology Abstracts, at the VI Interdepartmental Conference. St.Petersburg, pp. 55-56.
7. Kisilev Yu.G., V.B.Mitk'o (1994) International and National importance of Russian seismoacoustic-oceanographic research of Arctic Basin. Proceedings of the conference "Regional informatics-94", St.Petersburg, pp. 225-239.
8. Mitk'o V.B., S. Parchevsky (1996) International cooperation problems in hydroacoustics technology field. Thesis of Reports "Conversional technology of hydroacoustics-96", St.Petersburg, pp. 25-26.
9. Turovcev O.N. (1994) Development's ways of Information technologies, providing creation of means for ocean assimilation. Thesis of Reports "Regional informatics-94", St.Petersburg, pp. 171-172.
10. Vorobjov V.I., Yu.G.Kisilev, V.B.Mitk'o, V.A.Solodov (1995) Banks and Bases of Data about natural objects of earth, seas, oceans. Ideology. Strategy. Contraction System. Thesis of Reports "Regional informatics-95", St.Petersburg, pp. 276-277.

**CHAPTER XV:**

***ROBOT CONTROL***

THIS CHAPTER INCLUDES PAPERS  
PRESENTED AT THE CONFERENCE SESSION:

**ROBOT CONTROL**

Organized by: *Prof. Iliya V. Miroshnik,*  
*Prof. Felix M. Kulakov.*

# ASTATISM IN NONLINEAR CONTROL SYSTEMS WITH APPLICATION TO ROBOTICS<sup>1</sup>

Anatoli A. Pervozvanski, Leonid B. Freidovich

Saint-Petersburg State Technical University  
Department of Mechanics and Control Processes  
29 Politehnicheskaja str., 195251 St. Petersburg, RUSSIA  
E-mail: control1@citadel.stu.neva.ru

**Abstract.** The notion of astatism broadly used in classical linear control theory is extended to nonlinear systems. Some basic assertions concerning with the properties of astatic systems are presented. A special attention is paid to robust control problems of Lagrangian systems and robotic manipulators as a particular case. It is shown that PID control ensure robust stabilization of a desired position and tracking with a bounded error if the desired velocities are small enough.

**Keywords.** astatism, robot control, *PID* controllers.

**1. Introduction.** The astatism is an extensively used notion in classical linear theory. Astatic systems have zero steady-state error if disturbances are constant or tend to constant values. Moreover, for linear systems the astatism ensure a bounded reaction under any unbounded disturbances having bounded derivatives. In linear case the astatism conditions are very simple. It is necessary and sufficient to have stable transfer function with a numerator having zero root. It is of practical interest to show what is a nonlinear astatic system, to state some conditions ensuring the astatism and to clear up if the properties of linear astatic systems are saved in nonlinear case. The paper gives some answers for these questions. Moreover, it is demonstrated that the general results can be used for the explanation of robustness of PID feedback in non-linear Lagrangian systems and in robotics especially.

**2. Astatism and boundedness of reactions.** Let a system be presented in the form

$$\dot{x} = f(x, w(t)), \quad x(0) = x_0; \quad y = g(x, w(t)) \quad (1)$$

where  $x$  is the state vector,  $y$  is the output vector,  $w(t)$  is the vector of disturbances (inputs), and  $f(x, w)$  satisfies conditions ensuring existence and uniqueness of solutions to (1) under any admissible  $w(t) \in W$ ,  $t \in [0, \infty)$ .

**Definition.** The system (1) is **internally astatic** if it has an isolated equilibrium point  $x = 0$  and this equilibrium is uasymptotically stable under any  $w(t) = w = \text{const}, w \in W$ .

The system (1) is **input-output astatic** if it has an asymptotically stable solution  $x = x_\infty(w)$  under any  $w(t) = w = \text{const}, w \in W$  and

$$g(x_\infty(w), w) = 0.$$

---

<sup>1</sup>This work was partially supported by INTAS, 94-965 and by RFBR, 97-01-01159

Let  $f$  and  $g$  be linear in  $x$  and  $w$ , or more precisely, let

$$f(x, w) = Ax + Bw, \quad g(x, w) = Cx + Dw$$

where  $A, B, C, D$  are constant matrices. Then the definition presented above coincides with the definition of classical linear theory (see, f.e. [1]) and the astatism conditions are equivalent the following ones:

- $A$  is stable (Hurwitzian),
- $A^{-1}B = 0$  (for internal astatism) or  $D - CA^{-1}B = 0$  (for input-output astatism).

However, if  $f$  is linear in  $x$  only, i.e.

$$f(x, w) = A(w)x + B(w), \quad (2)$$

then the astatism conditions have to include the stability of  $A(w)$  under all  $w \in W$ , i.e. the parametric robustness condition.

It is clear now that the generalized notion of astatism introduced above is much more sophisticated than the traditional notion of linear theory.

It is evident that if the linearized system having structure of right-hand side in the form (2) (with  $A(w) = \frac{\partial f}{\partial x}(0, w)$ ,  $B(w) = \frac{\partial f}{\partial w}(0, w)$ ) is internally astatic then the original non-linear system (1) is internally astatic too.

The basic problem under consideration is the following one. Let the system (1) be internally astatic. Is it true that the output  $y(t)$  is bounded under any  $w(t) \in W$  which are unbounded but have a bounded derivative?

**Theorem 1<sup>2</sup>.** *Let the system (1) be internally astatic on a set of time-constant disturbances  $w \in W$  and moreover, there exists a Lyapunov function  $V(x, w)$  such that*

$$V(x, w) \rightarrow \infty \quad \text{under} \quad \|x\| \rightarrow \infty \quad \text{for any} \quad w \in W \quad (3)$$

*and its time derivative  $\dot{V}$  defined on trajectories of the system (1) under  $w = \text{const}$  satisfies the conditions*

$$\dot{V} \leq -\beta\|x\|^2, \quad \left\| \frac{\partial V}{\partial w} \right\| \leq \gamma\|x\|^2 \quad \text{under} \quad \|x\| \leq R, \quad w \in W \quad (4)$$

*Let for any  $\delta < \beta\gamma^{-1}$*

$$\|\dot{w}(t)\| \leq \delta, \quad w(t) \in W \quad (5)$$

*then there exist  $R_0 > 0$  such that*

$$\|x_0\| \leq R_0 \implies \lim_{t \rightarrow \infty} \|x(t)\| = 0.$$

---

<sup>2</sup>A similar assertion is contained in [2, Ch.5]

**Theorem 2.** Let  $x = 0$  be asymptotically stable equilibrium of the system (1) under any constant  $w \in W$  and moreover there exists a Lyapunov function  $V(x, w)$  satisfying (3) (i.e. radially unbounded) and

$$\dot{V} \leq -Q(x), \quad \text{under } \|x\| \leq R$$

where  $Q(x) > 0, \|x\| \neq 0, Q(0) = 0, \frac{Q(x)}{\|x\|} \rightarrow \infty$  under  $\|x\| \rightarrow \infty$ , and

$$\left\| \frac{\partial V}{\partial w} \right\| \leq c\|x\|, \quad c = \text{const} > 0, \quad \text{under any } w \in W \quad (6)$$

Then, under any  $\Delta > 0$ , one can show  $\delta > 0$  and  $R_0 > 0$  such that

$$\overline{\lim}_{t \rightarrow \infty} \|x\| \leq \Delta$$

if  $w(t)$  satisfying (5) and  $\|x_0\| \leq R_0$

The Theorem 2 gives some sufficient conditions under which the astatism yields a boundedness of reactions if the disturbances have a bounded rate of changements. Naturally, the conditions are much harder and "more local" then in the linear case.

Note that the conditions of theorems 1,2 concerning with a "regular" behavior of Lyapunov functions in  $w$  are nontrivial.

Let  $f(x, w)$  be continuously differentiable in  $x$  and  $w$ , and uniformly Lipschitzian in  $x$  under all  $w \in W$ . Let the system (1) have an asymptotically stable solution under  $w = \text{const} \in W$ . Then following Massera theorem (see, f.e. [3]), one can state only that there exists a smooth Lyapunov function  $V(x, w)$  such that under  $\|x\| \leq R$ :

$$a(\|x\|, w) \leq V(x, w) \leq b(\|x\|, w), \quad \dot{V}(x, w) \leq -c(\|x\|, w),$$

$$\left\| \frac{\partial V}{\partial x} \right\| \leq d(w), \quad \left\| \frac{\partial V}{\partial w} \right\| \leq e(w),$$

where  $a, b, c$  are smooth positive anywhere (except zero  $x$ ) functions monotonically increasing with  $\|x\|$  (i.e. class  $\mathcal{K}$  functions) and  $d, e$  are non-negative smooth function. Sometimes one can show a limited upper bounds which are independent on  $w$ ; and it is possible not only for a bounded  $W$ . In those cases any solutions to (1) are **uniformly ultimately bounded**, i.e.  $x(t)$  converges in a ball if  $\|\dot{w}\|$  and  $\|x_0\|$  are small enough. In other terminology the system (1) is **dissipative**. Proof of this assertion is analogous to the proof of Malkin theorem on dissipativity under nonvanishing bounded disturbances (see, f.e. [3]).

**3. Astatic servomechanisms.** In many cases the description (1) does not allow to express completely the disturbance influence on the system behavior. In particular, it takes place in the tracking problems where a desired trajectory plays role of a disturbance, and zero value of tracking errors is the desired equilibrium point (or it is required to ensure uniform ultimate boundedness of those errors).

Let, f.e., it be desired that the state  $x(t)$  of the closed-loop system

$$\dot{x} = F(x, u), \quad u = k(x, x_d),$$

where  $k(x, x_d)$  is a static feedback control law ( $F(x_d, k(x_d, x_d)) \equiv 0$ ), converges to a differentiable process  $x_d(t)$ , i.e. the output  $y(t) = x(t) - x_d(t)$  tends to zero. Denoting

$$y = x - x_d \quad \text{and} \quad f(x, x_d) = F(x, k(x, x_d))$$

one obtain

$$\dot{y} = f(y + x_d, x_d) - \dot{x}_d$$

If one return to the symbols introduced in the astatism definition, i.e. ( $y \rightarrow x$ ,  $x_d \rightarrow w$ ), one has

$$\dot{x} = f(x + w, w) - \dot{w} \quad (f(w, w) \equiv 0)$$

One can see now that the right-hand side depends not only on  $w$  as in (1) but on  $\dot{w}$  also.

Note that the output coincides here with the state of the system and hence the notion of internal astatism coincides with the notion of input-output astatism.

In more general case if one desires to estimate the tracking errors for the closed-loop system described by the differential equations of  $n$ -th order

$$x^{(n)} = F(x, \dots, x^{(n-1)}, k(x, \dots, x^{(n-1)}, x_d)) \equiv f(x, \dots, x^{(n-1)}, x_d)$$

one has to consider behavior of solutions to the equations of the following type

$$x^{(n)} = f(x + w, \dots, x^{(n-1)} + w^{(n-1)}, w) - w^{(n)}$$

where  $x, w$  stand now for the errors and the desired trajectory.

The examples shown above demonstrate that the generalized description

$$\dot{x} = f_0(x, w) + f_1(x, w, w', \dots, w^{(k)}) \quad (7)$$

where  $f_0(0, w) \equiv 0$  and  $f_1(x, w, 0, \dots, 0) \equiv 0$  be useful in applications.

Note that the astatism property is defined by  $f_0(x, w)$  only. However the system reaction depend on  $f_1$  too.

**Theorem 3.** *Let the system*

$$\dot{x} = f_0(x, w) \quad (8)$$

*be astatic and moreover there exists a radially unbounded Lyapunov function  $V(x, w)$  such that*

$$\left\| \frac{\partial V}{\partial x} \right\| \leq c_x \|x\|, \quad \left\| \frac{\partial V}{\partial w} \right\| \leq c_w \|x\|, \quad w \in W, \quad \|x\| \leq R$$

*and its time derivative along trajectories of the system (8) under  $w = \text{const} \in W$  satisfy*

$$\dot{V} \leq -Q(x), \quad \text{under } \|x\| \leq R$$

*where  $Q(x) > 0$ ,  $\|x\| \neq 0$ ,  $Q(0) = 0$ ,  $\frac{Q(x)}{\|x\|} \rightarrow \infty$  under  $\|x\| \rightarrow \infty$ .*

*Let  $f_1$  be Lipschitzian in all arguments.*

Then there exist  $\delta > 0, R_0 > 0$ , such that if

$$\|w', \dots, w^{(k)}\| \leq \delta, \quad w(t) \in W, \quad \|x_0\| \leq R_0,$$

then for any solution to the system (7)

$$\overline{\lim}_{t \rightarrow \infty} \|x(t)\| \leq \Delta,$$

under any given  $\Delta > 0$ .

**Remark.** If for any  $w \in W$  and  $\|x\| \leq R$ :

$$\|f_1(x, w, w', \dots, w^{(k)})\| \leq (L + \beta_2 \|x\|) f_2(w', \dots, w^{(k)}),$$

where  $f_2$  is a function such that there exists a continuous function  $\bar{f}_2(\delta)$  vanishing under  $\delta \rightarrow 0$  and satisfying the inequality

$$\|w', \dots, w^{(k)}\| \leq \delta \implies \|f_2(w', \dots, w^{(k)})\| \leq \bar{f}_2(\delta),$$

$f_1(x, w, \dots, w^{(k)})$  is not necessary to be Lipschitzian, and  $Q(x) = \beta \|x\|^2$ , then  $\overline{\lim}_{t \rightarrow \infty} \|x(t)\| \leq \Delta = \frac{\bar{f}_2(\delta)L}{\beta - \beta_2 \bar{f}_2(\delta)} \rightarrow 0$  under  $\delta \rightarrow 0$ . This remark was used in [4].

**4. Integral feedback and astatism.** Consider the following system

$$\dot{z} = f(z, u, w), \quad z(0) = z_0 \quad (9)$$

where  $u$  is a control. Choose  $u$  in the form of integral feedback

$$\dot{u} = -\mu \psi(z); \quad u(0) = 0 \quad (10)$$

where  $\mu > 0$ ,  $\psi(0) = 0$ ,  $z^T \psi(z) \neq 0$  under  $z \neq 0$ .

Under  $w = \text{const}$  the closed-loop system (9), (10) has the unique equilibrium

$$z = 0, \quad u = u_\infty(w) = \text{const}.$$

The system is astatic if the equilibrium is asymptotically stable. Let under zero control and  $w = \text{const} \in W$  the system (9) have an asymptotically stable solution

$$z = z_\infty(w), \quad f(z_\infty(w), 0, w) \equiv 0.$$

The integral feedback (10) is introduced to shift that equilibrium to the desired position  $z = 0$  independently on any  $w = \text{const} \in W$ .

First of all, as in the linear theory, one has to know if the stability is not destroyed.

**Theorem 4.** Let  $f$  and  $\psi$  be twice differentiable functions. Let, under any  $w \in W$  and  $u \in U$ ,  $U$  be bounded and the equation

$$f(z, u, w) = 0$$

have a root  $z_\infty(u, w)$  which is locally exponentially stable equilibrium of the system (9) under  $w = \text{const} \in W$ ,  $u = \text{const} \in U$ . Let the system

$$\dot{u} = -\mu \psi(z_\infty(u, w)) \quad (11)$$

have a locally exponentially stable solution.

Then there exists  $\bar{\mu} > 0$ , such that under any  $\mu \in (0; \bar{\mu}]$ , the closed-loop system (9), (10) is input-output astatic with input  $w$  and output  $z$ .

**Proof** to the theorem 4 is based on classical results by Tikhonov (see, f.e. [5, 1]) and Klimushev [6]. In fact, let us introduce a "slow" time  $\tau = \mu t$ . Then (9), (10) can be rewritten in a singular perturbed form

$$\frac{du}{d\tau} = -\psi(z), \quad \mu \frac{dz}{d\tau} = f(z, u, w) \quad (12)$$

Under a small  $\mu$ , the control  $u$  is a "slow" variable in reference to the object state  $z(t)$ . The system (11) is the reduced one in reference to (12) and it gives an appropriate description of the "slow" variables. The conditions of the Theorem ensure that existence of the exponentially stable solution to the reduced system yields the convergence of  $u(t)$  defined by the original system to a constant value. However, due to the properties of  $\psi(z)$  it is possible only if  $z(t) \rightarrow 0$ .

**Remark 1.** If the system (9) has globally asymptotically stable and locally exponentially stable equilibrium (i.e. globally asymptotically stable hyperbolic equilibrium) under  $u = \text{const} \in U$  and the system (11) also has globally asymptotically stable and locally exponentially stable solution under  $w = \text{const} \in W$  then the closed-loop system (9), (10) has globally stable equilibrium under positive  $\mu$ , small enough. It follows from Hoppensteadt theorem [7].

**Remark 2.** Under some properties of the functions  $f, \psi$  ensuring the uniqueness of zero equilibrium point, the condition  $z^T \psi(z) \neq 0$  under  $z \neq 0$  may be omitted. Moreover all results are true if  $f, \psi$  smoothly depend on  $\mu$ .

**5. Astatism of controlled Lagrangian system and a slow tracking.** Consider a dynamical system described by the Lagrangian equation

$$A(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = Q \quad (13)$$

where, by definition,  $q$  is  $n$ -vector of generalized coordinates,  $A(q)$  is a positive definite inertia matrix,  $C(q, \dot{q})\dot{q}$  is a vector of centrifugal and Coriolis forces such that

$$\dot{A}(q) = C(q, \dot{q}) + C^T(q, \dot{q})$$

and  $g(q)$  is a vector of gravitational forces,

Let

$$Q = w + v$$

where  $v$  is a control and  $w$  is an external disturbance. Introduce a proportional-differential (PD) feedback, i.e.

$$v = -K_p(q - q_d) - K_v\dot{q} + \dot{q}_d \quad (14)$$

where  $K_p, K_v$  are positive definite matrix of gains and  $q_d = \text{const}$  defines a desired position.

Under  $\hat{u} = \text{const}$ , one can show [8] that the system (13), (14) has globally asymptotically stable (GAS) equilibrium  $q = \bar{q}$ ,  $\dot{q} = 0$ , satisfying the condition

$$g(\bar{q}) = w + \hat{u} - K_p(\bar{q} - q_d) \quad (15)$$

if only

$$K_p > \alpha I, \quad (16)$$

where  $\alpha$  is a Lipschitz constant for  $g(q)$ .

Introduce now  $\hat{u}$  as an additional integral feedback

$$\dot{\hat{u}} = -F(u), \quad \dot{u} = \mu(q - q_d) \quad (17)$$

where  $F(u) = [F_1(u_1), \dots, F_n(u_n)]^T$  and  $F_i(u_i)$  are twice continuously differentiable functions such that

$$\begin{aligned} u_i F_i(u_i) &> 0 \quad \text{under } u_i \neq 0, \quad i = 1 \dots n \\ |F_i(u_i)| &\geq \bar{F} \quad \text{under } |u_i| > \bar{U} \quad \text{and} \quad \frac{dF_i(u_i)}{du_i} > 0 \quad \text{under } |u_i| \leq \bar{U} \\ \bar{F} &> \max_{1 \leq i \leq n} \{|w_i| + |g_i(q_d)|\}, \quad w \in W \quad \forall i = 1 \dots n \end{aligned} \quad (18)$$

**Theorem 5.** Under the conditions (16), (18) and  $\mu > 0$  small enough the closed-loop system (13), (14), (17) has GAS equilibrium  $(q, u) = (q_d, u_\infty) \equiv (q_d, -F^{-1}[g(q_d) - w])$ .

The result follows from the Theorem 4 and Remark 1 if one uses special types of Lyapunov functions (energy like one as in [9, 10, 11] and Lur'e - Postnikov like one:  $[V(u, w) = \int_0^{u-u_\infty} \{F(s + u_\infty) - F(u_\infty)\}^T ds]$ ) for the fast and reduced systems corresponding to (13), (14) and (17) with  $q = \bar{q}$  given by (15) and for their first approximation systems.

It proves the astatism of the Lagrangian system with PID feedback in reference to constant external forces.

For the particular case of linear feedback that result was shown early in [12].

Using the Theorem 3 and the Lyapunov function presented in [4] one can show now that PID feedback ensures a tracking of a desired trajectory  $q_d(t)$  with a bounded error if  $q_d(t)$  is changed slowly enough.

**Theorem 6.** Let  $q_d(t)$  be twice differentiable and there exist constants

$$a_1 > 0, \quad \alpha, a_2, c_1, c_2, d_1, d_2 \geq 0$$

such that for all  $x, y, z \in \mathbb{R}^n$

$$\begin{aligned} a_1 I &\leq A(x) \leq (a_2 + c_2 \|x\| + d_2 \|x\|^2) I, \\ \|C(x, y)z\| &\leq (c_1 + d_1 \|x\|) \|y\| \|z\|, \quad \|g(x) - g(y)\| \leq \alpha \|x - y\|; \end{aligned}$$

and either the desired motion is bounded or  $d_1, c_2, d_2 = 0$ .

Moreover, let there exist  $\varepsilon > 0$  and  $R > 0$  such that

$$K_v > \varepsilon \max_{\|x\| \leq R} \{A(x + q_d) + (c_1 + d_1 \|x + q_d\|) \|x\| I\}, \quad \varepsilon(K_p - \alpha I) > K_i > 0$$

and there exist  $\delta_1, \delta_2 > 0$  such that

$$\|\dot{q}_d\| \leq \delta_1, \quad \|\ddot{q}_d\| \leq \delta_2$$

Then all solutions to the system (13), (14), (17) are uniformly ultimately bounded.

**6. Conclusion.** The results presented in the paper allow to extend the useful notion of astatism to nonlinear systems. It is necessary to have in mind that in non-linear systems the astatism does not ensure a boundedness of tracking errors if the desired velocities are very large. However, some numerical simulations for robotic manipulators show that the upper level of admissible velocities may have practically reasonable high value.

## References

1. Pervozvanski, A. A. (1986). Theory of automatic control. Nauka, Moscow. (In Russian).
2. Khalil, H. W. (1992). Nonlinear systems. Macmillan, New York.
3. Rouche, N., P. Habets, M. Laloy (1977). Stability theory by Liapunov's direct method. Springer-Verlag, New York.
4. Freidovich, L. B., A. A. Pervozvanski (1997). Some estimates of performance for PID-like control of robotic manipulators. Prepr. of IFAC Symposium on Robot Control. Nantes.
5. Vasil'eva, A. B., V. F. Butuzov (1973). Asymptotic expansion of solutions to singularly perturbed equations. Nauka, Moscow. (In Russian).
6. Klimushev, A. I. (1973). Stability of nonlinear systems of differential equations with small parameter multiplying the derivatives. Trudy Ural'skogo Politekh. Inst., (211), 44-54. (In Russian).
7. Hoppensteadt, F. (1974). Asymptotic stability in singular perturbation problems. II. J. Diff. Equat., 15 (3), 510-521.
8. Tomei, P. (1991). Adaptive PD Controller for Robot Manipulators. IEEE Trans. Robot. Autom, 7 (4), 565-570.
9. Pozharicki, G. K. (1961). On asymptotic stability of equilibria and stationary movements of mechanical systems with partial dissipation. Prikl. Mat. & Mekh., 25 (4), 657-667. Engl. transl. in Appl. Math. & Mech.
10. Salvadori, L. (1966). Sull'estensione di sistemi dissipativi del criteris di stabilita del Routh. Recherche Mat., 15, 162-167.
11. Takegaki, M., S. Arimoto (1981). A new feedback method for dynamic control of manipulators. J. of Dyn. Syst., Meas., and Control, 102, 109-125.
12. Burkov, I. V., A. A. Pervozvanski, L. B. Freidovich (1996). Global asymptotic stabilization of Lagrangian systems via PID regulators. Abst. of 4th Intern. Seminar "Stability and oscillations of nonlinear control systems", 50, Moscow.

# COMPARATIVE STUDY OF METHODS FOR ENERGY-OPTIMAL GAIT GENERATION FOR BIPED ROBOTS

L. Roussel, C. Canudas-de-Wit and A. Goswami\*

Laboratoire d'Automatique de Grenoble  
UMR-CNRS 5528, ENSIEG-INPG  
B.P. 46, 38402, Grenoble, France  
e-mail: roussel@lag.ensieg.inpg.fr

\* INRIA Rhône-Alpes, ZIRST, 655 ave. de l'Europe  
38330 Montbonnot St Martin, France

**Abstract** In this paper we compare three methods for the energy optimal gait generation for biped robots during the single support phase. The first approach searches for unconstrained trajectories generated by piecewise constant inputs, the second approach constrains the Cartesian trajectories of the swing foot and the hip of the robot to a class of time-polynomial functions, and the last method approximates the robot joint trajectories by a truncated Fourier frequency series. Using a simplified robot dynamics that ignore the centripetal and Coriolis terms, these methods are compared according to the input energy and the initial mechanical energy. The numerical study presented here shows that for an equivalent amount of computational burden, the unconstrained method provides motions with the lowest input energy. Furthermore, it also provides the initial velocities that generate ballistic motions with almost zero input energy.

**Keywords** Biped robot, energy optimal gait, piecewise-constant control.

## Introduction

Recently, many studies have been devoted to locomotion, path planning and control of biped robots [1]-[8]. The main motivations for using walking robots rather than the more conventional wheeled robots for certain tasks are their locomotion capability on irregular surfaces and their versatility in negotiating larger obstacles.

Since mobile robots need to carry their own energy source a lower rate of energy consumption would directly contribute to a longer work cycle. In particular, the current generation legged robots require more control energy than the comparable wheeled robots and this is an important issue to be resolved before the use of legged robots is practically viable. So, the interest of characterizing low energy trajectories appears natural [1] [2] [3] [11]. It is not too unreasonable to expect that there are locomotion gaits for biped robots that consume very little energy. This is reinforced by the biomechanical analysis of natural human gait [10] and practical [9] and numerical [7] results of simple passive biped robot models that can walk down unaided on an inclined slope.

The problem of finding these low energy gaits for a more complex robot model is not trivial and only a few partial analytical results are available up to now, see [6]. Only few papers deal with the energy optimal gait generation while time optimal problem is more commonly treated. Previous works searching for numerical solutions were based on time-polynomial approximations [2] [3], or Fourier expansions [1], or on a combination of both [11]. A nice and quite complete treatment of the application of the optimal programming

to human locomotion is given in [5], where penalty functions are used to minimize the total mechanical work done. This technique is now superseded by more recent numerical optimization algorithms. It can be noticed that the concatenation of several phases, with impulsive loads, makes the optimal control problem even harder.

In [4], look for ballistic trajectories through impulsive control which brings the system to required initial conditions to perform the natural gait. This impulsive control is supposed to be instantaneous and it is applied during the impact phase.

In this paper we propose an alternative method to generate ballistic motion of the biped by supposing that a control exists all through during the motion (as opposed to the methods of [4] [8]). The optimization process generates the initial and terminal velocities that correspond to the minimum-energy motions. This approach is compared to the two previously proposed methods based on an optimization over a restricted class of joint trajectory functions.

## Problem formulation

The complete dynamics of a biped robot faithfully mimicking the human gait will be rather involved. The human gait cycle is divided into two phases: the single support phase or the swing phase (one foot on the ground and the other foot swinging) and the double support phase (both feet on the ground). The transition from the single support to the double support phase, also called the *contact phase*, is associated with the heel of the front foot impacting with the ground. The transition from the double support to the single support phase of the next step, also called the *take-off phase*, is caused when the toe of the rear foot leaves the ground. The dynamic equations of a robot consisting of all the described phases is composed of ordinary differential equations for swing stage and algebraic equations for the transition stages (the latter are usually modeled as instantaneous phenomena). Moreover, the topology of the kinematic chain making up the robot changes from the single support to the double support phase complicating further the differential equations.

It is not an easy task to choose a kinematic model for the biped that captures the essence of the anthropomorphic gait while keeping the model reasonably simple to allow intuitive insights about its behavior. Admitting the fact that the simplifications may sacrifice some of the subtleties of human motion, we have converged upon a planar four degrees-of-freedom (DOF) biped mechanism as shown in Figure 1. We have assumed that in this model the trunk will be upright during the walk. This seems reasonable because the trunk's maximal excursion from the vertical axis is about 20mm at the pelvis point, as reported in [11]. We will consider that the center of mass of the trunk is located at the hip, that is at the center of rotation of the third joint.

The foot of the swing leg is considered massless thereby obviating motor in the swing leg ankle. This guarantees that our robot model always has 4 actuated joints. In spite of this commonly used assumption (see [3] and [11]) which substantially simplifies the model, foot articulations (tarsus and metatarsus) play an important role during the transition phase in generating and absorbing a significant amount of energy.

The  $n$ -DOF model biped robot is obtained by the Euler-Lagrange equations:

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{u} \quad (1)$$

where the vector  $\mathbf{q} \in R^n$  describes the generalized joint coordinates,  $\mathbf{H}(\mathbf{q})$  is the inertia matrix,  $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$  is the matrix of centripetal acceleration and Coriolis terms,  $\mathbf{g}(\mathbf{q})$  is the gravity vector, and  $\mathbf{u}$  is the input torque vector.

From the energy point of view, the forces due to variation of  $\mathbf{H}(\mathbf{q})$  are workless and hence they contribute very little to changes in the energy levels since they have no impact

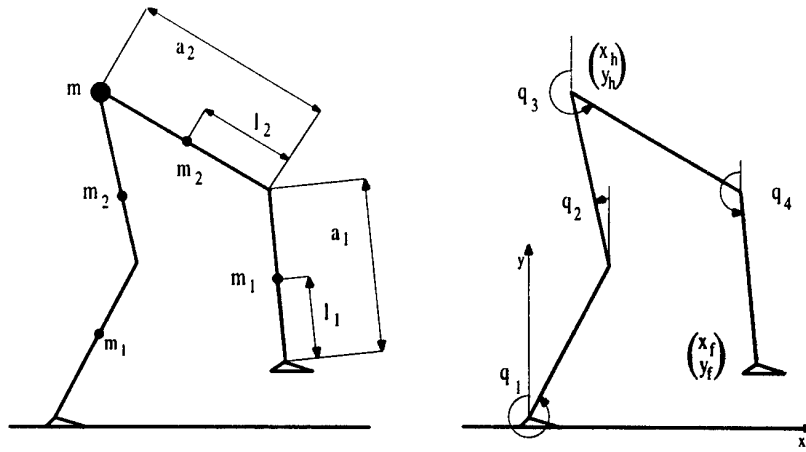


Figure 1: Simplified structure of the 4-DOF biped robot used for our study.

on the time variation of the system Hamiltonian (i.e.  $\dot{q}^T \left\{ \frac{1}{2} \dot{H}(q) - C(q, \dot{q}) \right\} \dot{q} = 0$ ). They will then not be taken into account in our study. This assumption is reinforced by the fact that gear ratios of the D-C actuators are large enough so that coupling and position dependent terms of the inertia matrix can be ignored. The main nonlinearities considered by our model are the gravity term. The simplified biped dynamics is:

$$H\ddot{q} + g(q) = u \quad (2)$$

where  $H$  is a diagonal constant matrix.

The robot dynamics (2), can also be expressed in a state-space description:

$$\dot{x} = f(x, u) \quad t \in [0, T] \quad (3)$$

where  $x = [x_1^T, x_2^T]^T$ , the state vector, is composed of the position vector,  $x_1$ , and the velocity vector  $x_2$ . Biped motion should be defined within an admissible set of joint positions  $\Omega_x$ :  $x_1 \in \Omega_x$  that describes a region in the joint configuration space, where some physical restrictions are imposed (i.e. legs should not cross the ground level, avoid singular configuration, etc.). The specified boundary conditions for the optimization are the initial and the terminal joint angles,  $x_1(0)$  and  $x_1(T)$ , respectively, and the time interval of the swing phase  $T$ . The initial and final velocities  $x_2(0)$  and  $x_2(T)$  are free however. They represent an additional degree of freedom giving the possibility for the optimization procedure to generate low-energy trajectories (with zero cost).

For optimization we will use the following cost criterion:

$$J = \int_0^T u^T u dt \quad (4)$$

the minimization of which can be shown to be equivalent to the minimization of the injected energy to the robot, other losses are neglected.

We should also define the admissible set  $\mathcal{U}$  of feasible control  $u(t)$  which defines a class of bounded signals with finite energy in the time interval  $[0, T]$ :  $u \in \mathcal{U} \triangleq \mathcal{L}_2^e \cap \mathcal{L}_\infty^e$  where  $\mathcal{L}_2^e$ , and  $\mathcal{L}_\infty^e$  stand for the extended  $\mathcal{L}_2$ , and  $\mathcal{L}_\infty$  spaces, respectively.

**Problem 1** Given the initial and final joint angles  $\mathbf{x}_1(0) = \mathbf{x}_{10} \in \Omega_x$ ,  $\mathbf{x}_1(T) = \mathbf{x}_{1T} \in \Omega_x$  and the time interval  $T$ , the problem is to find the optimal sequence  $\mathbf{u}^*(t) \in \mathcal{U}$ , minimizing the cost function  $J$  (eq 4), such that it steers the system (3) from  $\mathbf{x}_{10}$  to  $\mathbf{x}_{1T}$  complying with the restriction  $\mathbf{x}_1(t) \in \Omega_x$ . Or equivalently:

$$\begin{cases} \min_{\mathbf{u} \in \mathcal{U}} J(\mathbf{u}) = \int_0^T \mathbf{u}^T \mathbf{u} dt \\ \text{under } \begin{cases} \dot{\mathbf{x}}(t) = f(\mathbf{x}(t), \mathbf{u}(t)) \\ \mathbf{x}_1(t) \in \Omega_x \end{cases} \\ \text{given } (\mathbf{x}_{10}, \mathbf{x}_{1T}, T) \end{cases} \quad (5)$$

### Optimization methods

In this section, two classes of optimization methods will be discussed. In the first, the control input  $\mathbf{u}(t)$  is assumed to be a series of constant values during the time-intervals  $\Delta t = T/N$ . We refer to this method as the unconstrained piecewise constant control, since the sequence of  $\mathbf{q}(t)$  produced by this control input is not explicitly restricted to belong to any particular time or frequency function. In the second class of optimization methods, called the constrained methods, the Cartesian vector  $\mathbf{y}(t)$  or, equivalently, the joint position vector  $\mathbf{q}(t)$  is constrained to belong to a time-series polynomial expansion or to a Fourier frequency approximation. The control input in both the classes obey the inverse dynamic equations.

**Unconstrained and piecewise constant method.** The dynamic optimization problem 1, can be transformed into a static optimization problem as follows. First, assume that the control sequence  $\mathbf{u}(t)$  is piecewise constant. Let  $N$  be the number of time-intervals and  $\Delta t = T/N$  the time-length of these intervals. The only restriction on the sequence  $\{\mathbf{u}(k)\}_{k=0}^{N-1}$  is that each element belongs to  $\mathcal{U}$ .

Let  $\mathbf{U} \in R^{n \times N}$  be the input matrix gathering the input vector sequence  $\mathbf{u}(k)$ , i.e.

$$\mathbf{U} = [\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_{N-1}] \quad (6)$$

then the cost function (4) can be rewritten as:

$$\mathcal{C} = \sum_{k=0}^{N-1} \mathbf{u}(k)^T \mathbf{u}(k) \Delta t \quad (7)$$

Then, by approximating  $\dot{\mathbf{x}}$  as,

$$\dot{\mathbf{x}} = \frac{\mathbf{x}(k+1) - \mathbf{x}(k)}{\Delta t}$$

in the state equation (3), we can obtain the following implicit discrete-time nonlinear representation

$$\mathbf{x}(k+1) = \mathbf{x}(k) + \Delta t f(\mathbf{u}(k), \mathbf{x}(k))$$

By induction, it is possible to express the state  $\mathbf{x}$  at the instant  $N$  as a function of the initial state  $\mathbf{x}(0)$  and the series  $\mathbf{u}(0), \mathbf{u}(1), \dots, \mathbf{u}(N-1)$ , i.e.

$$\mathbf{x}(N) = F(\mathbf{x}(0), \mathbf{u}(0), \dots, \mathbf{u}(N-1)) = F(\mathbf{x}(0), \mathbf{U}) \quad (8)$$

where the operator  $F$  is defined as:

$$F = f \circ f \cdots \circ f(\mathbf{x}(0), \mathbf{u}(0)) \quad (9)$$

The dynamic optimization problem can now be transformed into the following static optimization one.

**Problem 2** Given the initial and final joint angles  $\mathbf{x}_1(0) \in \Omega_x$ ,  $\mathbf{x}_1(N) \in \Omega_x$  and the time interval  $T$ , the problem is to find the optimal value for  $\mathbf{u}^*(t) \in \mathcal{U}$ , minimizing the cost function  $\mathcal{C}$  (7), such that it steers the system (3) from  $\mathbf{x}_1(0)$  to  $\mathbf{x}_1(N)$  complying with the restriction  $\mathbf{x}_1 \in \Omega_x$ . Or equivalently:

$$\left\{ \begin{array}{l} \min_{\mathbf{U} \in \mathcal{U}} \mathcal{C}(\mathbf{u}) = \sum_{k=0}^{N-1} \mathbf{u}(k)^T \mathbf{u}(k) \Delta t \\ \text{under } \left\{ \begin{array}{l} \mathbf{x}(N) = F(\mathbf{x}(0), \mathbf{U}) \\ \mathbf{x}_1 \in \Omega_x \end{array} \right. \\ \text{given } (\mathbf{x}_1(0), \mathbf{x}_1(N), T, N) \end{array} \right. \quad (10)$$

Note that, as above, the initial and final velocity are free in our problem, and should thus be generated as a result of optimization. Note also that the discretization has been performed on the  $2n$  dimensional state vector  $\mathbf{x}$  and not on the equation (2) to reduce discretization errors due to approximation of the second-order time derivative.

**Polynomial Approximation.** Let  $\mathbf{y}(t)$  be defined as:

$$\mathbf{y}(t) = \begin{pmatrix} z_h \\ x_h \\ z_f \\ x_f \end{pmatrix}$$

where  $(z_h, x_h)$ , and  $(z_f, x_f)$ , are the Cartesian coordinates of the hip and foot, respectively (see Fig. 1), then robot motion can be specified in terms of these coordinates. They are related to the joint angles by the mapping  $\mathbf{y} = W(\mathbf{q})$ . As proposed in [3] and [2], the optimal trajectory in  $\mathbf{y}(t)$  can be assumed to be approximated by an  $m$ -order time-polynomial of the form:

$$\mathbf{y}(t) = \mathbf{p}_0 + \mathbf{p}_1 t + \cdots + \mathbf{p}_m t^m.$$

With initial and final position given,  $\mathbf{p}_0$  is uniquely defined. The remaining parameters  $\mathbf{P} = [\mathbf{p}_1 \cdots \mathbf{p}_m]$  should be determined by the optimization procedure as explained below.

From the inverse kinematics we have:

$$\mathbf{q}(t) = W^{-1}(\mathbf{y}) ; \dot{\mathbf{q}} = \mathbf{J}^{-1} \dot{\mathbf{y}} ; \ddot{\mathbf{q}} = \mathbf{J}^{-1} [\ddot{\mathbf{y}} - \dot{\mathbf{J}} \mathbf{J}^{-1} \dot{\mathbf{y}}] \quad (11)$$

where  $\mathbf{J}$  is the Jacobian matrix which is full rank (in this case where  $\mathbf{J}$  is quadratic, it will be non-singular) for all  $\mathbf{q} \in \Omega_x$ . Note that singular configurations are never reached during the human walk. Combining (2) and (11), and using the time approximation given above,  $\mathbf{u}(t)$  can be written as a function of the unknown parameters  $\mathbf{P}$ :

$$\mathbf{u} = \mathbf{H} \mathbf{J}^{-1} \ddot{\mathbf{y}} - \mathbf{H} \mathbf{J}^{-1} \dot{\mathbf{J}} \mathbf{J}^{-1} \dot{\mathbf{y}} + \mathbf{g}(W^{-1}(\mathbf{y})) \quad (12)$$

$$= \Phi_p(\mathbf{P}, t) \quad (13)$$

Problem 1, can then be reformulated as:

**Problem 3** Given the initial and final joint angles  $\mathbf{x}_1(0) = \mathbf{x}_{10} \in \Omega_x$ ,  $\mathbf{x}_1(T) = \mathbf{x}_{1T} \in \Omega_x$  and the time interval  $T$ , the problem is to find the optimal parameters  $\mathbf{P}^*$ , minimizing

the cost function  $J$  (eq 4), such that it steers the system (3) from  $\mathbf{x}_{10}$  to  $\mathbf{x}_{1T}$  complying with the restriction  $\mathbf{x}_1(t) \in \Omega_x$ . Or equivalently:

$$\left\{ \begin{array}{l} \min_{\mathbf{P}} \sum_{k=0}^{N-1} \Phi_p(\mathbf{P}, k)^T \Phi_p(\mathbf{P}, k) \Delta t \\ \text{under } \mathbf{x}_1 \in \Omega_x \\ \text{given } (\mathbf{x}_1(0), \mathbf{x}_1(N), T, N) \end{array} \right. \quad (14)$$

where  $\mathcal{U}_p$  is the set of functions included in  $\mathcal{U}$  generated by equation (13) for all  $\mathbf{P} \in R^{n \times m}$ . Note also that the initial and final conditions in the joint coordinates are obtained (uniquely) from the initial conditions on the Cartesian coordinates.

**Fourier Approximation.** Instead of constraining the Cartesian trajectories of the robot one can also constrain its joint trajectories and one straightforward way to do this is with a truncated Fourier series. This approximation may be called the frequency approximation, and it can be expressed as:

$$\mathbf{q}(t) = \mathbf{a}_0 + \sum_{k=1}^K \mathbf{a}_k \cos(k\omega t) + \sum_{k=1}^K \mathbf{b}_k \sin(k\omega t)$$

where the  $\mathbf{a}_i$  and the  $\mathbf{b}_i$  are the Fourier coefficients and  $\omega$  is the step base frequency. If only the single support phase is considered then  $\omega = 2\pi/T$ . However, in this case the trajectory has an important bias between initial and final joint positions. This type of trajectories is thus poorly approximated by only using the expansion given above. To solve this problem it has been suggested in [11] to add a time-polynomial to absorb differences between the initial and final conditions without over-increasing the order of the Fourier approximation. For instance, adding a first-order polynomial gives,

$$\begin{aligned} \mathbf{q}(t) &= \mathbf{a}_0 + \mathbf{c}t + \sum_{k=1}^K (\mathbf{a}_k \cos(k\omega t) + \mathbf{b}_k \sin(k\omega t)) \\ &= \phi_f(\mathbf{D}, t) \end{aligned} \quad (15)$$

$\mathbf{D} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_K, \mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_K, \mathbf{c}]$  is the unknown parameter matrix to be determined.

The vectors  $\dot{\mathbf{q}}$  and  $\ddot{\mathbf{q}}$  are computed from (15) and substituted into (2). They enable the control  $\mathbf{u}$  to be written as:

$$\mathbf{u} = \Phi_f(\mathbf{D}, t) \quad (16)$$

This equation defines, for all matrices  $\mathbf{D}$  with bounded entries, a class of signals  $\mathcal{U}_f \subset \mathcal{U}$ , in which the search for optimal  $\mathbf{u}^*$  will be performed.

Our problem is now formulated as follows:

**Problem 4** Given the initial and final joint angles  $\mathbf{x}_1(0) = \mathbf{x}_{10} \in \Omega_x$ ,  $\mathbf{x}_1(T) = \mathbf{x}_{1T} \in \Omega_x$  and the time interval  $T$ , the problem is to find the optimal parameters  $\mathbf{D}^*$ , minimizing the cost function  $J$  (eq 4), such that it steers the system (3) from  $\mathbf{x}_{10}$  to  $\mathbf{x}_{1T}$  complying with the restriction  $\mathbf{x}_1(t) \in \Omega_x$ . Or equivalently:

$$\left\{ \begin{array}{l} \min_{\mathbf{D}} \sum_{k=0}^{N-1} \Phi_f(\mathbf{D}, k)^T \Phi_f(\mathbf{D}, k) \Delta t \\ \text{under } \mathbf{x}_1(k) \in \Omega_x \\ \text{given } (\mathbf{x}_1(0), \mathbf{x}_1(N), T, N) \end{array} \right. \quad (17)$$

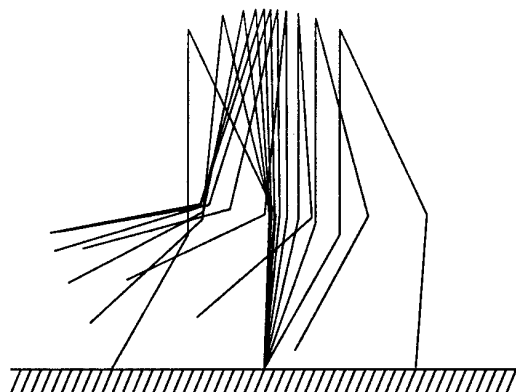


Figure 2: Walk with piecewise constant control.

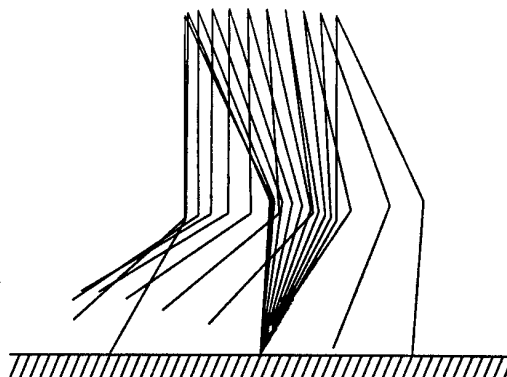


Figure 3: Walk with polynomial approximation.

## Results.

The optimization method used here is a Sequential Method Programming based on Kuhn-Tucker equations. This method solves at each iteration a Quadratic Programming subproblem (QP) which involves a quadratic approximation of the Lagrangian function composed by the cost function and the constraints which are premultiplied by the Lagrange multipliers.

In order to compare the different optimization methods, the following comparison criteria are used:

- **Input Energy.** The cost function  $J$  (eq 4) represents the total control energy needed to perform the single support phase. This measure quantifies the energy consumption of the biped over one step without considering the energy consumed during the transition from one step to the next.
- **Initial Energy.** If the injected energy is zero during the time step interval (i.e., the motion is passive in the sense that it is generated by a zero control input), it is important to take note of the initial value of the energy. This quantity represents the energy level necessary to bring the system to the initial state required to perform the ballistic gait. The total energy needed during a step is the sum of the energy during the single-support phase and the energy during transition.
- **Computational Burden.** The CPU computation time and the number of parameters to be optimized are also considered as a function of the total amount of computation burden associated to each optimization method.

For the simulation results presented in this section, we have used as the walk constants the step length  $S = 0.6m$  and the step period  $T = 1s$ . Initial and final positions are chosen to be symmetrical.

In figures (2) (3) and (4) we can observe the optimal gait obtained respectively with piecewise constant time control(PCTC), polynomial approximation and Fourier expansion respectively. Some differences can be observed; the foot trajectories are similar in Fourier expansion and the polynomial approximation, but quite different in the PCTC, whereas the hip trajectories are similar in the Fourier expansion and the PCTC, but different in the polynomial approximation. These trajectories are shown in more detail in figure (5).

The control cost ( $J$ ) and initial energy ( $\mathcal{H}(0)$ ) associated with each of these methods are shown in Table I. It can be seen that the energy consumption is close to zero with

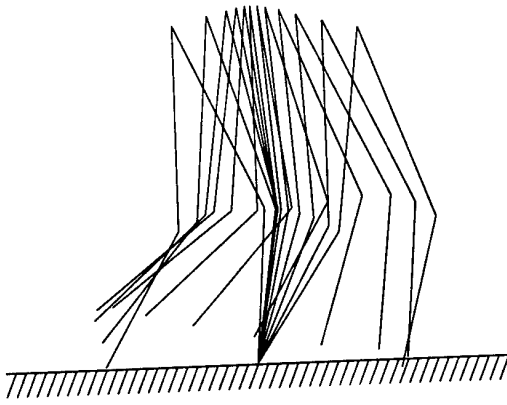


Figure 4: Walk with Fourier series approximation.

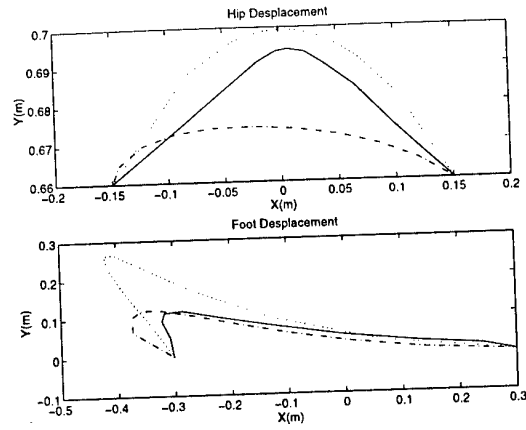


Figure 5: Hip end Foot trajectories – Fourier method — · — Polynomial method  
... Piecewise constant method.

METHOD	Cost $J$ ( $N^2.m^2.s$ )	$\mathcal{H}_0$ ( $N^2.m^2.s$ )	Par. No.	CPU time $s$
Piecewise constant	$2.10^{-7}$	53.2	$n \times N = 40$	37
Polynomial Approximation	2121	47.9	$n \times m = 20$	137
Fourier Expansion	1565	48.1	$n \times (2K + 2) = 40$	451

Table I: Comparison of the Optimization Methods Based on the Three Proposed Criteria. the

piecewise constant control. This implies that this formulation finds initial conditions of velocity so that the gait is passive during the single support phase. The energy required to achieve these initial conditions is approximately the same as the one required in the other two methods. However, the initial directions of the velocity joint vectors turn out to be quite different. In PCTC optimal gait, the swing leg behavior is like a pendulum: the initial velocity carries the foot sufficiently high in order to arrive at the final position through a “free” movement. Fig(6) shows the time evolutions of the control inputs.

The number of optimization parameters of these methods are quite equivalently (see Table I, column 4), however CPU-time of the PCTC method is much smaller than in the other two. By looking at this table, the PCTC appears superior to the other two methods in all the comparison criteria.

### Conclusions and Further Extensions.

This paper presented and compared three methods for optimal-energy gait generation for biped robots. These methods are: piecewise constant inputs, time-polynomial approximation and Fourier expansion. The comparison was performed on the basis of injected energy, initial energy and computational burden. The numerical study presented here shows that the piecewise constant input method is superior to the other two approaches in terms of energy and CPU-time. Moreover, this method was able to find initial velocities that generate ballistic motions with almost zero injected energy. Hence the method was able to find passive motions. The study was only concerned with the single support phase and the transition phases. We have already developed a model and are currently studying a complete gait cycle including the double support phase. It is interesting to investigate the possibility of finding other basis functions to approximate the walking gaits, and to

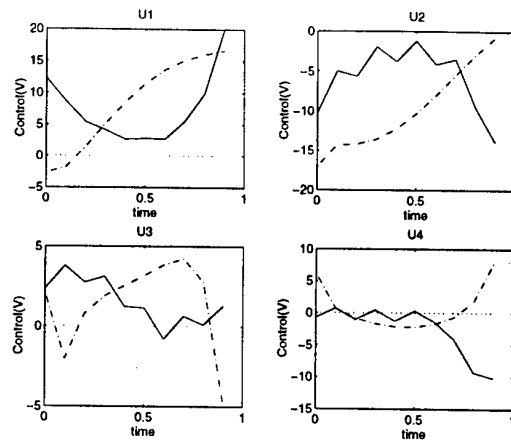


Figure 6: Input control during a step  
 — Fourier method    - - - Polynomial method  
 . . . Piecewise constant method.

study the potential benefits of introducing passive elements in the joint articulations [1], which appear as a natural way of enhancing a passive walk.

## References

- [1] G. Cabodevilla, N. Chaillet, and G. Abba. Near optimal gait for a biped robot. In *Proc. of the AMS'95*, Karlsruhe, Germany, 1995.
- [2] P.H. Channon, S.H. Hopkins, and D.T. Pham. Simulation and optimization of gait for a bipedal robot. *Math. Comput. Modelling*, 14:463–467, 1990.
- [3] P.H. Channon, S.H. Hopkins, and D.T. Pham. Derivation of optimal walking motions for a bipedal walking robot. *Robotica*, 10:165–172, 1992.
- [4] C. Chevallereau, A. Formalsky, F. Goselin, and B. Perrin. Ballistic motion for a quadruped robot. In *Proceedings of WAC*, 1996.
- [5] C.K. Chow and D.H. Jacobson. Studies of human locomotion via optimal programming. *Mathematical Biosciences*, 10:239–306, 1971.
- [6] C. Francois and C. Samson. Running with constant energy. In *Proceedings of IEEE International Conference on Robotics and Automation*, pages 131–136, San Diego, USA, 1994.
- [7] A. Goswami, B. Espiau, and A. Keramane. Limit cycles and their stability in a passive bipedal gait. In *Proceedings of IEEE International Conference on Robotics and Automation*, pages 246–251, Mineapolis, USA, 1996.
- [8] A.A. Grishin, A.M. Formalsky, A.V. Lensky, and S.V. Zhitomirsky. Dynamic walking of a vehicle with two telescopic legs controlled by two drives. *The International Journal of Robotics Research*, 13(2):137–147, 1994.
- [9] T. McGeer. Passive dynamic walking. *The International Journal of Robotics Research*, 9(2):62–82, 1990.

- [10] T.A. McMahon. *Muscles, Reflexes, and Locomotion*. Princeton University Press, 1984.
- [11] V. Yen and M.L. Nagurka. Suboptimal trajectory planning of a five-link human locomotion model. In *Biomechanics of Normal and Prosthetic Gait*, pages 17–22, Boston, MA, 1987. ASME Winter Annual Meeting.

# AUTONOMOUS MICROROBOTS

U. Rembold and S. Fatikow

University of Karlsruhe, Institute for Real-Time Computer Systems and Robotics  
Kaiserstr. 12 (Geb. 40.28), 76128 Karlsruhe, Germany  
Phone: +49-721-3656, Fax: +49-721-606740, E-Mail: fatikow@ira.uka.de

**Abstract:** Robots have been a subject of research for almost half a century. In order to make robots more versatile they must be able to operate in a semi-structured work place where unforeseen events occur and where sensor data are incomplete. An entire research community has been working on this problem and many unique autonomous and so-called intelligent robots have been conceived and built. Most of these efforts are concerned with robots that operate in the macroworld where they take on chores that could also be handled by humans. However, there is the microworld in which manipulation and handling tasks are very difficult and for which a human has no tools and where the work area is so small that fine manipulation is almost impossible by hand. This paper is concerned with autonomous robots that can operate in a microworld, where microassembly operations, microsurgery or integrated circuit testing and repair is done. For independent operations, these robots need special sensors and an efficient computer architecture that hosts the planner and executor. There are also special drive systems and effectors necessary for micromotions and micromanipulations, respectively. An attempt is made to describe these components and the problems encountered to configure them to a microrobot. As an example of advanced microrobots, the design and functions of several autonomous microrobots of the University of Karlsruhe are shown; they employ different locomotion and object handling principles. The paper also includes a discussion of the typical operating problems caused by the microworld and of future research that has to be done to conceive and build efficient microrobots.

**Keywords:** autonomous microrobots, piezoelectric actuators, micromanipulation, robot control, microassembly station

## 1. Introduction

The concepts of a robot originate from the human desire to do away with hard and dangerous work and have such jobs done by mechanical means. Robots have many capabilities with regard to force, speed, reproducibility and endurance, superior to those of human operators. Robots have improved many production processes and have led to their rationalization. However, there are also other robots which, compared to industrial robots, provide advanced functions and take on the tasks of a servant. These robots are known as service robots, whereby the concept of "service" comprises a multitude of activities such as service in medicine, inspection, maintenance, household, entertainment etc. In order to meet future demands and to offer services, robots must be mobile, easy to handle by the user, and be adaptable; all this requires a certain amount of intelligence.

In many applications it is very important to be able to precisely manipulate and position different types of objects having a resolution in the  $\mu\text{m}$  or  $\text{nm}$  range. Microassembly in production, cell manipulations in biomedical research, eye and plastic surgery are just a few good examples. For this reason, the "flexible microrobots" or "desktop micromanipulation systems" have currently become of interest. Like conventional robots, microrobots are very complex systems using various types of microactuators and microsensors. They are provided with algorithms for intelligent signal and information processing. Microrobots also move, apply forces, manipulate objects, etc. Some drive principles from the macroworld can be applied to the

microworld, but a possible scale down effect must be taken into account; otherwise, performance data may be calculated for a scaled-down micromachine which are not realistic. With the exception of the specific microworld problems and the size difference, the design criteria and steps to build micro- or macrorobots can in many cases be identical. Analogous to the design and making of a macromachine, the functional components of a microrobot must be first produced to the desired dimensions and internal structures and they are then assembled and fine tuned. Finally, tests are run to make sure the robot is functioning correctly. On the other hand, as with most developments based on microsystem technology (MST), manufacturing and operational problems occur in microrobots due to their tiny dimensions. Microrobots pose a great challenge to MST researchers; there is a continuous increase in research activities directed to new application fields which can not be handled by conventional robots due to conceptual limitations. Great importance is placed into the ability of microrobots to move over long distances at adequate speed, to finely manipulate different types of objects, to be robust and able to operate in hazardous environments, and to be functional over long periods of time without needing maintenance.

The slow adoption of MST by industry has made it clear that many problems exist in mass-producing microsystems. Most batch processes are rarely applicable for the production of complex microsystems which consist of microcomponents made of different materials and which are manufactured using different techniques. This means that individual components must be very exactly assembled in

one or more steps to form a desired microsystem. Often it is necessary to combine conventional and microcomponents which requires very accurate fine tuning and a high flexibility on part of the assembly system. The microassembly systems which exist today are rather large, are usually only fit for a specific task and depend on the manual agility of an operator. For microrobots the assembly of microsystems (i.e. non-destructive transportation, precise manipulation, and exact positioning of tiny objects that have dimensions in the nm- and  $\mu\text{m}$ -range) is one of the most promising applications.

## 2. Manipulation of Microobjects

As mentioned, with the growing variety of design elements there is a definite trend toward the development of hybrid microsystems. During the design of a hybrid microsystem assembly-specific requirements must be considered besides task-specific ones. It must be known if suitable interconnecting technologies are available, how the assembly steps can be automated and what are the assembly related costs. The automation of highly precise assembly processes will make it easier to realize low cost operable microsystems. In order to make it possible to produce microsystems and components in lot sizes or to mass-produce them, it is absolutely necessary to introduce flexible, highly precise and fast microassembly methods.

Adapting conventional manipulation methods to the requirements of the microworld is only partially possible:

- 1) Microscopically small objects cannot be handled easily as bulk material. They entangle or jam and can be damaged during transport or when isolating single components;
- 2) Microobjects have a limited robustness;
- 3) Factors like dust, humidity, vibrations or temperature change may have an adverse effect on handling;
- 4) Microobjects are subject to undesirable adhesion forces;
- 5) Sensor information from the microworld to the macro-world is difficult to transmit.

Although the manipulations vary from application to application, approximately the same operations are used in every case, such as grip, transport, position, release, adjust, fix in place and processing steps like cutting, soldering, gluing, removal of impurities, etc. In order to be able to carry out these operations, corresponding tools are needed, such as microknives, microneedles to affix microobjects, microdosing jets for gluing, microlaser devices for soldering, welding or cutting, different types of microgrippers, microscrapers, adjustment tools, etc. Microgrippers play a special role, since they considerably influence the manipulation capabilities of a robot. Microgrippers can clamp, make frictional connections or adhere to material, depending on the physical and geometrical properties of an object. Adapting a gripper to the shape of the object to be gripped is often the best solution in the microworld. This allows handling of a workpiece having a complex shape,

such as a gear; thereby the gripper securely attaches to the contour of the part. For small, smooth parts, a suction pipette might be a practical tool. If the upper surface of a workpiece must not be touched/gripped due to technological reasons, it can be protected by a corresponding form-fit of the pipette hole. For contour clamping and frictional connections in manipulations involving fragile parts, elastic grippers made of soft plastics are preferred over metal grippers. Due to the variety of task-specific gripping tools in automated micromanipulation systems, a suitable gripper exchanger system might be necessary.

## 3. Microassembly by Microrobots

The development of automated microrobot-based assembly facilities for microsystems is still in the beginning stage. Current micromanipulation systems depend on the skill of the operator, which is limited by his own skills to handle small objects and the tools available to him. Although the human hand is a very versatile instrument, and has an almost unsurpassed dexterity, it does not have unlimited abilities to manipulate and to assemble small microsystems without suitable aids. Therefore, the further development of MST depends on the availability of flexible micromanipulation stations, which allow components to be automatically assembled, thereby reducing production costs and simultaneously obtaining high quality. For this reason, the manufacturing engineer is looking for flexible microrobots having both transportation and manipulation skills. Automatically controlled with the help of visual and force sensors, these robots may free humans from the tedious task of having to manipulate miniscule objects directly. A microrobot-based assembly station should be quickly adaptable to different types of products with the support of sensors and suitable control and planning units.

The spectrum of tasks in microassembly ranges from simple preparatory operations like applying adhesives, drawing adjustment marks, cleaning objects, etc. to the performance of the final assembly and inspection of the finished microsystem. A well conceived microassembly station must be able to automatically accomplish the following steps with its microrobots:

- 1) preparation of the parts to be assembled;
- 2) transportation of parts;
- 3) positioning and fixing of parts;
- 4) connecting the parts;
- 5) testing and measuring the finished microsystem.

Typically, in a conventional automatic or semi-automatic assembly station, standardized mechanical parts are assembled in well-defined work positions. The robots performing the work are usually of multi-axis arm design or they are gantry systems, usually driven by d.c. motors. Today, it is being attempted to use these type of familiar systems for handling and assembling of miniaturized components with dimensions in the millimeter range [1-2].

With increasing workpiece miniaturization, however, it becomes more and more difficult to use conventional manipulation robots for assembling microsystems. The manipulation accuracy is mechanically limited for conventional robots, since disturbing influences which are often negligible in the macroworld, such as fabrication defects, friction, thermal expansion or computational errors, play an important part in the microworld. Furthermore, these robots are subject to mechanical wear and must undergo regular maintenance, which makes them expensive. The next point is that the positioning accuracy and the tolerances of the microcomponents lie in the nm range, a few orders of magnitude smaller than in conventional assembly. These accuracy requirements can only be obtained with manipulators having highly precise direct drives utilizing MST and advanced closed-loop control. New microrobot-based desktop stations may offer the desired features and versatility.

The advantages offered by microrobots can only be used if a microassembly desktop station also has high-resolution sensors. Since the handling forces encountered when manipulating microobjects are often very small and lie within the range of 1 nN to 1  $\mu$ N, the required force sensors are difficult to build. For this reason, often only optical sensors offer a feasible solution to obtain a position feedback in the assembly station. The entire assembly process is done either under a light microscope equipped with a high-resolution CCD-camera or in a vacuum chamber of a scanning electron microscope. Laser measurement instruments are also suited for determining the position and orientation of a microrobot or the parts. Sometimes the applied forces can be inferred by optically examining the deformation of a tool tip. In the future, new types of force or contact sensors will improve the capabilities of a desktop station.

A new concept of an automated micromanipulation desktop station, which uses piezoelectric microrobots, was discussed in [3]. Within this station, it is possible to perform an entire assembly process under the automated light microscope, which is equipped with a RS232 interface. The microscope has an automatic positioning table with two translational degrees of freedom (x-y plane). On top of the table, a glass plate is fixed. By controlling the movement of the table, every part of the plate, which serves as a working area of the station, can be brought in the field of view of the microscope. The station has a central computer (Pentium PC) which is used for task-specific assembly planning. With it the operational steps are defined and carried out successively. The commands of the central computer are further processed on a lower control level, by a parallel computer system which was reported in [19]. Here, the commands are resolved into a defined command sequence to activate the system components, such as the robots, microscope and positioning table. The central computer is coupled with the parallel computer system of the robots over serial and parallel interfaces. In the multiprocessor system, the generated commands are executed in parallel, which makes the realtime operation of the micro-

assembly station possible. This way the 2D movements of the positioning table, the microscope functions (changing of objectives, focussing and adjusting of the light) and all robots' piezoactuators are controlled.

In order to automatically control the manipulation processes in the desktop station, there is sensor feedback from a CCD camera. The camera and the microscope form a local sensor system, which supplies visual information of fine manipulations with the microobjects to the central computer; for this purpose the position of the microobjects and that of the robot tools must be determined. The coarse positioning of the robot on the glass base is supervised with a global sensor system, consisting of another CCD camera and a laser measurement system. The information of both these sensor systems is used to generate the commands for the robot, microscope and the positioning table. Vision is supported by a frame grabber in connection with a fast real-time image processing systems. The vision parameters are passed on to the parallel computer system, where they are used as set points for the control loop.

In such a microassembly station, all tools and techniques necessary for an automated assembly of a microsystem must first be determined, so that the station can be set up for a task-specific operation sequence. The specified techniques and tools must take the geometry of the components of the microsystem into consideration, as well as their physical properties, such as rigidity, texture and temperature stability. Therefore, the planning phase of an automated microassembly involves many problems and requires a high degree of competence at the higher levels of the planning system [4]. Pure "top-down" planning in a microassembly station seems to be impossible, since the selected robots and their manipulation tools determine both the flexibility and the degree of automation of the station, i.e. they determine its performance limits. One possible planning strategy is the "meet-in-the-middle"-strategy allowing the intermediate interface between the "top-down" and "bottom-up" planning algorithms to be on the tool level. After all, the determination of the task-specific sequences of the elementary operations and the selection of the necessary tools for carrying out the assembly operations (grippers, probes, dosage systems, etc.) are the last step of assembly planning and, at the same time, the interface between the planning and the control levels. On the other hand, the involved tools and the operations require that the microrobots have specific functional properties, which greatly influences the robots' design.

#### 4. New concepts of micromanipulation robots

Two micromanipulation robots, MINIMAN (MINIaturized MANipulator) and PROHAM (Piezoelectric RObot for HANDling of Microobjects), developed for use in a microassembly station, were reported in [5-6]. The design of their piezoactuators and the control principles were reported

in [7]. These robots have a micromanipulating unit integrated in their platform, which makes them capable of moving and manipulating. Various assembling tools can easily be used and exchanged. However, the robots are not free from problems, such as the relatively high drive voltage of the robots or the instability of grasp-and-hold operations. In the following we will be discussing principles of several new microrobots which have been conceived and built at the Institute for Real-Time Computer Systems and Robotics (University of Karlsruhe).

#### 4.1 An one-legged robot as a positioning unit

**4.1.1 Design and actuation principle.** The piezoelectric tube-shaped robot legs of MINIMAN and PROHAM need a drive voltage of  $\pm 150\text{V}$ , which requires the use of large and expensive electrical components. The design of a multilayered piezoactuator, which can be operated with low voltages of  $\pm 20\text{V}$ , has lead to the conception of a positioning unit with three degrees of freedom. Beside the high performance of the piezoceramics, the low drive voltage gives the possibility of using simpler electronic components. They are smaller and can be placed on the robot platform itself, and they are much cheaper than the high voltage ceramics used for the above mentioned robots.

As a driving actuator, a multilayered ceramic actuator produced by Hoechst CeramTec has been chosen because of its good performance. This multilayered actuator is only able to make an one-dimensional movement. When applying electric voltage between the electrodes of the actuator (Fig. 1), the piezoceramic material changes its length.

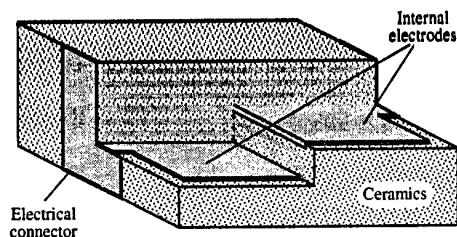


Fig. 1: View of the multilayered actuator

The main goal was to find a design for a positioning robot unit (robot platform), in which the one-dimensional movement of its actuators can be converted into a two-dimensional movement and a rotation. Figure 2 shows the design of the unit and its actuation principle.

The size of the positioning unit is about  $3\text{cm} \times 3\text{cm} \times 3.5\text{cm}$ . It has only one leg, which is mounted on a circular foot-plate, which has three glass ball-feet on its lower side. The upper end of the leg is attached to a tilt-plate, which is driven by three multilayer piezoactuators positioned in a triangle. When the leg moves, the tilt-plate inclines,

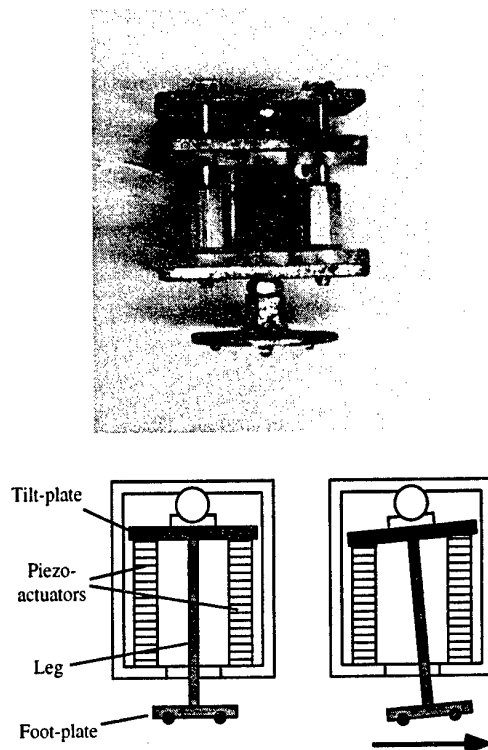


Fig. 2: The one-legged robot and its actuation principle

whereby the tilt-line is always perpendicular to the direction of the movement (Fig. 3). If one piezoactuator elongates, the leg moves in this direction. The motion principle of the unit relies on a sequence of tiny jumps. A slow displacement of the unit in the drive direction is followed by a fast motion of the leg. Thereby, the unit does not move backwards due to the low friction between the ball-feet and the ground resulting in a sliding motion when the leg is moved at high speed (slip-stick motion principle).

**4.1.2 Motion control: Translation.** As described in the last section a translatory movement is obtained when one multilayered piezoactuator is oscillating fast in one direction and slowly in the other. The oscillation can be described by an asymmetrical saw-tooth function. To increase the step length, the other two actuators of the robot leg should oscillate with an opposite phase. The design of the unit offers the possibility of supporting gliding of the leg on the ground, which currently can not be achieved by the microrobot constructions described in [5-6]. During the fast forward movement, the center of gravity is moving upwards. This is achieved when the center of gravity lies in front of the tilt-line in direction of the movement.

The translatory movement is not only possible in the three directions of the actuators, but in every direction. The following expressions describe the amplitudes of the voltage functions  $V_1$ ,  $V_2$ , and  $V_3$  which must be applied to

the leg actuators so that the desired movements take place. These functions are also saw-tooth functions because of the direct proportionality between the voltage and the elongation of the piezoactuators:

$$\begin{aligned} V1 : V2 : V3 &= a1 : a2 : a3 \\ a1 &= \sqrt{3} / 3 * l * \sin(90^\circ - \alpha) + d \\ a2 &= \sqrt{3} / 3 * l * \cos(60^\circ - \alpha) - d \\ a3 &= \sqrt{3} / 3 * l * \cos(60^\circ + \alpha) - d \end{aligned}$$

Here,  $a1$ ,  $a2$  and  $a3$  represent the distances of the actuators to the tilt-line (Fig. 3),  $l$  is the distance between two actuators,  $d$  is the distance between the center of gravity and the tilt-line (it must be determined experimentally) and  $\alpha$  is the angle between the first actuator and the direction of the motion. The points of contact between the actuators and the tilt-plate are marked with A, B and C.

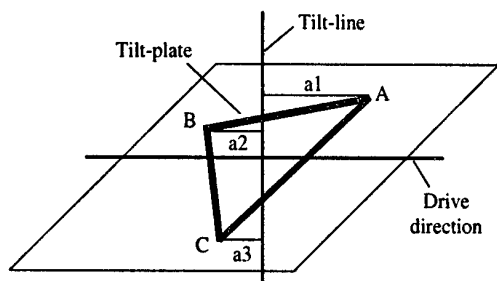


Fig. 3: Tilt-plate motion

**4.1.3 Motion control: Rotation.** A correct rotation with only one leg is not possible without torsion. The positioning unit is able to turn itself, but at the same time it follows a circular trajectory. A turn is possible because the glass balls are arranged under the actuators so that the movement of each actuator is applied to the corresponding ball below. If two actuators oscillate with opposite phases we will obtain the desired rotation but with a movement along a circular trajectory with a radius of  $0.86 \times l$  (Fig 4).

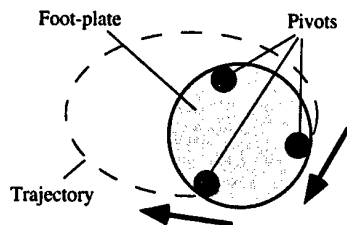


Fig. 4: Rotation of the unit (bottom view)

A more accurate rotation can be obtained if after every step the next two multilayered actuators in the direction of movement oscillate. In this case, the pair of oscillating ac-

tuators also rotate with every step and the radius of the circular trajectory is minimal.

**4.1.4 Analysis and future work.** If only one multilayered actuator is used, the asymmetric shape of the positioning unit causes a derivation of up to 15% of the trajectory. The highest accuracy is achieved if all actuators are used simultaneously and the distance between the center of gravity and the tilt-line is optimal. The maximum velocity for the translatory movement is about 0.9mm/s. The maximum step length is  $36\mu\text{m}$  and the minimal one is  $0.142\mu\text{m}$ .

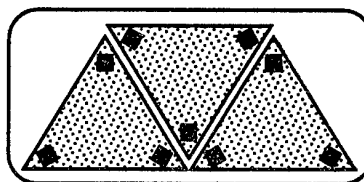


Fig. 5: Top view of a platform with three one-legged units

A symmetrical form of the unit may improve its behavior and increase the accuracy of both translatory and rotatory motion. The unit can be further reduced: a size of about  $1.5\text{cm} \times 1.5\text{cm} \times 3\text{cm}$  is possible. An interesting solution seems to be a combination of three one-leg units to a three-leg platform (Fig. 5). It would have the ability of moving in any direction with a very high accuracy and would turn well. The size of such a platform could be about  $3.5\text{cm} \times 1.5\text{cm} \times 3\text{cm}$ .

## 4.2 The positioning unit SPIDER-I

**4.2.1 Design and actuation principle.** Another micropositioning unit, the so called SPIDER-I, is driven by bimorphic piezoactuators. A simple bimorphic piezoactuator is composed of two bonded piezoceramic sheets (Fig. 6). When contracting one side and extending the other, the bimorphic structure bends.

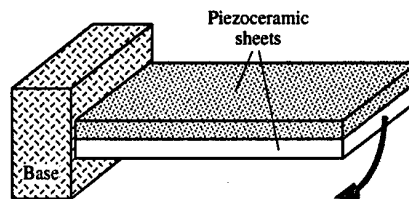


Fig. 6: Bimorphic structure

In order to obtain a high stability of the positioning unit, three actuators (legs) with permanent contact with the ground are used. Each leg is composed of two perpendicular bimorphic actuators (Fig. 7). With this design, the robot has three degrees of freedom (two translatory and one

rotatoric). A uniform omnidirectional movement is possible if the legs are arranged in a 120° angular pattern, as shown in the figure.

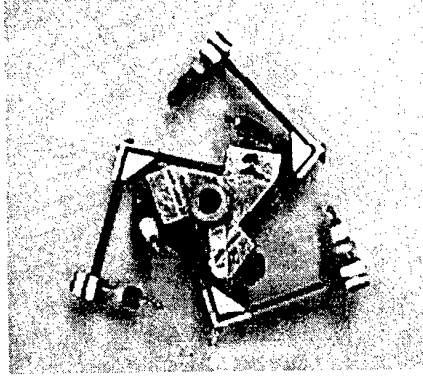


Fig. 7: The three-leg robot SPIDER-I

The slip-stick motion of the platform is generated by a simultaneous slow movement of the legs in the same direction, which is then followed by a fast return to the start position. During the slow movement, the platform is pushed forward due to the friction between the legs and the ground. In the second phase, the reduced slide friction allows the legs to return without moving the platform.

**4.2.2 Motion control.** The movement of one leg results from adding the movement of its inner and outer piezoactuators (Fig. 8). To generate a leg motion in direction of  $R_n$  to the point  $(x_n, y_n)$ , the actuators have to bend as follows:

$$d_i = -x_n; d_o = y_n + d_i \frac{l_o}{l_i} = y_n - x_r \frac{l_o}{l_i}$$

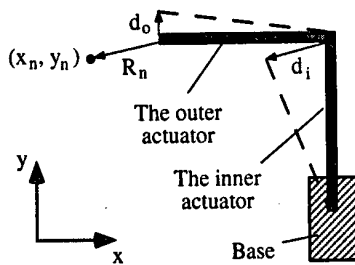


Fig. 8: Motion principle of one of the robot legs

To perform a translational robot motion in direction of  $R$  to the point  $(x_r, y_r)$  (Fig. 9), every leg has to move in direction of  $R_n$ ,  $n=1..3$  with:

$$R_n = \begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} \cos(360^\circ - \delta_n) & -\sin(360^\circ - \delta_n) \\ \sin(360^\circ - \delta_n) & \cos(360^\circ - \delta_n) \end{pmatrix} * \begin{pmatrix} x_r \\ y_r \end{pmatrix}$$

where angles  $\delta_n$  depend on the actuators' orientation. For the robot design in Figure 7:  $\delta_1=180^\circ$ ,  $\delta_2=300^\circ$ ,  $\delta_3=60^\circ$ .

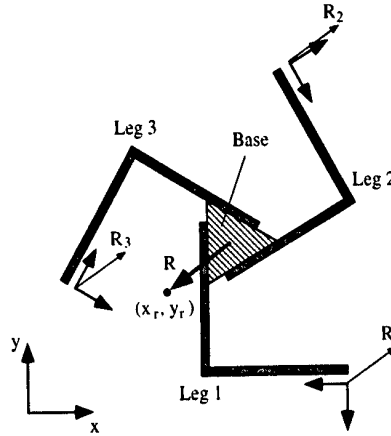


Fig. 9: Motion principle of the robot

A rotation is controlled in a similar way. To obtain continuous walking of the robot, the motion cycles consisting of slow and fast motions have to be repeated. Thereby, the drive voltage for each of the bimorphic actuators takes the form of a sawtooth-function. The applied voltage frequency determines the speed of the robot and the voltage amplitude determines the bending of the bimorphic actuator. Evidently, the voltage amplitude has to be proportional to the corresponding step length  $d$ .

Because only the simultaneous movement of all actuators can guarantee an optimal trajectory of SPIDER-I, the amplitude values have to be scaled in an appropriate way. To do this, the maximal value of  $d$  is set to 1 and then the other values are changed while preserving the aspired ratio. The amplitude of the voltage is calculated as follows:

$$V_x = V_{ref} * d_x$$

The reference voltage  $V_{ref}$  defines the step length. It must be set to 20V to obtain a maximum step.

**4.2.3 Analysis and future work.** This robot can move in any direction with a linear speed of up to 2 cm/min and a rotational speed of up to 1 rpm; the drive voltage is in the range of -20V ... +20V. A movement accuracy of 175nm can presently be achieved. The lower weight of this device compared to that of MINIMAN affects the precision of the motion so that a weight adjustment must be made. In order to do this, the torsional moment on the inner bimorphic actuators must be reduced. A shorter bimorphic structure or a change of the angle between the inner and the outer actuator of a leg would reduce the lever arm. This more compact arrangement of the actuators would lead to further miniaturization of the robot because the diameter of the platform can be reduced from 6cm to less than 4cm.

### 4.3 The micromanipulation robot SPIDER-II

**4.3.1 Design and actuation principle.** Like the previous robot, the new micromanipulation robot SPIDER-II employs bimorphic piezoactuators. In contrast to SPIDER-I, here the piezoactuators used (manufactured by Philips) have better characteristics than the actuators of SPIDER-I and can be driven by a voltage of  $\pm 60V$ .

The micromanipulation robot includes six piezolegs and a manipulator, which are fastened directly to the electronics board; the latter serves as a robot base (Fig. 10).

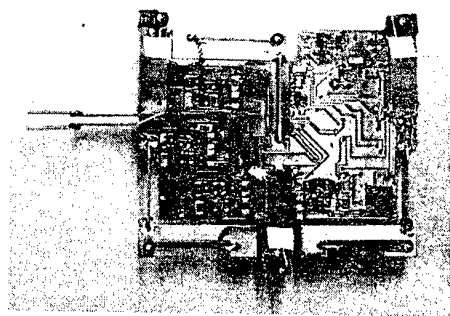
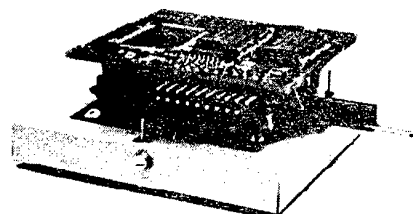


Fig. 10: The robot SPIDER-II (front and bottom view)

The robot has three "moving" legs to move the robot along the x- and y-axis and three "lifting" legs to raise and lower the robot along the z-axis. Each of the lifting legs consists of one bimorphic actuator whereas each of the moving legs includes, like SPIDER-I, two perpendicular bimorphic actuators. Each actuator can be bent in any position in a range of  $\pm 260\mu m$  with a resolution of 4nm.

The design of a moving leg is shown in Figure 11. The piezoactuators for the x-movement are bounded to the robot's body. Each of these actuators has a lever on its end, which in turn carries an outer piezoactuator for the y-movement; the latter actuator has a metal ball-foot fastened to its tip. This arrangement was chosen in order to minimize torsion, which may destroy the extremely brittle piezoceramics easily. The robot moves in the following way. At the beginning of a step, the moving legs rest on the

ground. When a voltage is applied to them in an appropriate way they start to bend backwards. This will push the robot forward. After this motion has been completed, the lifting legs are lowered, which raises the robot. In this robot position the moving legs, which do not touch the ground any more, are moved forward without a movement of the robot. Afterwards, the robot is lowered again by the lifting legs and completes one step forward.

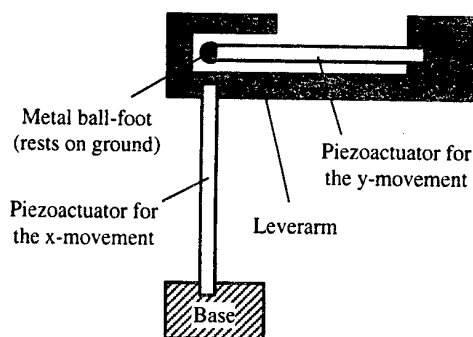


Fig. 11: Design of a SPIDER-II leg (top view)

The manipulator is driven by three piezoceramic actuators, which are of the same kind as the leg actuators (Fig. 12). The bimorphic actuator that is fastened to the robot's body is used to raise and lower the manipulator arm, while the other two actuators serve as a two-fingered gripper. Different tools may be attached to the manipulator in order to handle miscellaneous objects.

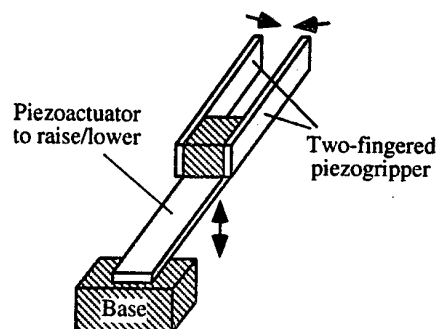


Fig. 12: Design of the SPIDER-II manipulator

The piezoactuators need a 60V differential power supply. They are controlled by a signal with pulse width modulation (PWM), which is generated by the C167 controller. The signal is amplified by a transistor circuit. The electronic components are small so that it was possible to integrate them into the robot board; thereby, no wires are needed to connect the electronics with the legs. All the actuators are controlled via a fast synchronous serial interface with 1 Mbits/second, which ensures real-time capability.

**4.3.2 Analysis and future work.** Compared to the robot MINIMAN [5-6], the robot size was reduced from 176cm<sup>3</sup> down to 105cm<sup>3</sup> (including the robot and its controller). The drive voltage was reduced from 300V to 60V, which in turn led to a much simpler design of the electronic circuits. Instead of 38 wires only 6 connect the robot and the the controller unit. Since MINIMAN sometimes has trouble dragging these wires behind it, this is an important improvement. A human operator can interact with the robot via a graphical interface implemented on a PC.

In contrast to the slip-stick motion, the SPIDER-II legs are at all times tightly connected to the ground by friction so that the robot motion is always predictable and very stable. The maximum span of objects to be grasped is 3mm and the maximum grasp force that can be applied by the robot manipulator is 0.23N. The new piezoceramics used allows low drive voltages and has excellent bending capabilities, but at the same time it has some drawbacks. The actuators are expensive and rather brittle: dropping the robot from about 3cm height led to the destruction of several actuators. Additionally, the piezoceramics has material tolerances of up to 10% and shows a hysteresis with a width of up to 10% of the nominal value. After changing the voltage, the actuator will have settled to its final position only after several seconds. To eliminate these problems a closed-loop control using the sensor information from a CCD-camera is being developed. The camera image is fed into a computer, which extracts features by image recognition and guides the robot in its task. This will enable the robot to complete a task without monitoring by a human.

## 5. Conclusions

Problems with the use of microrobots and micromanipulators were discussed in this paper. Presently, no easy solutions exist for assembling microparts, especially when one considers the available hardware and software and the high costs involved. It can be clearly seen, however, that the availability of very versatile automated microassembly stations will greatly contribute to the long-awaited industrial breakthrough of MST.

Several new piezoelectric micromanipulation robots developed for use in a microassembly desktop station were presented. The one-legged robot employs multilayered piezoactuators, which are driven by a voltage of  $\pm 20V$ . This robot may serve as a positioning unit. The three-legged robot SPIDER-I is driven by bimorphic piezoactuators with a voltage of  $\pm 20V$ . This robot may be used for positioning tasks, too. In contrast to these two units, the six-legged robot SPIDER-II is a complete micromanipulation robot which is capable both of moving over long distances and of handling microscopic objects. The robot includes six legs and a manipulator, which are driven by bimorphic piezoactuators with a voltage of  $\pm 60V$ .

All the presented robots have different abilities so that they can cooperate and perform microassembly tasks in a flexible microassembly desktop station. Force control of the robots and intelligent information processing via the desktop station are aspects of future work. Special tools, such as micro needles and micro pipettes for the manipulation of biological cells, micro grippers and micro glue applicators for the micro assembly, and special probes to measure voltage, current and temperature on a silicon chip, must be developed and tested as well.

## Acknowledgement

Part of the work was done by researches of the State Aviation Technical University of Ufa (Russia) whose travel was funded by the NATO grant HTECH.CRG 951002.

## References

- [1] U. Gengenbach et al., "Ein System zur automatischen Montage von Mikrosystemen", *im Tagungsband des 2. Statuskolloquiums des Projektes Mikrosystemtechnik, Forschungszentrum Karlsruhe*, 1995, S. 62-66.
- [2] O. Frick, "Fertigungsgerechte Montage- und Fügeverfahren für Mikrosysteme", *im Tagungsband des 2. Int. Kongresses für Mikrosysteme und Präzisionstechnik*, 1995, S. 44-51.
- [3] S. Fatikow and U. Rembold, "An Automated Micro-robot-Based Desktop Station for Micro Assembly and Handling of Microobjects", *in Proceedings of the IEEE Conference on Emerging Technologies and Factory Automation*, 1996, pp. 586-592.
- [4] S. Fatikow and R. Munassypov, "An Intelligent Micromanipulation Cell for Industrial and Biomedical Applications Based on a Piezoelectric Microrobot", *in Proceedings of the 5th Int. Conference on Micro Electro, Opto, Mechanical Systems and Components*, 1996, pp. 826-828.
- [5] B. Magnussen, S. Fatikow and U. Rembold, "Actuation in Microsystems: Problem Field Overview and Practical Example of the Piezoelectric Robot for Handling of Microobjects", *in Proceedings of the INRIA/IEEE Conference on Emerging Technologies and Factory Automation*, 1995, Tome 3, pp. 21-27.
- [6] S. Fatikow, B. Magnussen and U. Rembold, "A Piezoelectric Mobile Robot for Handling of Microobjects", *in Proceedings of the Int. Symposium on Microsystems, Intelligent Materials and Robots*, 1995, pp. 189-192.
- [7] R. Munassypov, B. Grossmann, B. Magnussen and S. Fatikow, "Development and Control of Piezoelectric Actuators for the Mobile Micromanipulation System", *in Proceedings of the Int. Conference on New Actuators*, 1996, pp. 213-216.

# ROBOT-LIKE SYSTEM FOR PHYSICAL IMITATION OF WEIGHTLESSNESS ON EARTH

Felix M. Kulakov, Victor B. Naumov, Evgeniy N. Smirnov, Alexey A. Evlampiev,  
Victor A. Shaluha and Vladimir N. Neokesariyskiy.

Russian Academy of Science

St. Petersburg Institute for Informatics and Automation, Robotics Laboratory

14th line, 39

St. Petersburg 199178, RUSSIA

e-mail: kul@mail.ias.spb.su

**Abstract.** In the article the methods of design of informational control system of untraditionally used robots are considered. Such an application is useful at the decision of the number not simple problems in various areas of science and engineering, in particular, in space research. One of these tasks is physical imitation of movement of bodies by applying to them forces.

**Keywords.** Robots, space robotics, untraditional applications, force-torque sensor.

**Introduction.** The development of effective methods of imitation on Earth of the movement of the rigid bodies in weightlessness condition is a rather urgent problem. The successful decision of this problem facilitates improvement and check on Earth of various assembly and other work, which need to be made on spacecraft on stationary orbits.

The known methods of imitation of movement of firm bodies under weightlessness, based on use aerostatic, hydraulic support, support on air pillow, have a lot of defects, among which main is the difficulty of 3D movement imitating of the bodies. As to imitation weightlessness by means of use of hydrosphere, in which body is immersed, in this case the main difficulties are connected to the large resistance of environment, which depends from the speed of body movement.

The proposed approach (the brief description of it is made in [1]) has not mentioned lacks. It is based on using of the robot-like device, in special gripper of which the body, under the form and size conterminous with the real simulated body is placed (the massinertional characteristics of it should not without fail coincide with the real). As such device it is possible to use the manipulator with six degrees of freedom, equipped by monitoring drives and wrist sensor, with the help of which are measured all six projections of resulting forces and torques, acting on body. The informational control system of the manipulator should generate the controls for six manipulator drives using the function of applied forces and torques (which provide such moving of the taken body) as though this moving occurs in weightlessness under the action of these applied forces and torques.

**Principle of weightlessness imitation.** At applying to the taken by the manipulator gripper to a body in the point A the force  $F$  and the torque  $M$  (Fig. 1) force-torque sensor measures the force  $F_s = F$  and the torque  $M_s = F \times r_a + M$ , where  $r_a$  - the vector, which appropriates to the point A of the applying force  $F$ , the  $M$  - the torque, acting on the body concerning the mass centre.

On these values, knowing the position of the mass centre of the body, in system of coordinates,

$$M_0 = M + F \times (r_a - r_c) = M + F \times r_a - F \times r_c = M_s - F_s \times r_c$$

The applied to the body force is directly measured by the sensor  $F = F_s$ . The equations of body movement have the kind:

$$\begin{aligned} m\dot{v}' + \omega \times v &= F_s = F; \\ I\dot{\omega}' + \omega \times I\omega &= M_s - F_s \times r_c = M_0 \end{aligned} \quad (1)$$

where  $\omega = (\theta'_1, \theta'_2, \theta'_3)$  and  $v = \alpha X'$  - the vectors of angular speed of the body and the linear speed of its mass centre in the system of coordinates, connected with the body,  $\alpha$  -  $3 \times 3$  - the matrix of the directing cosines, determining the rotation of the body in stationary system of coordinates,  $X = (x_1, x_2, x_3)$  - the vector of the mass centre position of the body in stationary system of coordinates,  $\theta_1, \theta_2, \theta_3$  - the angles of small rotation of the body.

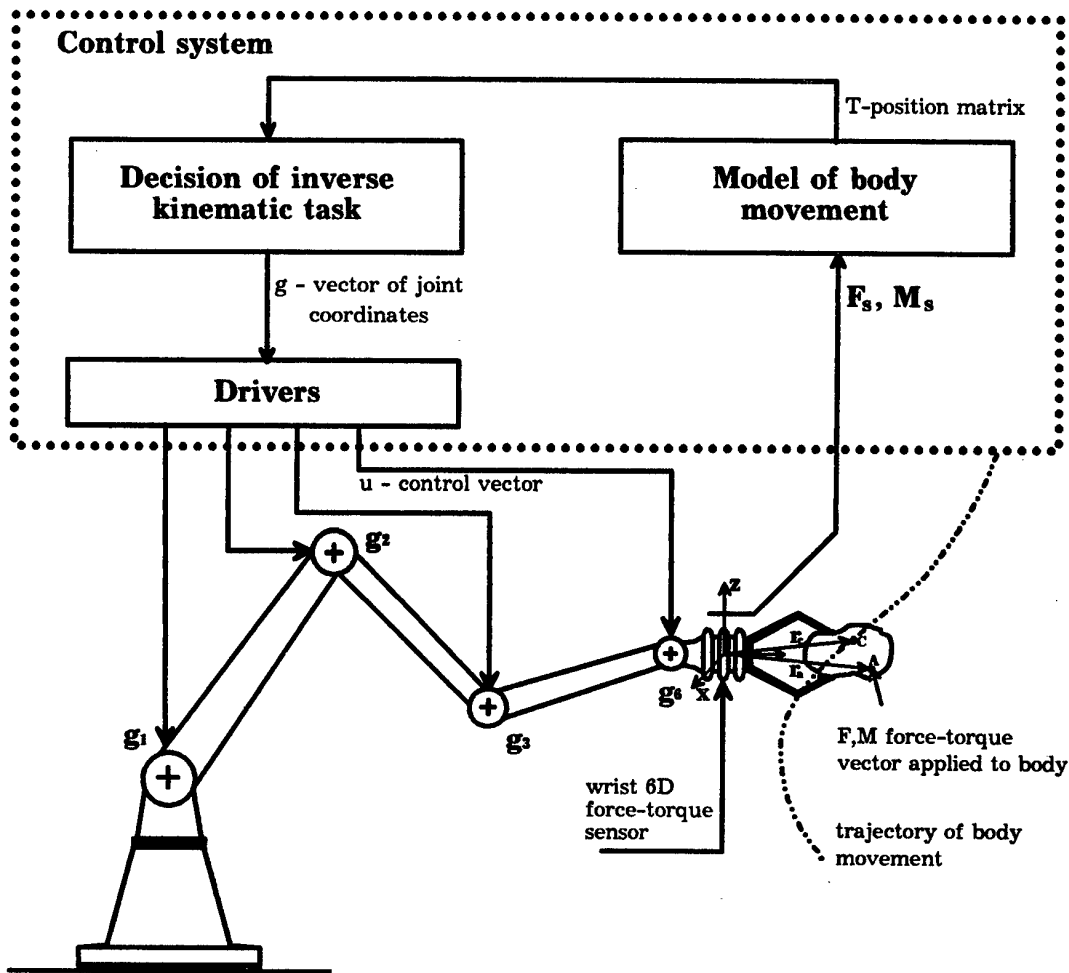


Fig.1 Weightlessness simulation

This equation is completely determined, if the mass  $m$  and the matrix of inertia  $I$  of the body, the values of the force-torque  $F_s, M_s$  and the position of mass centre of the body in sensor system of coordinates, defined by the vector  $r_c$  are known. Thus, knowing these values, and also the initial values  $\omega = \omega_{bg}, v = v_{bg}$  are possible to calculate the laws of variation in time of the

values  $\omega$  using one of methods of numerical integration of differential equations. As the vector of the linear and angular speed of the body  $v\omega = (v_1, v_2, v_3, \omega_1, \omega_2, \omega_3)$ , rigidly taken by the manipulator gripper, is the vector of speeds of its gripper, for pole of which mass centre of the body is accepted, this vector can be submitted as

$$v\omega = J(g)g',$$

where  $J(g)$  -  $6 \times 6$  - Jacobean matrix of the manipulator, being the function of the vector  $g = (g_1, \dots, g_6)$  of the joint coordinates. For the given particular manipulator this matrix is known.

Then the law of change of the joint speeds vector  $g'$  of the manipulator (at which body under action of applied force and torque moves how as if it is in weightlessness) can be expressed through received law  $v\omega(t)$  of change in time of the vector of speeds of a body as:

$$g' = J^{-1}(g) v\omega(t) \quad (2)$$

And the law of changing of the joint coordinates of the manipulator can be received as the result of the integration of Eq. 2:

$$g = \int_0^t J^{-1}(g) v\omega(t) dt \quad (3)$$

The calculated values of joint coordinates of the manipulator are possible to use as desirable values for tracking drives, fulfilling these values, therefore all six coordinates of the manipulator with given accuracy will repeat these desirable values. Thus, the system of imitation of body movement under the action of the applied to it forces functions as follows. Before the beginning of modeling in the information system values of the mass  $M$  and the matrix of inertia  $I$  of the modeled body are entered. These values should not without fail coincide with the mass of the real body. Besides, the position of mass centre of the body  $r_c$  in system of coordinates, connected to the sensor is entered. These data are necessary for reception of a left-hand part of the equations of dynamics of the body Eq. (1).

At applying to the body the force  $F$  and the torque  $M$  at the time moment  $t_0$  the sensor measures the torque  $M_s = F \times r_c + M$  and the force  $F_s = F$ . The software package of the weightlessness imitation system using this data and the position of mass centre of the body in system of coordinates of the sensor calculates the values  $F_s$ , applied to mass centre and the torque  $M_0$  of a body (the right part of equations of dynamics of the body Eq. (1)), executes the integration of equations of dynamics, i.e. calculates the values of linear speeds, the centre of masses of a body  $v = v_1, v_2, v_3$ , and its angular speeds  $\omega = (\omega_1, \omega_2, \omega_3)$ , in the moment torque  $t_1 = t_0 + \Delta t$ , where  $\Delta t$  - the step of calculations. At last, using these data the values of the joint coordinates  $g_i$  of the mechanisms by calculation of the integral (3) on the interval from  $t_0$  up to  $t_0 + \Delta t$  are defined. These values are the tasks for the tracking drives on the interval from  $t_1$  up to  $t_1 + \Delta t$ .

At the moment of time  $t_1$  the cycle of calculation, which is similar to the calculation on the previous interval, repeats once more, i.e. the measurement  $M_s$  and  $F_s$ , the calculation on these data of values  $F$  and  $M_0$ , finding of the values  $v_1, v_2, v_3$  and  $\omega_1, \omega_2, \omega_3$  for the time moment  $t_2 = t_1 + \Delta t$ , the calculation of values appropriate to them of coordinates  $g_i$ , which are the tasks

for the tracking drives on interval from  $t_2$  up to  $t_2 + \Delta t$ , is carried out. In the moment  $t_2$  the cycle of calculations repeats and etc. Obviously, than it is less the interval  $\Delta t$ , that it is less the delay in transfer of the tasks to the drive, and, hence, that exact imitation of the body movement in weightlessness under the action of applied forces. So, we considered the functions of the informational control system of robot-like device for bodies movement imitation in weightlessness conditions.

The probable updating of the generation procedure of the law of time variation for the mechanism coordinates is a procedure, based on the representation of body position in space with the help so-called similar (4x4) matrix of the position of body

$$T = \left| \begin{array}{ccc|c} & & & x_1 \\ & \alpha & & x_2 \\ & & & x_3 \\ \hline 0 & 0 & 0 & 1 \end{array} \right|,$$

In this case the new body position is characterized by the matrix  $T_{i+1} = T_i + \Delta T_i$ . The matrix  $\Delta T_i$  is

$$\Delta T_i = \left| \begin{array}{ccc|c} 1 & -\theta_3 & \theta_2 & \Delta x_1 \\ \theta_3 & 1 & -\theta_1 & \Delta x_2 \\ -\theta_2 & \theta_1 & 1 & \Delta x_3 \\ \hline 0 & 0 & 0 & 1 \end{array} \right|, \quad (4)$$

if there are the rotation of the body ( $\theta_1, \theta_2, \theta_3$ ) and displacement of its mass centre  $\Delta x = (\Delta x_1, \Delta x_2, \Delta x_3)$  concerning its current position, which is characterized by the matrix  $T_i$ , are small.

The vectors of small rotation  $\theta$  and small displacement  $\Delta x$  on  $i$  ( $i=1,2,\dots$ ) interval  $t_i < t < t_i + \Delta t$  can be determined by the equations

$$m\Delta x'' + \theta' \times \Delta x' = F_s, \quad (5)$$

$$I\theta'' + \theta' \times I\theta' = M_s - F_s \times r_c$$

$$\text{for } t = t_i: \Delta x(t_i) = \theta(t_i) = 0, \Delta x'(t_i) = \Delta x'(t_i)|_{i-1}, \quad \theta'(t_i) = \theta'(t_i)|_{i-1},$$

where  $\Delta x'(t_i)|_{i-1}, \theta'(t_i)|_{i-1}$  - the values  $\Delta x', \theta'$  at the end of previous ( $i-1$ ) interval.

Thus, the integration of Eq. (5) defines the values  $\Delta x$  and  $\theta$  for intervals 0,1,2,... and etc. Using these values it is possible to construct the sequence of matrixes  $T_0, T_1, T_2, \dots$  etc., determining the trajectory of the body movement.

Knowing the matrix  $T$  for the any time moment, it is possible to decide an inverse kinematic task, i.e. to define values of the joint coordinates of the mechanism by decision known algebraic equations, connecting elements of the matrixes of rotation  $T$  with joint coordinates of the manipulator.

**Weightlessness imitation method testing.** For check of serviceability of the proposed method the following hard- and software are used:

- the robot with the six degrees of freedom (type "PUMA" with the carrying capacity of 5 kg); this hardware executes the functions of the special mechanism, equipped with drives and special gripper, in which simulated body is located;
- the robot control rack for the robot type "SFERA 36", in which are used for purposes of weightlessness imitation realization the control system for the tracking drive of the robot,
- the wrist force-torque sensor, with help of which three projections of the forces and torques, acting on a body are measured; the sensor sensitivity: 0.02mB/N by force and 0.6 mB/(Nxm) by torque, pliant of construction: 0.05mB/N by force and 0.2 rad/(Nxm) by torque;
- PC AT 486, which connects by COM port (19200 baud) with "SFERA-36" and wrist sensor; with the PC using the procedure of integration (Eq. (1)) and the forming of joint coordinates, which are the inputs for tracking drives, are realized and also the program interface between human and weightlessness imitation device is organized.

The computer software includes:

- the program module of the formation and integration of the dynamics Eq. (1) for vectors  $v_0$  of mass centre of the small displacement of the body and its small rotation, and also the formation on them using Eq. (3) the values of joint coordinates vector  $g_i$ ;
- the program interface, ensuring the input and editing of the parameters of the simulated body (mass, matrix of inertia, position of centre mass), which are necessary for Eq. (1) formation;
- the service program module for the sequential channel of communication with the exchange speed 19200 baud between PC and robot control rack and between the wrist sensor and PC.

In given physical model it was possible to decrease to the acceptable minimum the total time, expended for change of the force and of the torque, put to body, the calculation of the values  $\Delta x$  and  $\theta$ , finding on them the increments of the joint coordinates  $g_i$ , and also the transfer of these values for of tracking drives. It makes 16 msec.

With help of the created system (Photo 1) the physical modeling of movement of bodies with weightlessness imitation was conducted.



Photo 1. Test of method of weightlessness simulation

The masses of bodies were the following: 0.050 ton, 0.20 ton, 0.600 ton, 1.2 ton, 3 ton, 5 ton, 10 ton; the matrixes of inertia were chosen so, that with abovestated mass they corresponded to the bodies of the cubic form, executed from aluminum.

The results of modeling have shown the following:

- the total error of the deviation of the trajectory imitated of movement is increased with reduction of the mass of the imitated body and makes 3% from the length of the way, passed by the mass centre of a body, during the time 15 sec for the body 1.2 ton and 1% for the body 10 ton;
- the imitation of movement of bodies of small mass has appeared inconvenient because of the fast exit of the body over the borders of the zone of free moving, stipulated by the construction of the robot;
- the above-stated accuracy of imitation of the movement of a body has a place at short-term applying of the forces, not more than 1.5 sec.

In case of grasping and deduction of the body in current of long-duration time there is the infringement of the system action, since the processes, stipulated by the dynamic properties of manipulator control system, begin to act. It demands the perfection of the imitation system.

### ***References***

1. Kulakov, F. M. (1996, April). Some Russian Researches on Robotics. Robotics and Autonomous System J. Vol.21, N3, Elsevier, Amsterdam-Lausanne-NewYork-Oxford-Tokyo.

# PECULIARITIES OF CONTROL BY ROBOTIC SYSTEMS USING ALLOYS WITH SHAPE MEMORY EFFECT

S.V.Panteleev, A.F.Pervishko, A.I.Priadko, S.I.Semenov, I.E.Viahhi

Institute of Robotic and Technical Cybernetics

Tikhoretsky pr., 21, 194064,

St-Petersburg, Russia

e-mail: vahhi@mail.rtc.neva.ru

**Abstract.** Various robotic system are developed, made and investigated. A force drives in these systems are made from single crystals CuAlNi with shape memory effect. The basic theoretical researches are spent and special control system for these devices is developed. It can supervise target parameters of devices in any point and at any time.

**Keywords.** Shape memory effect, robotic system, control system, actuator, force element.

**Introduction.** At the last time alloys with shape memory effect (SME) are spread as application in various technical devices. Shape memory is a physical phenomenon by which a plastically deformed metal is restored to its original shape by a solid state phase change caused by heating. The explanation for the shape memory response is found in the strong crystallographic relationship between the phase stable at low temperature, called martensite, and the phase stable at high temperature, called austenite.

Drives, in which force elements are made of alloys with SME have begun to be used in industrial robots and mechanotronics from middle of 80's due to the numerous advantages, basic of which are [1]:

- Large value of relation of carrying capacity to own weight.
- Simplicity of a design.
- Large deformation and ability to make difficult movements.
- Absence gear mechanism.
- Good remote control.
- Presence of function of a sensitive element.

Our Institute possesses experience of work on development rotary and linear drives of robotic systems (RS) using as force working elements from single crystals CuAlNi with SME [2]. In result of theoretical and practical researches general structure of the top level of management by robotic complexes is developed. On its basis is developed and manufacturing control system for RS with SME.

## ***Chapter 1. GENERAL STRUCTURE OF THE TOP LEVEL OF MANAGEMENT BY ROBOTIC COMPLEXES***

### ***Terminology.***

- The operating zone - zone, in which RS can carry out the whole complex of working operations.
- Model of a operating zone - mathematical, logic, verbal or other description of a operating zone and being in it of objects (including RS).
- The pseudocamera - set video-data, generated by results of processing model of a operating zone. Emulates work of the real camera.
- 3D-stage - Visual display of model of a operating zone on the screen of the monitor (as set of projections, kind with real or pseudo-camera and etc.).
- The videoprocessor - hardware or programm system, forming on the information with videocameras and sensors model of a operating zone and a 3D-stage.
- Target direction - set "object-action", perceived on a rule "to carry out action over object". The choice of object is made by its allocation on a 3D-stage.

***The block diagram, possible modes of operations.*** The generalized block diagram of RS, controlled by target direction, is show on Fig. 1.

The 3D-stage generated by the videoprocessor is show on the monitor of the personal computer. Specifying by cursor of object of a 3D-stage, the operator causes for it the menu of typical operations (for example, Fig. 2). The choice of operation finishes formation of target direction, which by the personal computer is translated in

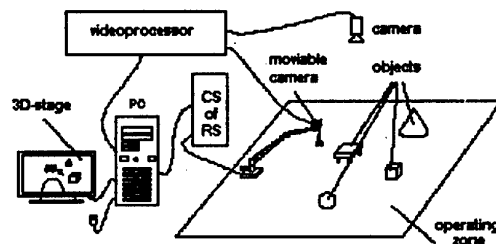


Fig.1 General structur scheme of control by target direction

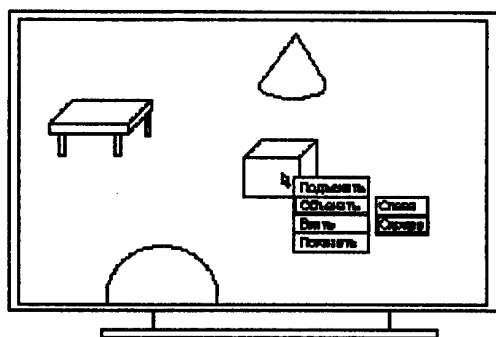


Fig.2. Typical operation menu of system "Target direction"

tasks for drives RS. If the typical operation has not enough for the decision of a problem, following mode of any mobile element of RS (link of the manipulator, gripper and etc.) to a movement of the cursor on a 3D - stage is possible. We shall consider an example (see Fig. 3): the manipulator is in a position 1 and it is necessary push object 3 from a surface of object

1. By including a following mode of gripper to moving the cursor, we relocate it (ONLY on a 3D-stage) to a position 2. The videoprocessor simulates new model of a working stage and refresh a 3D-stage. If the final result suits the operator, he gives a command on real improvement of such movement, if do not - operation is cancelled. Mode of the simple direction of coordinates (general or absolute) target point, law of change of a trajectory and etc. is also possible.

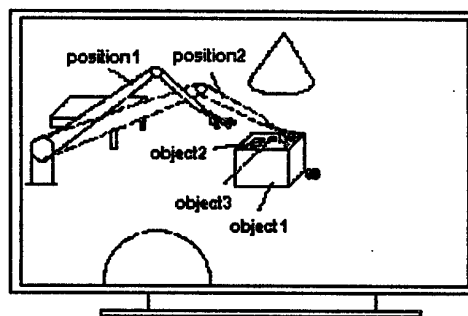


Fig.3. Following mode

*The videoprocessor.* The videoprocessor during construction of model of a operating zone should develop hypotheses about the form of invisible surfaces and, in a general kind, about

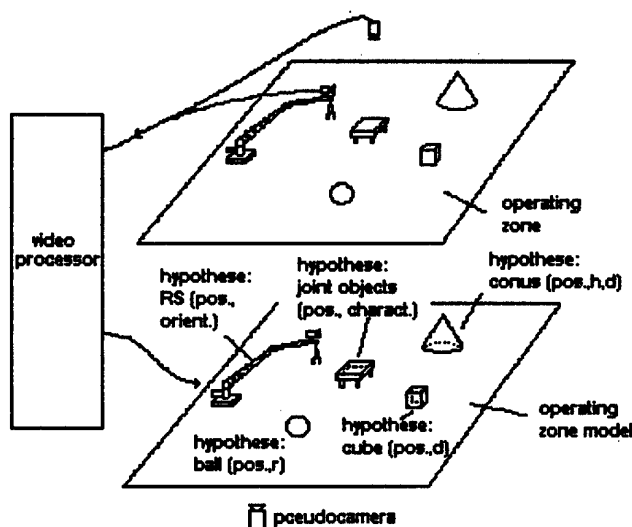


Fig.4. Modeling of operating zone

the form of objects in a operating zone (Fig.4). Those hypotheses can be change after change of position and/or orientations of videocameras and sensors. The greatest problem at creation of the videoprocessor - presence of a mobile videocamera, located on object of a operating zone - the manipulator. Accordingly at a movement of the manipulator internal parameters of the videoprocessor (base

of sight, orientation of optical axes and etc.) change, that strongly complicates its optimization on speed. The second problem - problem of qualitative recognition of separate objects and

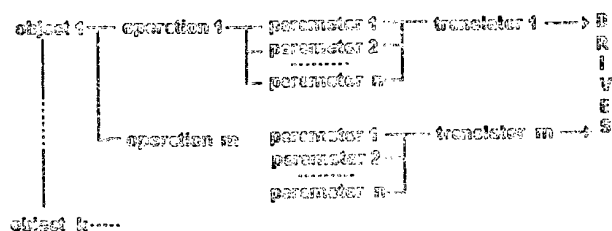
their characteristic and/or of base points for determination of coordinates of object in 3dimension space from two flat images from videocameras. The problem of identification of objects and determination of their coordinates can be much simplified by use distance sensors, concurrent to cameras. The third problem - problem of speed. Processing the information from telecameras and sensors, modeling of a operating zone and formation of a 3D-stage owe, whenever possible, to come true in real time scale for maintenance of constant adequacy of the image on the monitor to a real conditions.

*Typical operations.* The set of typical operations in system of target direction should satisfy at least to two requirements:

- To depend from type/size/position of object (for example, it is senseless to give a command as " to grasp ball ", if gripper is designed for gripping of objects with parallel surfaces.
- To have functional completeness of actions (for example, for cube: to grasp, to show, to approach in, to go round from the right / at the left, to cancel operation), carried out without further specification.

For maintenance of the first requirement presence of knowledge about interrelation of objects and operations allowable over them is necessary. These knowledge can to collect and actualing while in process of exploitation of RS. For maintenance of the second requirement it is necessary to set for each typical operation a set of parameters by default. Thus the parameters can be changed by the operator at any moment, i.e. should be optional, but not directive.

The most convenient to realize a subsystem of typical operations as database:



The translator translate the name of operations over object in a set of influences on a drive of RS in view of meanings of parameters.

*Following mode.* In a following mode of elements of RS to a movement of the cursor on a 3D-stage there is the problem of logic connection of a movement of the cursor and allocated element of RS on the flat image and in real space. The most simple way of the decision - simultaneous display three projections of a operating zone and RS, thus for moving the cursor in each projection independent combinations of keys are used. Way is more convenient for the operator, at which except three projections a kind from the pseudocamera is present also. Position and orientation of pseudocamera may be change by operator. The movement in a plane of the image of the pseudocamera comes true by the mouse without pressed keys, and on the third axis - by mouse with pressed (for example, right) key. As in a following mode all actions will be carried out in two stages (modelling of movement / agree/ real movement), during modeling a movement it is necessary to remember the information on position of links, their linear and angular speeds - i.e. it is necessary to have the complete description of a trajectory of a movement of a chosen element of RS. Mode of correction of a given trajectory before its real improvement should be stipulated.

*Mode of the simple direction.* In this mode a trajectory (or its target point) some link of RS is

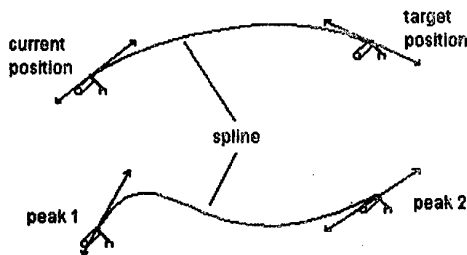


Fig.5. Trek direction as spline

set. The most convenient to set a trajectory as adjustable spline in three projections with the control by the view from the pseudocamera, as in a following mode (Fig. 5).

## **Chapter 2. PRINCIPLES OF CONTROL BY ROBOTIC SYSTEMS USING ALLOYS WITH SHAPE MEMORY EFFECT**

An analysis and a generalization of the experience in development of devices for various purposes based on SMA have given a possibility to develop a block diagram of controlled robotic systems(Fig.6). An operational body (OB) of the force element of SMA equipped with an electrical heater and a temperature sensor forms an electrical actuator (EA) which is the simplest structural unit of the robotic system. A transformation of the thermal energy into

mechanical one takes place inside operational body volume. Depending on an application, the force element is made in a form of a rod, plate, wire, or coiled spring.

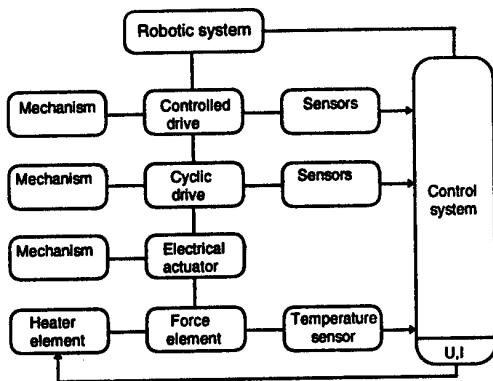


Fig.6. Block diagram of robotic system based on the application of SMA.

A system of electrical actuators connected by the corresponding mechanism forms the simplest cyclic drive (CD) which is able to fulfill its functions repeatedly under corresponding electrical actuator temperature control. Cyclic drive is built in such a way that electrical actuator operates in tension, compression, bending, or twisting, and due to this fact cyclic drive parameters can be easily

defined via kinematic and force calculations. Electrical actuators restore their shape during martensitic transformation. Thus, cyclic drive transforms a translation displacement of actuators  $x$  or their rotation angles  $j$  and corresponding forces  $f$  and torques  $m$  into a movement of the output element of cyclic drive -  $X, F$  with corresponding forces  $F$  and torque  $M$ . These transformations in general form can be written as follows:

$$[x, j, f, m] = [II] [X, F, F, M], \text{ where } [II] - \text{transformation operator.} \quad (1)$$

In robotic systems it is necessary to provide time control of the motions and/or forces generated. A controlled drive has to consist of :

- one or several cyclic drives;
- mechanisms of transmission, linking various elements of controlled robotic system;
- displacement and force sensors to realize coordinate and force control;
- control system which controls the drive displacement via temperature control of the force elements.

The peculiarity of construction of force-movement of management of drive with SME consist that knowledge of properties of a material with SME manages without force-movement of the sensor. The realization of this principle is shown on an example of gripper (Fig.7.), which is capable to keep subjects of various rigidity and durabilities. So, gripper is capable to develop any effort in limits of 0-300 N, with accuracy 2 N.

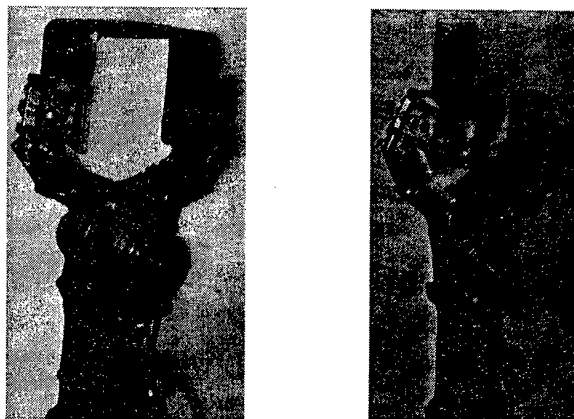


Fig.7. Adaptive finger-type gripper

Functioning a control system comes true as follows. The controller SAB-166 carrying out the instruction of the operator from the board of control based on the personal computer, realizes algorithm of management on the basis of signals of sensors of a feedback (sensors of temperature and moving). By defining a direction of given moving on size of a signal potentiometric sensor of moving, the controller give command of management on unit of the pulse-width modulator in conformity with the law of a two-channel proportional regulator of temperature. On achievement of given value of moving in view of a stop zone, the controller is switched on the proportional - differential law of regulation of temperature in view of an error on moving.

**Conclusions.** General structure of the top level of management by robotic complexes is developed. On it base are consider the principles of control by robotic systems using alloys with shape memory effect. Control system for this robotic system was developed, manufactured and reseached.

### **References**

1. Furuya Y., Shimada H., "Shape memory actuators for robotic applications", Engineering aspects of shape memory alloys / edited by T.W.Duerig, 1990, pp. 338-355.
2. Viakhi I., Panteleev S., Pulnev S., Priadko A., Judin V., "Application of materials with shape memory effect in robotic systems and devices operating under extreme conditions", International Conf. Instrumentstion in Ecology and Human Safety, St.Petersburg, 30 October-2 November 1996, pp. 54-55.

# DYNAMICS OF A CO-OPERATING ROBOTIC FIXTURE FOR SUPPORTING AUTOMATIC DEBURRING TASKS

R.C. Michelini, G.M. Acaccia, M. Callegari, R.M. Molfino, R.P. Razzoli  
*Industrial Robot Design Research Group, University of Genova*  
*Via all'Opera Pia, 15A - 16145 Genova, Italy*  
*e-mail: pmar@dimec.unige.it - web: www.pmar.dimec.unige.it*

**Abstract.** The paper deals with a cooperating robotic rig, to fit out a work-station, aiming at the automatic deburring and grinding of massive pieces with compliant three-dimensional surfaces. The operation is traditionally performed by manual operators due to the complexity of the tool paths, that need be tracked with high accuracy while preserving calibrated normal pressure. The rig is based on a moving platform, with six degrees of freedom, whose position and attitude is servoed to the task progression of the deburring operation. The finishing tool is carried by a six degrees of freedom arm and the combined motion of overall redundant equipment is exploited to supply the dexterity and versatility figures required by the deburring tasks. The outlines of the machining process are recalled, showing why robots with cooperation provide a proper solution; the development can be actually undertaken on condition to have the consistent characterisation of the equipment dynamics, to set the control coordination and fit the task programming up to the requested performance figures.

**Keywords.** Cooperating robots, redundant active systems, parallel manipulators, grinding and deburr finishing.

## INTRODUCTION

Instrumental robotics aims at enabling task-driven automatic sequences, depending on conditioning applications and requested performance figures. Their implementation is, on one side, simplified as compared to autonomous behaviour robot cases, due to the well structured environment the equipment has to interact with; it is, on the other hand, highly compelling since the issues need provide return on investment in intelligent automation, by carrying out the scheduled tasks with given accuracy, dexterity, efficiency and versatility figures while fulfilling economy of scope rules [1-3].

In short, task orientation is powerful help for setting robot's effectiveness, provided that functional models are stated, detailing the manipulation dynamics with the certainty ranges of the expected performance figures. The field of instrumental robotics is fitted up by talented solutions for factory automation; industrial manipulators support almost all work-cycles of the production processes [4]. Manufacturing enterprises, however, face some limitations at the stage of deburr finishing; most of the time, they still have resort to manual interventions when geometric constraints, edging compliance, shape variability and tolerated span represent highly demanding mix of requirements. The automated deburring, indeed, up now runs into deficiencies, that prevent accurate and efficient operation. The instrumental robots have perhaps developed by miming the interfacing at manual processes, even if no reason at all exists that the task-oriented solutions should develop, with reference to anthropocentric rules [5-9].

The switching to robotic equipment for precision deburring has, probably, to be reached by looking over again the task sequences account, to establish a set-up aiming at smooth engagement, position-and-force control, steady repetitiveness and, in general, at highly adaptive fit-outs based on skilful survey of the work progress to restore correctness. By robotising, once accuracy figures are achieved, productivity and tolerances are preserved according to total quality conditions, therefore assuring improved product finishing as compared to manual operation. The study moves from such ideas; the steps related to the functional modelling of robotic equipment are summarised in the following, since they are critical accomplishment for assessing the effectiveness of any consistent solution.

With a conventional anthropocentric approach, a deburring robot would present as a performant manipulating arm, with the finishing tool at the end-effector, conveniently actuated and

extensively sensorised. Of course the functionality of such equipment shows evident limitations that, eventually, a very skilful and trained operator can overcome, with craft and inventiveness, properly adapting operation modes to task progression.

Robotic equipment with co-operation is an important alternative to be considered [10]. The investigation addresses such goal; the deburring tool is operated by a six degrees-of-freedom arm; the work-piece is borne by a (similarly, six degrees-of-freedom) rig, whose mobilities are controlled, to interact with the machining end-effector. The rig, in this context, reduces to a platform, whose position and attitude are driven with due account of task-oriented requests. Functional innovation is 'co-operation': task setting is related to the ability of establishing work-sequences that depend on the deburring cycles to be executed. The dynamics of the bearing rig needs be programmed concurrently with the dynamics of the operating arm.

## RELATIONAL CONTEXT FOR MOTION SPECIFICATION

The co-operating engagement of the piece-supporting rig needs proper performance in terms of position tracking ability, reactive stiffness, attitude controllability, etc., in such a way to upgrade arm's accuracy, dexterity, efficiency and versatility according to requests. The mechanical architecture of this rig is based on a parallelly actuated platform, designed and realised by Romiti and Sorli at the Polytechnic of Turin [11] to help performing critical assemblage operations: the design has been revisited by Acaccia *et al.* [12], also in view of possible micro-robotics applications [13].

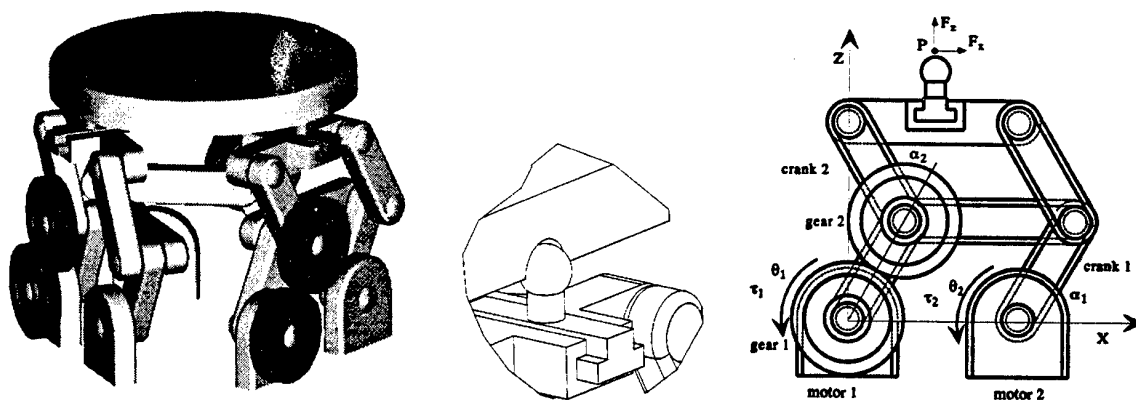


Fig. 1. In-parallel actuated robotic fixture: (a) solid model; (b) linking of table and upper parallelogram (particular); (c) side view of one platform's driver

Figure 1a shows a solid model of the fixture: it mainly consists of a plate (of radius " $r$ "), driven by moving the points  $P_1$ ,  $P_2$ ,  $P_3$ , placed at the vertices of an equilateral triangle. Each one of the three drivers is obtained by superimposing two planar parallelograms that move on a vertical plane: Fig. 1c shows the side view of one driver, while the top view of the platform is sketched in Fig. 2 in both central and perturbed configurations.

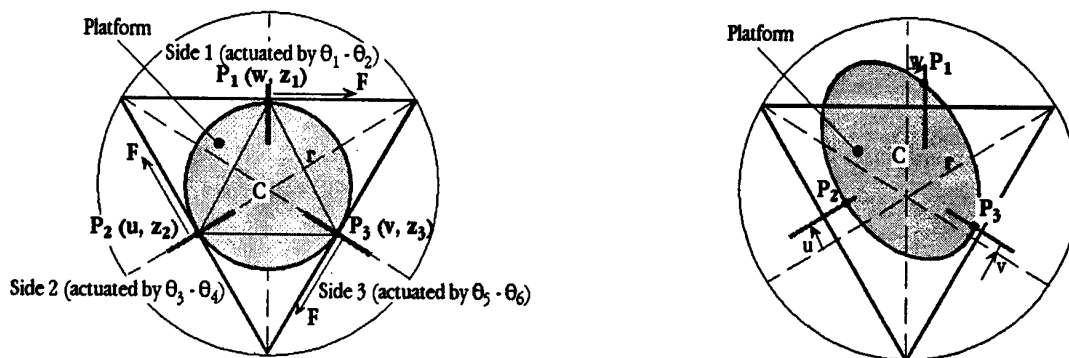


Fig. 2. Top view of the platform (central and perturbed configurations)

The plate is fixed to the upper parallelograms by the mentioned points  $P_1, P_2, P_3$ : the linking is assured by spherical bolts that are impinged at the lower side of the platform and can also slide along linear guideways, fixed to the top beam of the upper parallelogram, Fig. 1b. The guiding slots are placed orthogonally to their carrying beams, to form an angle of  $120^\circ$  between each other; the assembly with the plate results in a system with six degrees of freedom.

The coupled parallelograms have beams linked by ball bearings to reduce friction; they are driven by separate DC motors, solid with the rig base to reduce inertial effects. For position accuracy, the upper four bars, Fig. 1c, are actuated by a backlash-free gear train, not linked to the bottom cranks; the lower four bars are directly driven by the twin motor. The set-up repeats three times and the rig has six servo-motors to be controlled.

The rig behaviour is analysed [12] by assessing: · the actuation kinematics, using the geometric constraints to yield forward and backward mapping of work-space and control co-ordinates; · the open-loop dynamics, combining inertial terms and constrained motion to generate the reflected loads on the driving commands. The basic issues are hereafter summarised.

### Assessing the kinematical behaviour

The kinematical analysis of the parallel fixture is performed by first studying the platform kinematics and then considering the actuation system. A local frame  $\{\underline{\lambda}, \underline{\mu}, \underline{\nu}\}$  is attached to the platform itself and located at the centre point C, with the axes  $\underline{\lambda}$  and  $\underline{\mu}$  on the platform's plane and  $\underline{\nu}$  in the outward normal direction; three angles  $\psi, \theta, \phi$  are defined between this frame and a global frame in order to fix the orientation of the equipment; they define the classical RPR set, except for a fixed angular offset:

$$[R(\psi, \theta, \phi)] = [R_\nu(\psi + \pi)] \left[ R_\mu \left( \phi - \frac{\pi}{2} \right) \right] [R_\nu(\theta + \pi)] \quad (1)$$

The position of the platform is defined by giving the Cartesian coordinates of C, so that the work-space coordinates set  $\underline{x}$  is defined as:  $\underline{x} = [x_0 \ y_0 \ z_0 \ \psi \ \phi \ \theta]^T$ ; an internal coordinates set  $\underline{z}$  is given by assigning the absolute displacement of each slider's guide in its own reference plane, as shown in Fig. 2:  $\underline{z} = [z_1 \ z_2 \ z_3 \ u \ v \ w]^T$ .

The forward kinematic analysis is quite complex and gives out 8 different solutions for the direct transform:  $\underline{x} = \underline{f}(\underline{z})$  while the backward kinematics yields just one solution:  $\underline{z} = \underline{f}^{-1}(\underline{x})$ .

Once the Jacobian matrix is obtained (for a given configuration), the velocity analysis of the platform is easily performed, yielding:

$$\dot{\underline{x}} = [J(\underline{x}(\underline{z}))] \dot{\underline{z}} \quad \text{and} \quad \dot{\underline{z}} = [J(\underline{x}(\underline{z}))]^{-1} \dot{\underline{x}} \quad (2-3)$$

In the same way, the acceleration analysis is solved by the evaluation of the rate of the Jacobian:

$$\ddot{\underline{x}} = [J(\underline{x}(\underline{z}))] \ddot{\underline{z}} + [\dot{J}(\underline{x}(\underline{z}), (\dot{\underline{x}}(\underline{z}), \dot{\underline{z}}))] \dot{\underline{z}} \quad \text{and} \quad \ddot{\underline{z}} = [J(\underline{x}(\underline{z}))]^{-1} \ddot{\underline{x}} - [\dot{J}(\underline{x}(\underline{z}), (\dot{\underline{x}}(\underline{z}), \dot{\underline{z}}))] \dot{\underline{z}} \quad (4-5)$$

Of course, the time derivatives of the Euler angles can be better substituted by the angular velocity and acceleration vectors. The study of the actuation system is developed taking separately into account the three articulated mechanisms that drive the pertinent sides of the platform: an obvious choice for the work-space coordinates is the pair of displacements  $\{x_{p_i}, z_{p_i}\}$  of each slider's guide in the reference vertical plane; they correspond to the (internal) co-ordinates  $\{u, z_1\}, \{v, z_2\}, \{w, z_3\}$  for the three sides of the platform respectively. As for the internal coordinates, two sets of generalised coordinates are defined:  $\theta_1$  and  $\theta_2$  are the absolute rotations of the two motors, while  $\alpha_1$  and  $\alpha_2$  are the relative rotations of the two cranks respectively; they are simply related by:

$$\begin{cases} \dot{\alpha}_1 = \dot{\theta}_2 \\ \dot{\alpha}_2 = -i\dot{\theta}_1 + (i-1)\dot{\theta}_2 \end{cases} \quad \text{where: } i = \frac{\omega_{\text{follower}}}{\omega_{\text{driver}}} = \frac{R_1}{R_2} \quad (6)$$

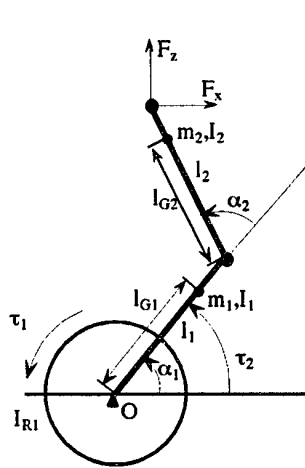
Calling  $\underline{q}$  the 6-dimensional vector of all the internal co-ordinates (either the  $\theta_i$  or the  $\alpha_i$  set), from the simple kinematics of the planetary actuation system, the pertinent solution of the direct kinematic transform is easily obtained:  $\underline{z} = \underline{g}(\underline{q})$  and, similarly, the 8 different solutions of the inverse kinematic transform are:  $\underline{q} = \underline{g}^{-1}(\underline{z})$ .

The velocity and acceleration analyses are straightforward and carried on according to the developed set-up. Ref. [14] provides the exact placement of the frames, the rigorous definition of the angles and all the mathematics that has been implied by the previous formulation.

### Assessing the dynamical behaviour

The dynamic model of the overall mechanical system is obtained again by assembling the separated models of the platform and the actuation system, whose dynamics is evaluated by considering separately the three articulated mechanisms that drive the related sides of the platform. It is noticed that the particular structure of the parallelograms allows a simple dynamic equivalence with a compound double-pendulum swinging in a vertical plane, Fig. 3; so that, in spite of the high number of links in the mechanism, all the masses and momenta of the articulated quadrilaterals have been easily included in the model.

Note that  $F_x$  and  $F_z$  (Fig. 1c) are the internal reaction forces at point  $P_i$  between the platform and the  $i$ -th actuation side ( $i=1,2,3$ ): since the point of impingement of this force is insignificant along the upper horizontal beam, due to the kinematics of the structure, it is considered to attack the equivalent model (shown in Fig. 3) at the distal point of the second crank.



$$\underline{\tau} = [H] \ddot{\theta} + \underline{V}_1 \dot{\theta}_1^2 + \underline{V}_2 \dot{\theta}_2^2 + \underline{V}_{12} \dot{\theta}_1 \dot{\theta}_2 + \underline{G} + [J]^T \underline{F}$$

$$[H] = \begin{bmatrix} i^2 m_2 l_{G_2}^2 + I_{R_1} + I_{M_1} + i^2 I_2 & -[i^2 m_2 l_{G_2}^2 + i m_2 l_1 l_{G_2} \cos(-i\theta_1 + (i-1)\theta_2) + i^2 I_2] \\ -[i^2 m_2 l_{G_2}^2 + i m_2 l_1 l_{G_2} \cos(-i\theta_1 + (i-1)\theta_2) + i^2 I_2] & i^2 (I_2 + m_2 l_{G_2}^2) + I_1 + I_{M_2} + m_2 l_1^2 + 2 i m_2 l_1 l_{G_2} \cos[-i\theta_1 + (i-1)\theta_2] \end{bmatrix}$$

$$\underline{V}_1 = \begin{pmatrix} 0 \\ -i^2 m_2 l_1 l_{G_2} \sin[-i\theta_1 + (i-1)\theta_2] \end{pmatrix} \quad \underline{V}_2 = \begin{pmatrix} -i m_2 l_1 l_{G_2} \sin[-i\theta_1 + (i-1)\theta_2] \\ (i-i^2) m_2 l_1 l_{G_2} \sin[-i\theta_1 + (i-1)\theta_2] \end{pmatrix}$$

$$\underline{V}_{12} = \begin{pmatrix} 0 \\ 2 i^2 m_2 l_1 l_{G_2} \sin[-i\theta_1 + (i-1)\theta_2] \end{pmatrix}$$

$$\underline{G} = \begin{pmatrix} -i m_2 l_{G_2} \cos[i(\theta_2 - \theta_1)] \\ i m_2 l_{G_2} \cos[i(\theta_2 - \theta_1)] + m_2 l_1 \cos \theta_2 + m_1 l_{G_1} \cos \theta_2 \end{pmatrix} \underline{g}$$

$$[J] = \begin{bmatrix} -i l_2 \sin[i(\theta_2 - \theta_1)] & [i l_2 \sin[i(\theta_2 - \theta_1)] + l_1 \sin \theta_2] \\ i l_2 \cos[i(\theta_2 - \theta_1)] & -i l_2 \cos[i(\theta_2 - \theta_1)] + l_1 \cos \theta_2 \end{bmatrix}$$

Fig. 3. Dynamic model of one actuation side

The dynamics of this structure can be easily solved using either Newton-Euler or Lagrange approach and actually both have been exploited to validate the final resulting equations. The motors torques are thereafter obtained as functions of the absolute rotations  $\theta_i$  and are shown in the table of Fig. 3. These equations can be written for each side, so that a set of 6 differential equations in the 6 variables  $\theta_i$  is obtained:

$$\begin{cases} \tau_1^{(i)} = f_1^{(i)}(\theta_1^{(i)}, \theta_2^{(i)}, \dot{\theta}_1^{(i)}, \dot{\theta}_2^{(i)}, \ddot{\theta}_1^{(i)}, \ddot{\theta}_2^{(i)}, F_x^{(i)}, F_z^{(i)}) \\ \tau_2^{(i)} = f_2^{(i)}(\theta_1^{(i)}, \theta_2^{(i)}, \dot{\theta}_1^{(i)}, \dot{\theta}_2^{(i)}, \ddot{\theta}_1^{(i)}, \ddot{\theta}_2^{(i)}, F_x^{(i)}, F_z^{(i)}) \end{cases} \quad (i=1,2,3) \quad (7)$$

The coupling effects  $F_x$  and  $F_z$  with the platform should be further explicited, by writing the dynamic equations of the plate-payload complex; the usual Newton-Euler equations are written as follows:

$$\Sigma \underline{F} = \underline{F}_{\text{reac}} + \underline{F}_{\text{ex}} + \underline{F}_g + \underline{F}_{\text{in}} = 0 \quad (8a)$$

$$\Sigma \underline{M}^{(G)} = \underline{M}_{\text{reac}}^{(G)} + \underline{M}_{\text{ex}}^{(G)} + \underline{M}_{\text{in}}^{(G)} = 0 \quad (8b)$$

$\underline{F}_{\text{ex}}$  and  $\underline{M}_{\text{ex}}^{(G)}$  are the effects (reduced to the centre of mass) of the mechanical interactions with the environment;  $\underline{F}_g$  is the total weight of the system and  $\underline{F}_{\text{in}}$ ,  $\underline{M}_{\text{in}}^{(G)}$  are respectively the vector of inertia forces and torques caused by the table movements.

Their components are:

$$\underline{F}_{\text{in}} = \begin{pmatrix} F_{x,\text{in}} \\ F_{y,\text{in}} \\ F_{z,\text{in}} \end{pmatrix} = -m \begin{pmatrix} \ddot{x}_G \\ \ddot{y}_G \\ \ddot{z}_G \end{pmatrix} \quad \text{and} \quad \underline{M}_{\text{in}}^{(G)} = \begin{pmatrix} M_{x,\text{in}} \\ M_{y,\text{in}} \\ M_{z,\text{in}} \end{pmatrix} = -\frac{d}{dt}([I]\underline{\omega}) = -(\underline{\omega} \wedge [I]\underline{\omega} + [I]\dot{\underline{\omega}}) \quad (9)$$

where:  $[I] = {}^0_L[R] [\bar{I}] {}^0_L[R]^T$ ;  $m$  and  $[\bar{I}]$  are respectively the mass and the central inertia tensor

of the whole part-platform system;  ${}^0_L[R]$  is the rotation matrix between the central (local) reference frame  $\{\underline{\lambda}, \underline{\mu}, \underline{\nu}\}$  and the world reference frame.

$\underline{F}_{\text{reac}}$  is the resultant of the internal reaction forces between the platform and the actuation system and is easily computed, taking into account the geometry of the platform itself (Fig. 2):

$$\underline{F}_{\text{reac}} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3 = \begin{pmatrix} F_{x1} - 0.5F_{x2} - 0.5F_{x3} \\ 0.5\sqrt{3}F_{x2} - 0.5\sqrt{3}F_{x3} \\ -F_{z1} - F_{z2} - F_{z3} \end{pmatrix} \quad (10)$$

The corresponding torques  $\underline{M}_{\text{reac}}^{(G)}$  are first computed about the centre of the platform C, and then transported to act around the overall centre of gravity G of table and workpiece:

$$\underline{M}_{\text{reac}}^{(G)} = - \begin{pmatrix} 0,5\sqrt{3}l_3(F_{x3} - F_{x2}) - F_{z1}(l_2 + r) + (0,5r - l_2)(F_{z2} + F_{z3}) \\ l_3(F_{x1} - 0,5F_{x2} - 0,5F_{x3}) + F_{z1}l_1 + F_{z2}(l_1 - 0,5\sqrt{3}r) + F_{z3}(l_1 + 0,5\sqrt{3}r) \\ -F_{x1}(r + l_2) + F_{x2}(-r + 0,5\sqrt{3}l_1 + 0,5l_2) + F_{x3}(-r - 0,5\sqrt{3}l_1 + 0,5l_2) \end{pmatrix} \quad (11)$$

where it has been called:  $(G - C) = (l_1 \ l_2 \ l_3)^T$ .

Substituting the above results (11) in Eq. (8), an algebraic system of six linear equations with the six variables  $F_{x1}$ ,  $F_{x2}$ ,  $F_{x3}$ ,  $F_{z1}$ ,  $F_{z2}$ ,  $F_{z3}$  is obtained; it can be solved to give the following result:

$$\begin{pmatrix} F_{x1} \\ F_{x2} \\ F_{x3} \end{pmatrix} = \frac{1}{3r} \left\{ \begin{pmatrix} l_2 - 2r \\ l_2 + r \\ l_2 + r \end{pmatrix} F_{x,\text{ex}} - \begin{pmatrix} l_1 \\ l_1 + \sqrt{3}r \\ l_1 - \sqrt{3}r \end{pmatrix} F_{y,\text{ex}} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} M_{z,\text{ex}} + \begin{pmatrix} l_2 - 2r \\ l_2 + r \\ l_2 + r \end{pmatrix} F_{x,\text{in}} - \begin{pmatrix} l_1 \\ l_1 + \sqrt{3}r \\ l_1 - \sqrt{3}r \end{pmatrix} F_{y,\text{in}} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} M_{z,\text{in}} \right\} \quad (12a)$$

$$\begin{pmatrix} F_{z1} \\ F_{z2} \\ F_{z3} \end{pmatrix} = \frac{1}{3r} \left\{ \sqrt{3} \begin{pmatrix} 0 \\ -l_3 \\ l_3 \end{pmatrix} F_{x,\text{ex}} + \begin{pmatrix} 2(d_z + l_3) \\ -(d_z + l_3) \\ -(d_z + l_3) \end{pmatrix} F_{y,\text{ex}} + \begin{pmatrix} r - 2l_2 \\ \sqrt{3}l_1 + l_2 + r \\ -\sqrt{3}l_1 + l_2 + r \end{pmatrix} F_{z,\text{ex}} + \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} M_{x,\text{ex}} + \sqrt{3} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} M_{y,\text{ex}} + \begin{pmatrix} -r + 2l_2 \\ -(\sqrt{3}l_1 + l_2 + r) \\ \sqrt{3}l_1 - l_2 - r \end{pmatrix} mg + \sqrt{3} \begin{pmatrix} 0 \\ -l_3 \\ l_3 \end{pmatrix} F_{x,\text{in}} + \begin{pmatrix} 2l_3 \\ -l_3 \\ -l_3 \end{pmatrix} F_{y,\text{in}} + \begin{pmatrix} r - 2l_2 \\ \sqrt{3}l_1 + l_2 + r \\ -\sqrt{3}l_1 + l_2 + r \end{pmatrix} F_{z,\text{in}} + \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} M_{x,\text{in}} + \sqrt{3} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} M_{y,\text{in}} \right\} \quad (12b)$$

The dynamic model must be finally assembled. It is noted that the interaction forces have thus been expressed in function of the inertial effects, so that, expliciting their dependence upon the motion of the platform, Eqs. (12) can be symbolically re-written as:

$$\begin{cases} \underline{F}_x^{(i)} = \underline{h}_x^{(i)}(\underline{x}, \underline{\dot{x}}, \underline{\ddot{x}}) \\ \underline{F}_z^{(i)} = \underline{h}_z^{(i)}(\underline{x}, \underline{\dot{x}}, \underline{\ddot{x}}) \end{cases} \quad i = 1, 2, 3 \quad (13)$$

In order to solve the motion, these equations should be expressed in function of the internal joints' displacements  $\theta_i$ , by means of the inverse kinematics transform; this transformation is very complex and can not be carried out explicitly. In any case, in implicit form it is obtained:

$$\begin{cases} \underline{F}_x^{(i)} = \underline{h}_x^{(i)}(\underline{\vartheta}, \underline{\dot{\vartheta}}, \underline{\ddot{\vartheta}}) \\ \underline{F}_z^{(i)} = \underline{h}_z^{(i)}(\underline{\vartheta}, \underline{\dot{\vartheta}}, \underline{\ddot{\vartheta}}) \end{cases} \quad i = 1, 2, 3 \quad (14)$$

Substituting such expression of  $\underline{F}_x^{(i)}$  and  $\underline{F}_z^{(i)}$  in Eq. (7), the final relation is worked out, expressing whole system's dynamics:

$$\underline{\tau} = \underline{\Theta}(\underline{\vartheta}, \underline{\dot{\vartheta}}, \underline{\ddot{\vartheta}}) \quad (15)$$

where  $\underline{\tau}$  is the array of motors' torques.

The characterising features of the co-operating robotised platform can be obtained by solving, through computer simulation, the previously recalled dynamical model. A simulation program has, thus, been developed to support studies on the parallel robot's kinematics and dynamics. It can be coupled with the solutions provided by the general purpose *SIRIxx* simulational environment [16], in order to demonstrate the actual behaviour of the overall robotic equipment with co-operation, devised to behave as automated deburring work-station. The simulator has been written in FORTRAN and C programming languages under a Unix operating system; the

user interface makes full use of the X-Window Xlib routines under Motif and the mathematical computations are based on the IMSL v.3.0 library by Visual Numerics.

In the *inverse dynamical analysis*, the robot's trajectory is assigned in the task space; in this case all the algebraic calculations involved in previous equations can be easily computed and the torques that are (theoretically) needed to perform the task are evaluated. This analysis has been useful to operate a first assessment of the motors that have to drive the platform.

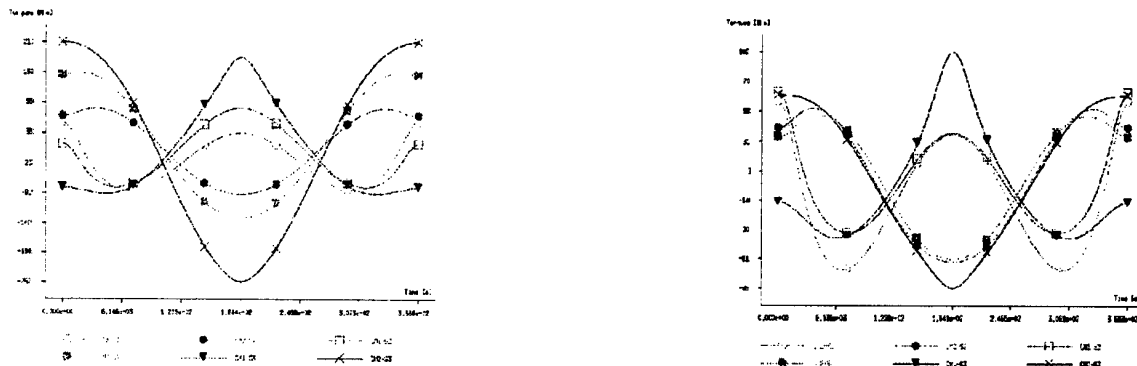


Fig. 4. Inverse dynamics: plot of required torques [N m].  
(a): full payload (5 Kg) - (b): no payload

In the *direct dynamical analysis*, on the other hand, the motors' torques are prescribed and the motion of the robot needs be resolved; this is useful for simulation purposes and for evaluating the performance of the system, along with the control. In this case a non-linear system of 6 differential equations of the second order should be solved numerically.

It must be noted that the equations are highly cross-coupled, so that a specific algorithm (based on the Petzold-Gear BDF method) has been used, in order to efficiently solve the DAE (Differential-Algebraic Equations) system: numerical problems showed up to be very difficult to be mastered anyway, even working in double precision format.

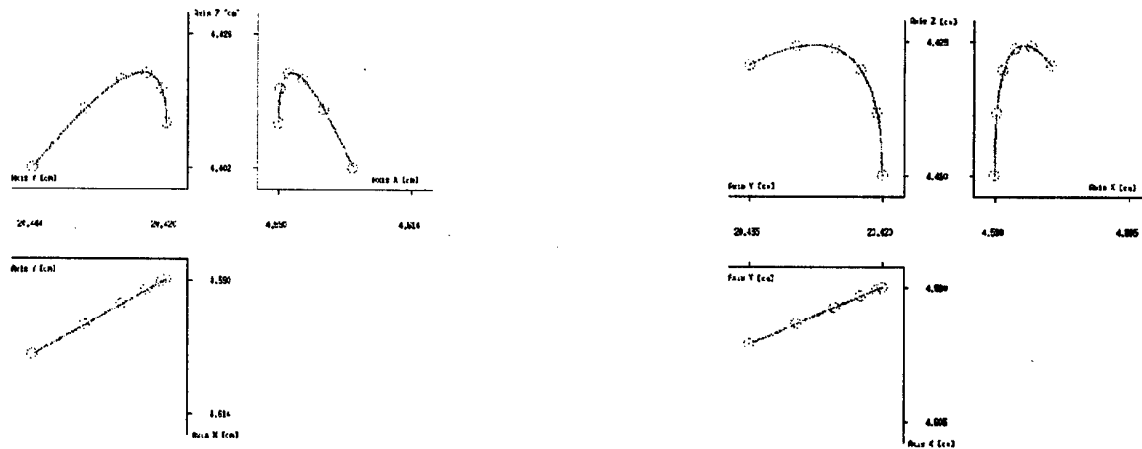


Fig. 5. Direct dynamics: orthographic projection of platform's free motion  
(initial vertical velocity of 0.1 m/s). (a): full payload (5 Kg) - (b): no payload

Some simulations have been carried out with the model of the platform that has been described: it should weight around 5 Kg and is supposed to be able to carry a maximum load of the same value. The working space is very restricted and poorly dextrous, but the platform should be requested to accomplish only small oscillations around its "central" poses. Both direct and inverse dynamics simulations have been performed: the imposed trajectories of inverse

contrivance, would become market driven option of factory automation.

For automation, the process should be first quantitatively modelled, starting by a proper estimation of burr size and shape [18-21]. To satisfy finishing results, the fixtured unit must be able to avoid the so-called "worst case burr". This is a maximum size burr, Fig. 6, occurring only quite occasionally. The amount of material to be removed per unit time, the so-called "material removal rate" (MRR), can be expressed by a simple balance:

$$\text{MRR} = (A_B + A_C) v_T = A_C (R_M + 1) v_T \quad (16)$$

where  $A_B$  is the burr's cross sectional area,  $A_C$  is the chamfer's cross sectional area and  $v_T$  is the tool speed along the edge to be deburred. Of course every parameter in Eq. (16) can be expressed in function of other parameters, such as contact forces, strength of the material, etc.

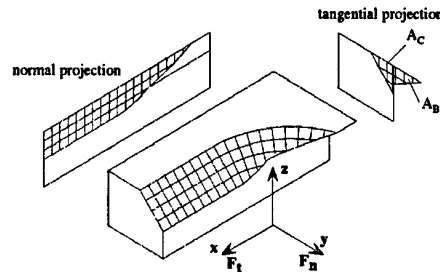


Fig. 6. Detail of deburring surface area

The tangential area ratio:  $R_M = A_B/A_C$ , typically varies between 0 and 2 depending on burr size, where value 2 refers to worst case burr. In the practice, large variations, causing diversified situations, need be faced. Process variability ranges should further be analysed, to fix standard reference figures. During deburring, normal  $F_n$  and tangential  $F_t$  forces stress, both, robot and piece. The current cross sectional area of burr and chamfer can be related to the normal and the tangential projection. Then, the variation of both normal and tangential forces ( $\Delta F_n$  and  $\Delta F_t$ ) depends on the respective projections of the cross sectional areas of burr and chamfer, that is:

$$\begin{cases} \Delta F_n = f_n(\Delta A_B/\Delta A_C)_n \\ \Delta F_t = f_t(\Delta A_B/\Delta A_C)_t \end{cases} \quad (17)$$

Now, chamfer normal and tangential projections are independent on burr size; on the contrary, the burr cross sectional area projections are related to the burr size and, thus, they vary; more precisely, the burr size greatly affects the tangential force  $F_t$ , whereas it has considerably smaller effect on the normal force  $F_n$ .

Moreover, a constant chamfer surface quality is required regardless the burr size; this means that the material removal rate (MRR) needs remain steady, otherwise the dressing process with small burrs suddenly changes to rough-machining process at occurrence of 'worst case burrs', yielding bad surface finishing. Since  $R_M$  varies between 0 and 2, the control system needs monitor the variations of  $F_t$  and  $F_n$  and modify the tool speed to keep the MRR constant. The option requires the control of the mutual positions of tool cutter and piece surface, with closure of independent force/torque feedbacks [22-25]. The analysis of the deburring process shows that the proper robotic solution should carry on, in parallel, the position/attitude and the force/torque modulation at the engagement boundary between cutting tool and compliant surface [17].

A six d.o.f. manipulator, without force feedback, suffers from considerable drawbacks, represented by three-dimensional vibrations, that upset the chamfer path. This results in unsatisfactory surface edge quality, which is important in precision deburring. The wavering behaviour depends on the discontinuities of the exchanged machining forces and on the compliance of piece supports and tool drivers. An attempt of preserving the finished surface quality has been sought through stochastic control [26] or by means of adaptive end-effectors.

Redundant mobilities can be added at the deburring front-end, in the way that the conventional 6 d.o.f. serial arm makes the main engagement (e.g. force control setting) and the extra mobilities carry out secondary compensation (e.g. position trimming). The set-up could be explored

through passive adaptation. The behaviour of the fixture can be described by a mass-spring-damper model, with normal direction mechanical impedance (ratio of contact force to end-effector deflection, as frequency function) given by:

$$F_n(s) = Q(s) X_n(s) \quad Q(s) = M s^2 + C_n s + K_n \quad (s = j\omega) \quad (18)$$

A large normal impedance causes the end-point to balance grinder forces, remaining close to the pre-set trajectory. Given the volume of the metal to be removed, the desired tolerance in the normal direction prescribes that the value of this impedance should not exceed conditions yielding burr excitation resonance. At the same time, it is necessary not to produce high tangential contact forces, since tool stall (or even breakage) may occur, with dangerous normal skips. It follows, Eqs. (16-17), that the end-effector needs operate all along with bounded interaction forces, which implies small tangential impedance. On the other hand, the uncertainties in the robot position are smoothed by a large compliance in the normal direction, at least within the robot resonance range. Summing up, the end-effector should show the following behaviour in the normal direction:

$\Rightarrow |Q(j\omega)|$  small for all  $\omega$  in the  $\omega_R$  band ( $\omega_R$  frequency range of robot's oscillations);

$\Rightarrow |Q(j\omega)|$  large for all  $\omega$  in the  $\omega_B$  band ( $\omega_B$  frequency range of the burr seen from end effector);

$\Rightarrow \omega_R < \sqrt{K_n/M} < \omega_B$

It is possible to design a passive end-effector with such dynamic characteristics, but it would be impossible to let it meet also the condition on the tangential direction (large compliance), because of the role played by the constant mass of the grinder, making equal the dynamic behaviour of the end-effector in both directions at high frequencies; when a large normal stiffness is chosen to improve the quality of the surface finish, then the end-effector will not be compliant enough to compensate for robot oscillations. This is why an active system is required to optimise the process parameters and to compensate for robot oscillations, while showing large stiffness in the normal direction.

### ***Precision deburring and co-operation***

Active dynamical systems can either operate by control redundancy or by functional cooperation. In the first case, independent position/attitude and force/torque sensors are used to accomplish (redundant) tool-tip control; the set-up still suffers deficiencies, as the uncoupling of normal and tangential behaviour is hindered by inertial effects of equal massic terms. As for functional cooperation, redundant mobilities are required, namely:

- addition of independently actuated members to the arm (serial d.o.f.) [27];
- inclusion of an actuated rig for holding the piece to be deburred (parallel d.o.f.) [17].

The first solution suffers of given snags: • low stiffness of the open chain, with critical control set-ups constraints, requiring nasty trimming and arduous pre-setting operations; • band limitation, in particular, with variable operation ranges, depending on extended mixes of pieces to be deburred.

The second solution offers several advantages:

- the redundant mobilities extend versatility and dexterity, enhancing robot accessibility along the surface edge to be deburred; the rig d.o.f. can be used to hold the piece in such a pose that favours the robot end-effector's work-trajectories;
- the adaptivity can be upgraded by intersecting paths operation modes, with efficient sweep up of the workspace and exploration of task planning which avoids collisions at engagements or undue penetration during deburring;
- the efficiency can be improved: the execution times can be reduced with low absolute speeds of each co-operating robot, but high relative speeds of the work-tip;
- the same accuracy can cover the full work-space; robot position/attitude along the main movements may be compensated by the position/attitude tracking of the co-operative rig, which is responsible for the servoed movements;
- critical tasks can be faced with repetitiveness: sharp corners, for instance, are tracked without considerable speed reduction by obtaining smooth paths by split tracks (this is important for precision deburring, when corner-rounding is not admissible);
- closed kinematic chain allows a lighter rig design, which results in lower mass inertia and better dynamic behaviour of the cooperating equipment used to rid of partner robot oscillations, as it is the case with precision deburring;

· further quality and efficiency betterments are obtained by adaptive job planning aiming at preventing worst case burr or, at least, at avoiding uttermost courses in the variability range of the removed burrs.

Obviously, there are certain drawbacks, too. Two robots, instead of one, result in higher costs. Coupled motion requires more sophisticated control, which further increases costs. Programming is more time-consuming compared to a single robot, particularly in case of adaptive path planning and tightly bound dynamics. These handicaps are reduced when proper standardisation is reached by the cooperating rig, the control architecture and the programming aid. As first instance, the parallelly actuated platform looks to offer quite an effective option; it is close-packed, easily powered, suited for position/attitude tracking. Control and programming burdens are drastically reduced by referring to CAD packages, such as the SIRI families of codes [16], and using them for the design, development, setting and fitting operations in virtual reality surroundings, all along the robots life-cycle, from conceptualisation, to all running phases.

## CONCLUDING REMARKS

The design of an instrumental robot starts by acknowledging sets of competing task-driven solutions, supporting the set-up of highly effective operation modes. Structured functional models need be established, so that the dynamics of each robotic equipment can be generated for the throughout assessment of the accuracy, dexterity, efficiency and versatility figures achieved by each particular solution. Once the functional models are available, the design procedure extensively exploits CAD-based environments granting virtual reality experimentation before, actually, building prototypal devices, that might reach the requested technical and economical effectiveness figures.

The success of this application of virtual reality approach critically depends on the functional model appropriateness; when reductive equivalencies (to lower the degrees of freedom), approximations (to suppress nonlinearities), motion constraints (to simplify cross-coupling effects), etc. are not properly stated, the generated dynamical behaviour fails to provide a correct reference to assess the robot operation performances, in terms of accuracy, dexterity efficiency and versatility. The paper has, therefore, shortly presented the reference frame to model the rig dynamics, before addressing the basic motivation of using robots with cooperation for automatic precision deburring.

The platform is driven, in parallel, by three independent actuators, each one displacing a side of an equilateral triangle, that specifies the reference plate. The individual driving block is composed by two superposed planar parallelograms, that move in a vertical plane. The table is fixed to the upper parallelograms by pivot points, that are actually ball joints; the ball, engaged with the platform, is carried by a slider running along a straight guide, integral with upper parallelogram's rods and orthogonal to the reference plate.

This kind of fixture deserves particular interest for its ability of accurate tracking, in position and attitude, any three dimensional surface. The rig, simultaneously controlled with the deburring arm, gives rise to a redundant mobilities set-up; joint force-and-displacement governing strategies can be enabled, to reach the very high versatility and dexterity figures of human operators while improving the efficiency achievements by the ability of operation continuity and the steady accuracy of the surface finishing by the combined force-and-displacement feedback loops. In the paper, the discussion is focusing the derivation of the functional models, to be used within the CAD-based environments for integrated-design purposes, in order to simultaneously implement the mechanical architecture and the control strategies of the instrumental robotic fixture.

## REFERENCES

1. Michelini, R.C. (1992) Decision anthropocentric manifold in robotic manufacturing. *Proc. of 4th ASME Intl. Symp. on Flexible Automation* (San Francisco, USA, July 12-15) 467-474.
2. Acaccia, G.M., C. Aiachini, M. Callegari, R.C. Michelini, R.M. Molino (1994) Dynamics shaping for the control of instrumental robots. *Proc. of 10th ISPE-IFAC Intl. Conf. CAD/CAM, Robotics and Factories of the Future: CARS&FoF* (Ottawa, Canada, Aug. 21-24) 514- 520.
3. Michelini, R.C., F. Pampagnin, R. Razzoli (1994) Lean engineering and performance assessment in flexible

- manufacturing. *Proc 27th ISATA Lean/Agile Manufacturing* (Aachen, Germany, Oct. 31-Nov. 4) 279-286.
4. Michelini, R.C., G.M. Acaccia, M. Callegari, R.M. Molfino, R.P. Razzoli (1996) Innovations in instrumental robotics: concurrence operations and co-operation; redundancy of mobility and control. *Proc. of 27th. Intl. Symp. on Industrial Robots: ISIR* (Milano, Italy, Oct. 6-8) 37-42.
  5. Stokic, D.M., D. Surdilovic (1990) Simulation and control of robotic deburring. *Intl. J. of Robotics and Automation*, 5(3) 107-114.
  6. Whitney, D.E., A.C. Edsall, A.B. Todtenkopf, T.R. Kurfess, A.R. Tate (1990) Development and Control of an Automated Robotic Weld Bead Grinding System. *ASME J. Dynamic Systems, Meas. and Control*, 112(2) 166-176.
  7. Ichinohe, M., K. Ohara, K. Yamaguchi, K. Maeda (1994) Development of Deburring Robot for Cast Iron with Vision and Force Sensing". *Proc. of 24th Intl. Symp. on Industrial Robots: ISIR*. 49-54.
  8. Asakawa, N., Y. Takeuch (1996) Automatic Deburring of Cast Iron Workpiece: Removal of Projection on a Convex Sculptured Surface. *Proc. of 3rd Japan-France Congress on Mechatronics* (Besancon, France) 686-690.
  9. Van Brussels, H., W. Persoons (1996) Robotic Deburring of Small Series of Castings, *CIRP Annals* 45(1) 405-410
  10. Acaccia, G.M., R.C. Michelini, R.M. Molfino, M.A. Recine (1991) Information reference set-up for the development of industrial multirobot systems. *Intl. J. Computer Applications in Technology*, 4(3) 137-148.
  11. Romiti, A., M. Sorli (1992) A parallel 6 d.o.f. manipulator for cooperative work between robots in deburring. *Proc. of the 23rd Intl. Symp. on Industrial Robotics: ISIR* (Barcelona, Spain) 437-442.
  12. Acaccia, G.M., P. Cagetti, M. Callegari, R.C. Michelini, R.M. Molfino (1994) Dynamic model of a robotised fixture for cooperative deburring operations. *2nd European Solid-Mechanics Conf. EUROMECH* (Genova, Italy, Sept. 12-16). Report R. 1129. Istituto Meccanica Applicata Macchine, Università di Genova.
  13. Acaccia, G.M., M. Callegari, R.C. Michelini, R.M. Molfino, R.P. Razzoli (1995) Dynamics of a multi-powered platform for task-steered instrumental robots. *Proc. IX World Conf. on Theory of Machines and Mechanisms: IFToMM* (Milano, Italy, Aug. 30-Sept. 2) 1816-1821.
  14. Acaccia, G. (1993) Progettazione integrata di sistemi robotici cooperanti per operazioni di finitura di superfici spaziali. *Final Report of Progetto Finalizzato Robotica* (goal 4.1.2), Consiglio Nazionale delle Ricerche, Roma. 2 luglio 1993.
  15. Sorli, M., M. Ceccarelli (1993) On the workspace of a 6 d.o.f. platform with three articulated double-parallellograms. *Proc. of Intl. Conf. on Advanced Robotics: ICAR* (Tokyo, Japan) 147-152.
  16. Acaccia, G.M., M. Callegari, R.C. Michelini, R.M. Molfino, R.P. Razzoli (1996) The design of robotic equipment for flexible automation. *Proc. of 2nd ECPD Intl. Conf. Advanced Robotics and Intelligent Automation* (Wien, Austria, Sept. 26-28) 554-559.
  17. Kazerooni, H., J.J. Bausch, B.M. Kramer (1986) An approach to automated deburring by robot manipulators. *ASME J. Dynamic Systems, Meas. and Control*, 108(4) 354-359.
  18. Hollowell, R., R. Guile (1987) An analysis of robotic chamfering and deburring. *ASME Winter Annual Meeting: Robotics Theory and Applications*.
  19. Kurfess, T.R., D.E. Whitney, M.L. Brown (1988) Verification of a dynamic grinding model. *ASME J. Dynamic Systems, Meas. and Control*, 110(4) 403-409.
  20. Liu, S., H. Asada (1992) Transferring Manipulative Skills to Robots: Representation and Acquisition of Tool Manipulative Skills Using a Process Dynamics Model. *ASME J. Dynamic Systems, Meas. and Control*, 114(2) 220-228.
  21. Persoons, W., P. Vanherck (1996) A Process Model for Robotics Cup Grinding. *CIRP Annals*, 45(1) 319-325.
  22. Bone, G.M., M.A. Elbestawi, R. Lingarkar, L. Liu (1991) Force control for robotic deburring. *ASME J. Dynamic Systems, Meas. and Control*, 113(3) 395-400.
  23. Kazerooni, H. (1987) Automated robotic deburring using electronic compliance: impedance control. *Proc. of IEEE Intl. Conf. on Robotics and Automation* (Raleigh, USA, March 31-Apr. 3) 1025-1032.
  24. Stepien, T.M., L. Sweet, M. Good, M. Tomizuka (1987) Control of tool-workpiece contact force with application to robotic deburring. *IEEE Trans. on Robotics and Automation* 3(1).
  25. Herr, M.G., H. Kazerooni (1991) Automated robotic deburring of parts using compliance control. *ASME J. Dynamic Systems, Meas. and Control*, 113(1) 60-66.
  26. Peklenik, J. (1964) Contributions to the theory of surface characterisation. *CIRP Annals*, 12, 173-178.
  27. Yoshikawa, T., K. Harada, H. Murakami (1994) Dynamic Hybrid Position/Force Control of Flexible Manipulators by Macro-Micro Manipulator Systems. *Proc. of ISCIE-ASME Intl. Conf. on Flexible Automation* (Kobe, Japan, July 11-18) 65-72.

## A DISTRIBUTED CONTROL ARCHITECTURE FOR AUTONOMOUS ROBOT SYSTEMS

Thomas Laengle, Tim C. Lueth, Ulrich Rembold, Heinz Woern

Institute for Real-Time Computer Systems and Robotics (IPR)  
University of Karlsruhe, D-76128 Karlsruhe, F.R. Germany  
E-mail: laengle@ira.uka.de

**Abstract.** The main advantage of distributed controlled robots and subsystems is the decentralized task execution by the system components. This way, properties for the design of flexible control architectures like modularity, fault-tolerance, integrability and extendibility are easy to obtain, further it is possible to use the concepts of distributed knowledge and decentralized world representation. On the other hand, coordination between the components, for example path planning for collision avoidance between both manipulators in a two-arm-system, is very difficult to guarantee. To explain these concepts, the Karlsruhe Autonomous Mobile Robot KAMRO is considered which is being developed at IPR. The robot system consists of several subcomponents like two manipulators, hand-eye-cameras, one overhead-camera and a mobile platform. Extensions to the distributed control architecture KAMARA (KAMROs Multi Agent Robot Architecture) are described that are responsible to overcome coordination problems, for example caused by the independent task execution of both manipulator systems. Further, it is explained which way the decentralized world representation can be used for parallel task execution. The described intelligent control architecture is to replace the former control architecture of the autonomous robot KAMRO.

**Keywords.** Autonomous Mobile Robot, Multi-Agent System, Distributed Architecture, Communication Mechanism, Cooperation and Negotiation

### 1. Introduction

During the past few years, the need for large scale and complex systems has become obvious. They are necessary to intelligently carry out tasks in the area of transportation, manufacturing, and maintenance. Examples are automatically guided transport systems containing many vehicles, complex flexible manufacturing cells, or eventually mobile manipulator systems, that could be used for autonomous service applications in an industrial setting [1, 2]. The main problem is often the design of the intelligent control structure (ICS) for such complex systems, and also for system components if the overall system consists of several separate systems.

Up to now, the control structures of such systems were usually designed as a hierarchical and centralized structure with a top-down process for planning and decision making. The number and complexity of the hierarchical layers determine the time the system requires for a reaction and also for the quality of a chosen action. In most cases, additional actuators or sensors have to be added during the development cycle to improve the capability of the overall system. In this case and, if the integration of component capabilities is required, it is easy to see the disadvantages of the hierarchical and centralized approach in comparison with the advantages existing at the initial system design process.

In contrast to this approach, distributed or decentralized control architectures [3 - 10] reveal their main advantages when it is necessary to enhance the system, to integrate components, and to maintain the system. The main disadvantage of not centralized architectures is having to make sure, that the system will fulfil an overall or global goal. On the other hand, the independent task execution by the system components causes problems in the area of coordination between the system agents. In this area, centralized control architectures show their main advantage. It will take a long time to have all these properties in an intelligent control architecture, but distributed and decentralized concepts will be the main approach for this goal.

In the second section, a taxonomy for intelligent control architectures is given and the distributed control architecture KAMARA for the mobile assembly robot KAMRO is shortly presented. Hereby, an agent model with local world representation is used. This model together with communication forms are explained in the third section. In distributed systems, task negotiation between the system components is necessary, see section four. In section five, an example is discussed to explain the concepts in detail. The article ends with an evaluation of the advantages of this new approach and conclusion for future work.

## 2. *Intelligent Control Architectures for Multi-Agent Systems*

From the definition of an agent, it is possible to describe and explain hierarchical systems (Fig. 1). An agent consists of three parts: *communicator*, *head* (for planning and action selection), and *body* (for action execution).

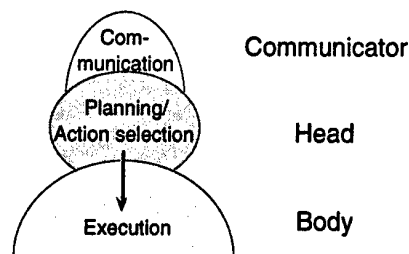


Fig. 1 Elements of an agent

The communicator connects the head to other agents on the same communication level or higher. The head is responsible for the action selection process for the body (centralized approach), organizes the communication among the body's agents actively or passively (distributed approach), or is only a frame to find a principle of order for the decentralized approach. The body itself consists of one or more executive components, which can be similarly considered as agents.

The executive components can be divided into three classes, as well as the components of the process for planning and action selection, i.e., the head of an agent (Fig. 2). The classification is as follows:

- *Centralized action selection:* Available information is centrally processed by a decision making component and transformed into an action for the agent's body (linear planner).

- *Distributed action selection:* Available information is processed by several decision making components, which communicate and negotiate to come to a decision. (Blackboard [11], Whiteboard [12])
- *Decentralized action selection:* The available information is processed independently by several decision making components and transformed locally in their own action decision for the agent's body (Motor schema [13]).

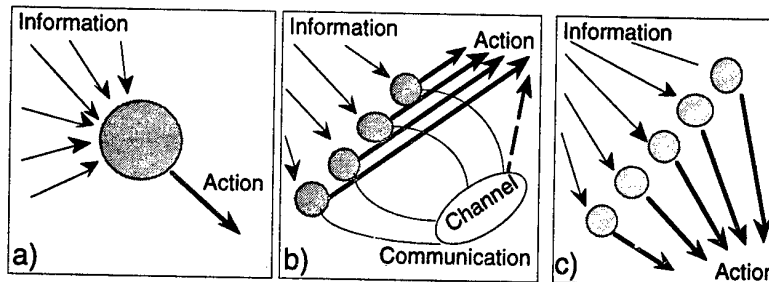


Fig. 2 Three different ways to make decisions, plan, or select actions

From an execution-oriented point of view, the presented taxonomy not only allows the classification of already described ICS for planning and control. It also describes multi-agent systems in a similar way as single systems. To explain these concepts, the Karlsruhe Autonomous Mobile Robot KAMRO (Fig. 3) is considered.



Fig. 3 The mobile robot KAMRO

The robot system consists of several subcomponents like two manipulators, hand-eye-cameras, one overhead-camera and a mobile platform. These agents have to communicate and negotiate with each other to collect the missing information to perform the desired task. The new control architecture, KAMARA (KAMRO's Multi-Agent Robot Architecture), for distributed intelligent robot systems and their components allows easier control in many directions and also easier component integration. The main topic in the following sections will be the problem of task distribution between the executive agents.

### 3. Agent Model and Communication Mechanism

As mentioned before, an agent  $A$  consists of a communicator, a head, and a body. In our system description, an agent, like a manipulator, is only capable of performing one task at a time. The reason for that is that its body is implemented as a single procedure. On the other side, a head with a communicator does not only has to control the body, but also has to communicate and negotiate with other agents or heads. An important reason for communication is the determination of the agent for executing an elementary operation. This means the head (and the communicator) has to deal with several different tasks at one time. Therefore, head and communicator are implemented as a variable set  $H, C$  of equal independent processes  $H, C$  for planning, communication, and negotiation (Fig. 4):

$$A = (C, H, B)$$

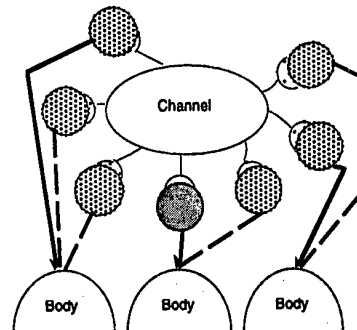


Fig. 4 Head and communicator can be several processes

The communication mechanism for all agents and for task distribution or task allocation is Blackboard-like. This mechanism holds all executable missions  $m$  in a mission set:

$$M = \{m_1, m_2, \dots, m_n\}$$

$M$  gets new missions  $M_{n+1}$  from the cell planner  $P$  or from the agents of KAMRO:

$$P, A: \mathcal{M} = \mathcal{M} \cup \mathcal{M}_{n+1}$$

Whether or not this communication mechanism is implemented as Blackboard, Whiteboard, or token ring, etc., is an implementation-oriented question (here, a Blackboard-architecture with a contract-net-like protocol is used).

In principle, this multi-agent architecture is also useful on the cell level. In this case the communication mechanism of one KAMRO robot is the head of a KAMRO-Agent (distributed

action selection architecture), and it is possible to use more than one KAMRO robot for complex tasks like carrying a large object with several robots or loaning one manipulator to a second robot (Fig. 5).

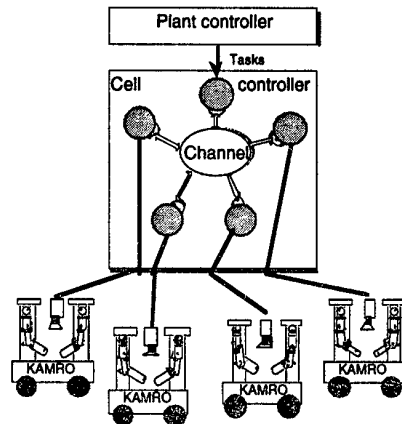


Fig. 5 Distributed cell controller view

Considering the distributed control architecture, it is easy to notice that between the system components, different cooperation forms are needed:

1. independent task execution,
2. asynchronous communication (teams), and
3. synchronous communication (special agents).

Teams are dynamic in nature, the number as well as the kind of agents may change during the task execution. For a defined space of time, cooperation is necessary to reach the goal. The communication for this kind of cooperation will be done on a high abstraction level by the agent's communicator. As an example, a team of the components camera and manipulator of the autonomous mobile assembly robot KAMRO for grasping and joining operations can be considered. In this case, the camera system must be able to correct the position of the manipulator if needed. Another example is the exchange of an obstacle between both manipulators, or a regrasping operation to change the gripping configuration of an object. In the first case, the camera and a manipulator must build a team to solve the described problem, in the second case, both manipulators together build a team. Because there is just brief information exchange that has not to be synchronized in a specific time interval, team building is sufficient. The communication form between the system agents is asynchronous. On that account, it is not possible to guarantee real-time constraints.

If two manipulators grasp a large or heavy part, and by this way close a cinematic chain, asynchronous communication between these system components is not sufficient. In this case, dependent on the desired control concept for the cinematic chain, a decentralized architecture (simple reflexive behaviour), a distributed architecture (master slave tasks), or a centralized architecture is required. In some cases (for example, complex two-arm manipulation tasks), a centralized robot controller is better than any other approach at this moment. This is the reason for an extension of the distributed control concept by the introduction of special executive agents. These special agents SA have, like all other agents, a head H and a communicator C. The body is allowed to allocate bodies of other agents, if available, and control them by special

communication channels with high transfer rates (see Fig. 6). During this time, the normal agents have no access to their bodies, since they are used by the special agent.

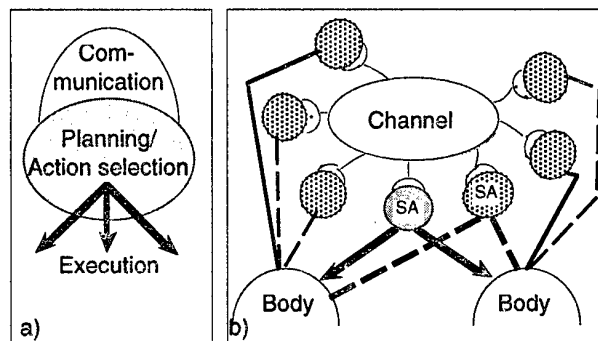


Fig. 6 Special agent: centralized planning for other agent bodies

In other words: if the information exchange between agents of the same team increases so dramatically that this results in a narrowing in the communication channels, then these agents have the possibilities to refer to a closer internal relationship. Because special agents change the structure of the control architecture while they are active, they should only be used if no other type of cooperation is suitable.

#### 4. Task Negotiation in KAMARA

If a new mission is assigned from the mission set  $M$ , it is possible that many agents are able to work on that task. As a consequence, an important problem that has to be solved is the negotiation among these agents that compete for the mission.

##### 4.1 Task Description

Because all agents are able to negotiate with the competing agents and the communication is managed by a blackboard architecture (mission set), it is necessary not only to represent the task itself, but also other information.

First of all, it is important to store the agent identification of the mediator and a list of all other candidates that compete for the mission. This way, it is possible to identify the mediator, and the mediator knows which other agents are involved in the negotiating process. Since the blackboard structure controls the information exchange among the system components, the evaluations of the competing agents are also stored in the task information block. As mentioned before, all agents that work on the subtasks have to send their solutions (or a signal) to the responsible initiators. Thus, the mission representation must also indicate the agent that appended the subtask to the mission set. To be specific, it is necessary to store the identification number of the desired head and communicator, since many of these heads and communicators can exist for an agent. Since an agent head that needs further information to perform its task is blocked until the desired information is present, a correspondence field informs this agent that the solution is stored on the blackboard structure. To identify the solution, the *mission identification number*  $n$  is used as a reference in the solution representation. Because the solution representation can be very complex and is not always known in advance, it is better not to integrate it into the mission description. Therefore, a mission is represented as a tuple

$$m=(n, I, R, P, t, A, V, E)$$

The field  $I$  contains a list of the *mission initiators*,  $t$  represents the *task* itself,  $R$  is a set of *receivers* that are interested in the mission solution,  $A$  is a list of the *competing agents* and  $V$  their *valuations*. The first candidate in the candidate list  $A$  is the mediator between the competing agents. All other entries are the candidates. When the mediator selects the agent that has to perform the task, a corresponding message is sent to all agents through the execution set  $E$ . In this field, all competing agents can see whether they are chosen to work on the mission or not. The set  $P$  contains *signals* to the initiators or receivers, which indicate that the mission solution is presented on the blackboard structure. The blocked initiator or receiver head waiting for information has to examine this field until a signal for them is presented. To overcome problems with older messages on the blackboard but still making the information available as long as necessary for other interested agents, the last receiver that fetches its information from the blackboard has to delete the mission and the corresponding answer from the mission set. This way, information is present as long as necessary and is deleted if all interested agents have received the solution.

#### 4.2 Selection of the Mediator

The negotiation procedure starts when a new mission appears in the mission set. The communicator of every agent, whether the body of this agent is already performing another mission or not, searches for tasks that it can solve in the mission set. One of these competing agents should negotiate with the rest of them. If the candidate list  $A$  is empty, the first competing agent head acts as mediator and stores its identification number into the first position of the field  $A$ . When another agent becomes interested in mission solving, it is obvious that this agent head should evaluate its problem solving ability and send it to the mediator by writing the information into the corresponding position of the valuation field  $V$ . The mediator calculates its own ability to work on the mission and waits an a priori defined time  $\tau$  for the evaluation of all other agents  $a_2, a_3, \dots, a_n$ , compares these evaluations and chooses the best agent  $a_i$  of the entire candidate list  $A=(a_1, a_2, \dots, a_n)$  to work on the mission. This way, the candidates that are not able to calculate their evaluation fast enough or are disabled are not involved in the negotiation process. Because all agents that have the ability to work on the mission are integrated in this selection procedure, it can be that an agent which was previously working on a different task can enter the competing process later when its body is "free".

#### 4.3 State Transition Diagram for the Agent Head

To briefly describe the above mentioned negotiation process, internal state transition diagrams for the agent head is presented. The state transition diagram of the agent head shown in Fig. 7 consists of four states: *mediate*, *calculate*, *ready* and *not existing*. If a new mission appears in the mission set, the communicator of an interested agent initiates a new process copy of the agent's head (state transition *not existing* to *ready*). The mediator head then changes his state to *mediate*, and all other candidates have to change their state to *calculate*.

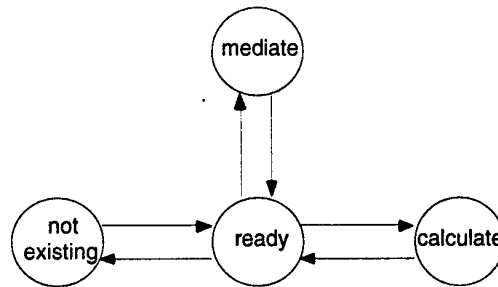


Fig. 7 State transition diagram for the agent head

In both states, *calculate* and *mediate*, the agent calculates its ability to solve the mission. All candidates that are in the state *calculate* store their calculated value in the corresponding position in the evaluation field V. When this field is complete or the defined time constant  $\tau$  is reached, the mediator selects the candidate that has the best ability to complete the mission and stores the execution information in the execution field E. Thereby, all candidates are informed whether they are chosen to work on the mission or not. The heads of all candidates that are not selected to work on the mission change their states back to *not existing*, the winning candidate changes its state to *ready*. If two (or more) agents send equivalent valuations and this is the highest in the negotiating process, the mediator randomly selects one of these competing agents. In contrast to that, the physical agent is only capable of performing one task at a time, thus the agent body is implemented as a single procedure.

## 5. Agents and Tasks in KAMARA

In the KAMARA system, there exist several agents that work together to perform the desired task. Consequently, a communication protocol between the agents is required. This language consists of operations that address an agent to perform a task and could be used by other agents to involve other agents in the solution process. In KAMARA, the agents perform the following operations:

- **Manipulator:** A manipulator is able to perform the implicit elementary operations PICK and PLACE.
- **Two-arm-manipulator:** A two-arm-manipulator is also able to perform the implicit elementary operations PICK and PLACE. Because this agent consists of two independent actuators that make up a superagent, the mission valuation of this agent is much higher than the calculated value of a single manipulator when picking up a heavy or large obstacle.
- **Manager:** This agent is responsible for the interpretation of a complex mission the system has to perform. It decomposes a complex task into its executable parts.
- **Database:** The database is able to offer world state information the agents need to perform their tasks.
- **Overhead camera:** This agent type is able to determine the position of obstacles by examination of a wide environmental area.
- **Hand camera:** This sensor type is able to determine exact relative object positions based on inexact absolute position estimation. This information is necessary for a manipulator just before performing a grasping operation. It can also be used to extract object positions like the overhead camera.

- *State controller:* This agent is responsible for the blackboard structure and the state of the other system components. One important task is to control the time a mission waits on the blackboard for execution. The other task is to control system component evaluation.

The communication and negotiation concept between agents to perform a mission will now be demonstrated using the example of an assembly task, the Cranfield Assembly Benchmark (Fig. 8). A Cranfield Benchmark task is represented by a precedence graph (Fig. 9) whose nodes consist of individual subtasks.

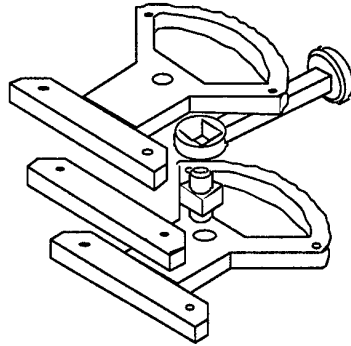


Fig. 8 Cranfield assembly benchmark

This precedence graph only describes the goals the system has to reach, whereas the executing agent has to decide how these goals can be achieved depending on the environment at execution time. Therefore, the agent head uses the system's sensor information to expand this implicit representation to an explicit one.

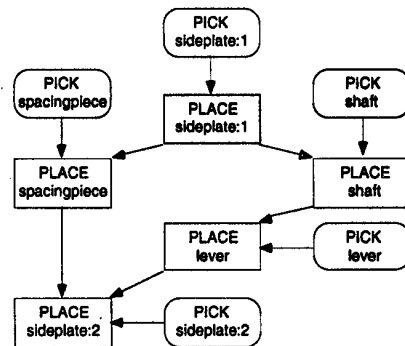


Fig. 9 Assembly precedence graph

Interpretation of a complex mission is only performed by the manager agent. This agent then competes for it. As a consequence, a new manager head process is involved in the system. This head is responsible for the whole execution process of this assembly mission. Thus, it starts with the task decomposition, taking the precedence graph into account. Referring to the above described example, only the operation PICK(sideplate) can be performed and is then appended to the mission set:

$m_I = (1, \{M\}, \{M\}, \text{nil}, \text{PICK}(\text{sideplate}), \text{nil}, \text{nil}, \text{nil})$

This mission, one implicit elementary operation, can be performed by many system components, for example all manipulators, perhaps also a special agent or a team of other agents. All these agents are involved in the solution process and have to calculate their ability to solve the mission. For example, both manipulators of KAMRO are interested in the mission, and the manipulator  $Mp_1$  is the first candidate that competes for it:

$$m_1 = (1, \{M\}, \{M\}, \text{nil}, \text{PICK}(\text{sideplate}), (Mp_1, Mp_2), \text{nil}, \text{nil})$$

As a consequence, two new processes of the agent's heads are started, the state transition to *mediate* is performed by agent head 1, and the second agent head switches to the state *calculate*. In these states, both manipulators start to calculate their mission solution valuation. Therefore, these system components need further information, for example, the position and the weight of the sideplate. Because both manipulators are not able to determine the position information, other agent types have to be involved. If, for example, the agent  $Mp_2$  is the initiator of the command, a new mission is appended to the mission set:

$$m_2 = (2, \{Mp_2\}, \{Mp_2\}, \text{nil}, \text{find\_position}(\text{sideplate}), \text{nil}, \text{nil}, \text{nil})$$

After a short time, manipulator  $Mp_1$  is also interested in the obstacle position:

$$m_2 = (2, \{Mp_1, Mp_2\}, \{Mp_1, Mp_2\}, \text{nil}, \text{find\_position}(\text{sideplate}), \text{nil}, \text{nil}, \text{nil})$$

Both agents also need weight information:

$$m_3 = (3, \{Mp_1, Mp_2\}, \{Mp_1, Mp_2\}, \text{nil}, \text{find\_weight}(\text{sideplate}), \text{nil}, \text{nil}, \text{nil})$$

In mission  $m_2$ , there are two types of system components that are able to calculate this position, and are thus interested: the overhead camera OK, and both hand-eye-cameras ( $HK_1$ ,  $HK_2$ ). As described above, a higher problem solving valuation for OK is calculated:

$$m_2 = (2, \{Mp_1, Mp_2\}, \{Mp_1, Mp_2\}, \text{nil}, \text{find\_position}(\text{sideplate}), (HK_1, OK, HK_2), (10\%, 95\%, 10\%), \text{nil})$$

On that account, the mediator  $HK_1$  negotiates between the candidates and modifies E to inform the competing agents whether they are chosen to work on the mission or not:

$$m_2 = (2, \{Mp_1, Mp_2\}, \{Mp_1, Mp_2\}, \text{nil}, \text{find\_position}(\text{sideplate}), (HK_1, OK, HK_2), (10\%, 95\%, 10\%), (0,1,0))$$

The component OK is able to calculate the position without further information. Because both  $Mp_1$  and  $Mp_2$  are waiting for the position and are registered in I and R,, both agents are appended to P :

$$m_4 = (4, \{OK\}, \text{nil}, \{Mp_1, Mp_2\}, \text{Answer}(2, (3.5;5.3;6)), \text{nil}, \text{nil}, \text{nil})$$

$Mp_1$  examines the blackboard, recognizes that there is information available and deletes its identification number from P:

$$m_4 = (4, \{OK\}, \text{nil}, \{Mp_2\}, \text{Answer}(2, (3.5;5.3;6)), \text{nil}, \text{nil}, \text{nil})$$

The initiating mission  $m_2$  stays on the blackboard structure so that another agent that demands this information can find the desired information immediately by use of a mission identification number. When  $a_2$  fetches the information from the blackboard, the post list is empty, and  $a_2$  thereby deletes message  $m_4$  and mission  $m_2$  from the blackboard. The database DB is able to calculate the object weight in an analog way:

$$m_5 = (5, \{DB\}, \text{nil}, \{Mp_1, Mp_2\}, \text{Answer}(3, (17g)), \text{nil}, \text{nil}, \text{nil})$$

Now, the competing manipulators are able to determine their ability to perform the desired PICK task under consideration of the distance to the object, weight of the object, and perhaps other information. The mediator compares all offers and starts the best qualified manipulator to perform the task, i.e.  $Mp_2$ , with use of the execution field :

$$m_1 = (1, \{M\}, \{M\}, \text{nil}, \text{PICK}(\text{sideplate}), (Mp_1, Mp_2), (30\%, 60\%), (0,1))$$

Agent  $Mp_2$  gets the execution signal and starts the PICK operation. Thereby, it sends a sequence of executable operations to its agent body. Immediately before execution of the grasping operation, it is necessary to determine the exact obstacle position. The integration of the hand camera to get the exact object position is also performed by the above described algorithm. This way, manipulator  $Mp_2$  holds all needed information to execute the grasping operation:

$$m_6 = (6, \{Mp_2\}, \text{nil}, \{M\}, \text{Finished}(1), \text{nil}, \text{nil}, \text{nil})$$

After execution of the PICK operation, the manager receives a signal that mission  $m_1$  is completed, and appends the next executable implicit elementary operation to the mission set. When the precedence graph is finished, the manager sends a signal to the cell planning system and leaves the system.

## 6. Conclusion

Due to its higher flexibility and robustness, the multi-agent approach, comparing to the centralized one, is more and more used for the control of the complex and inhomogeneous robot systems in recent time. One disadvantage of this approach, however, is the existence of the possibility of potential dead-lock among the interaction of different agents. Based on a communication platform realized for the agents in KAMARA, a dead-lock-free negotiation of task is guaranteed according to the enhanced Contract-Net-Protocol.

The integration of a new system component into a complex centralized system is often very difficult because it is not obvious where the system must be modified. In KAMARA, new system components can be added to the system easily because the system structure must not be modified. The integration is performed by the negotiation process.

## 7. Acknowledgement

This research work was performed at the Institute for Real-Time Computer Systems and Robotics (IPR), Prof. Dr.-Ing. U. Rembold and Prof. Dr.-Ing. R. Dillmann, Faculty for Computer Science, University of Karlsruhe and is funded by the nationally based research project on artificial intelligence (SFB 314) funded by the German Research Foundation (DFG).

## 8. References

1. Rembold, U.; T.C. Lueth; A. Hörmann (1993). *Advancement of Intelligent Machines*. ICAM JSME Int'l Conf. on Advanced Mechatronics, Tokyo, Japan, August 2-4, 1993, pp. 1-7.
2. Schraft, R.D. (1993). *Vom Industrieroboter zum Serviceroboter - Stand der Technik, Chancen und Grenzen*. ISRR VDI/VDE-GMA Intelligente Steuerung und Regelung von Robotern, Langen, Nov. 9.-10., 1993
3. Beni, G.; S. Hackwood (1992). *Coherent swarm motion under distributed control*. DARS Int. Symp. on Distributed Autonomous Robotic System, Wako, Japan, Sept. 21-22, 1992, 1992, pp. 39-52.
4. Camargo, R.F.; R. Chatila; R. Alami (1991). *A Distributed Evolvable Control Architecture for Mobile Robots*. ICAR Int. Conf. on Advanced Robotics, Pisa, Italy, June, 4-6, 1991, pp. 1646-1649.
5. Lueth, T.C.; Th. Laengle (1994). *Task Description, Decomposition and Allocation in a Distributed Autonomous Multi-Agent Robot System*. IROS IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, Munich, Germany, Sept 12-16 1994, pp. 1516-1523.
6. Noreils, F.R. (1992). *An Architecture for Cooperative and Autonomous Mobile Robots*. ICRA IEEE Int. Conf. on Robotics and Automation, Nice, France, 1992, pp. 2703-2710.
7. Ohmori, Y.; Y. Nakauchi; Y. Itoh; Y. Anzai (1992). *A Task Allocation Algorithm for Multiple Mobile Robot Environment*. 2nd ICARV Int. Conf. on Automation, Robotics and Computer Vision, Singapore, September 16-18, 1992, pp. RO-15.3
8. Ozaki, K.; H. Asama; Ishida, Y.; A. Matsumoto; H. Kaetsu; I. Endo (1992). *The Cooperation among Multiple Mobile Robots using Communication System*. DARS Int. Symp. on Distributed Autonomous Robotic System, Wako, Japan, Sept. 21-22, 1992
9. Tigli, J.Y.; M. Occello; M.C. Thomas (1993). *A Reactive Multi-Agents System As Mobile Robot Controller*. IROS IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, Yokohama, Japan, July 26-30, 1993, pp. 2008-2014.
10. Yuta, S.; S. Premvuti (1992). *Coordinating Autonomous and Centralized Decision Making to Achieve Cooperative Behaviors between Multiple Mobile Robots*. IROS IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, Raleigh, NC (USA), July 7-10, 1992, p. 1566 ff
11. Hayes-Roth (1985). *A blackboard architecture for control*. Artificial Intelligence 26, 1985, pp. 251-321.
12. Thorpe, C.E. (ed.) (1990). *Vision and Navigation - The Carnegie Mellon Navlab*, Kluwer Academic Publishers, Boston, 1990.
13. Arkin, R.C. (1989). *Motor Schema-Based Mobile Robot Navigation*. The International Journal of Robotics Research, 8, 4 (1989), pp. 92-112.

# PROBLEMS OF SENSING AND SPATIAL MOTION CONTROL IN ACTIVE ROBOTICS \*

Iliya Miroshnik, Gennady Boltunov and Vitaliy Shiegin

State Institute of Fine Mechanics and Optics  
Sablinskaya 14, St.Petersburg, 197101, RUSSIA  
e-mail: miro@cyber.beam.ifmo.ru

**Abstract.** Distinctive features of the active robotics including peculiarities of mechanical structures, sensory and control systems, as well as problems of their implementation are discussed. Redundant models of stationary manipulators are analyzed in details and sensor-based control of the spatial motion along different kinds of the physically detectable external objects are considered on the basis of coordinating control techniques more compatible with proximity measurements. They provide direct closed loop stabilization of the end-point relative position and well-coordinated behavior of multi-link mechanisms in the course of motion along external objects.

**Keywords.** Active robotics, spatial motion, sensor-based control, nonlinear systems.

## 1. Introduction

A new generation of intelligent robotics presented by active robots [1] is characterized by developed interaction with external objects, extended abilities of adaptation to complex, uncertain and mobile environment, high dexterity of manipulations accomplished by robotic mechanisms [1-5]. These properties provide high operational performance and versatility of active robots that implies realization of a variety of nontrivial locomotion tasks such as perfect obstacle avoidance and preventing collisions, penetrating into hard to reach domains of the working space and suitable approaching external objects, accurate motion along complex curvilinear trajectory and surfaces, maintaining a desired configuration of the robot kinematic chain and a desired position of objects grasped. The dexterity of active robots, their high adaptability and comparative autonomy are very attractive for many fields of application including automated manufacturing, space and underwater investigations, medicine and so on.

Active behavior of robots in complex environment is achieved by using advanced models and new kinematic schemes of stationary and mobile mechanisms, developed sensory systems providing adequate representation of environment and, especially, by new strategies of spatial motion control [5-10]. The recent research has demonstrated the efficiency of nonlinear control for maintaining pre-specified trajectory and spatial motion of mobile robots and stationary manipulators including restricted motion of robots interacting with environment [2, 5, 11-14]. The perfect interaction strategy and high-precision execution of locomotion tasks in the Cartesian space are usually prevented by such factors as the deficiency of the preliminary knowledge of the external objects and uncertainty of the current end-point posture caused by joint flexibility or even small errors of the kinematic chain geometry. In order to receive information on the external objects geometrical properties, define the robot attitude with respect to environments and correct the control

---

\*This work is supported by Russian Foundation of Basic Research (Grant 01.00432)

actions, different kinds of sensory systems from simplest force, tactile and optical instruments to the developed vision systems are used [3, 4, 6-9, 15]. The adequate choice of sensory systems, measurement schemes and control strategies extremely compatible with the measurements must be subject to the unified requirements of the locomotion control problem.

This paper concerns redundant models of stationary manipulators and sensor-based control of the spatial motion along different kinds of physically detectable external objects such as paths, counters and bounding surfaces of the workpieces and obstacles. The main attention is paid to proximity sensors providing the most accurate information on the robot relative attitude (deviations) in the vicinity of the plane or spatial objects and techniques of coordinating control. The latter, being developed on the basis of the non-linear control theory [16,17], is directly aimed at eliminating the spatial deviations and maintaining the required concordant interaction of different parts of complex dynamic systems [11-13, 17].

In order to verify the properties of the coordinating control system using proximity measurements, experimental results concerning robot motion along plane and spatial geometric objects are presented.

## 2. Models of robotic mechanisms and problem statement

The up-to-date models of robotic mechanisms are presented by multilink manipulators of classical and variable structure, car-like, cart-like and articulated wheeled robots, legged locomotion machines and platform-mounted arms. The required dexterity and versatility of the mechanisms are provided, first of all, due to additional bodies of the robot kinematic chain. Multi-link stationary and mobile robots possessing extra degree of freedom are kinematically redundant. The robot redundancy is the necessary condition for performing sophisticated locomotion tasks in complex environment. However this creates the known control difficulties related with kinematics calculation and noticeable flexibility of lengthy mechanisms which must be overcome by means of sensing and control [13,14]. In this paper, we consider stationary multi-link manipulators with  $m$  rotational joints. The dynamics of the robot is given by the Lagrangian equation

$$A(q)\ddot{q} + b(q, \dot{q}) = \mu, \quad (1)$$

where  $q = \{q_j\}$  is the vector of the joint (generalized) coordinates,  $\mu = \{\mu_j\}$  is the vector of generalized torques,  $j = [1, m]$ , and the inertia matrix  $A$  is invertible. The vector of the Cartesian coordinates  $y = \{y_j\} \in R^3$  of the robot end-point is defined by the equation of Direct Kinematics

$$y = c(q) \quad (2)$$

with the full rank Jacobian matrix

$$C_q(q) = \partial c / \partial q$$

Let the robot environment be prescribed by the geometrical object  $S \in R^3$  given by

$$\varphi(y) = 0, \quad (3)$$

We will distinguish the cases when (i)  $S$  is a smooth curve and  $\varphi$  is a smooth regular vector function of dimension 2 (ii)  $S$  is a surface and  $\varphi$  is a scalar function. The current deviation from the object  $S$  is introduced as

$$e = \varphi(y) \quad (4)$$

and the displacement along  $S$  (the longitudinal motion) is

$$s = \psi(y), \quad (5)$$

For the case (i),  $e = \text{col}(e_1, e_2)$  and  $s$  is a scalar variable associated with the length of the path; for the case (ii),  $e$  is a scalar variable and  $s = \text{col}(s_1, s_2)$  is the vector of surface local coordinates. The functions  $\varphi(\cdot)$  and  $\psi(\cdot)$  are assumed to be chosen such that  $e(t)$  specifies orthogonal deviations from the object  $S$ , the Jacobian matrix of coordinate transformation (4), (5)

$$M = \begin{vmatrix} \partial\psi/\partial y \\ \partial\varphi/\partial y \end{vmatrix}$$

is invertible ( $\det M \neq 0$ ) and, moreover,  $\|\partial\varphi/\partial y\| = 1$  [5,12-13].

For the robot interacting with a rigid or compliant surface (see Fig.1), the right part of equation (1) can be written in the form [3]

$$\mu = u + C_q^T (\partial\varphi/\partial y)^T f \quad (6)$$

where  $u = \{u_j\}$  is the vector of controlling torques (inputs),  $f$  is the value of the contact force depending on the deviation  $e$ .

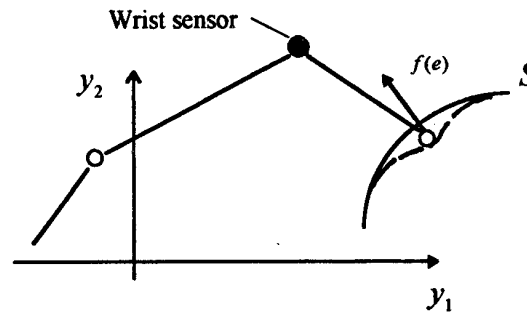


Fig. 1 Interaction with compliant object

Supposing that  $m > 3$ , we will consider the redundant robot (1), whose kinematics unable to be uniquely defined by the end-point position  $y$ . Then we introduce  $r$  intermediate principal points of the robotic mechanism  $y^i = \{y_j\}^i$  and a set the Direct Kinematics equations

$$y^i = c^i(q), \quad i = \overline{1, r} \quad (7)$$

where  $y^r = y$ . For the sake of simplicity, we suppose that  $3r = m$  and, in the robot operating zone, the composite matrix

$$\{C_q^i(q)\} = \{\partial c^i/\partial q\}$$

is invertible.

The correct problem statement for the redundant robot implies a sufficient number of additional holonomic restrictions. Being given in the form

$$\varphi^i(y^i) = 0 \quad (8)$$

they define, in the Cartesian space, the objects  $S^i$  (curves and surfaces) and make it possible to reduce the robot d.o.f. The fulfillment of the additional restrictions (8) by means of the appropriate control actions provides coordinated motion of the robot and maintenance its desired configuration in the course of the end-point displacement along the object  $S = S^r$ .

According to the conventional methodology [11-13, 17], we introduce the current deviations from the objects  $S^i$  as

$$e^i = \varphi^i(y^i) \quad (9)$$

and the displacement along  $S^i$

$$s^i = \psi^i(y^i) \quad (10)$$

where  $\varphi^i(\cdot)$  and  $\psi^i(\cdot)$  are smooth functions assumed to be chosen such that [5, 11-13] in the vicinity of  $S^i$  the Jacobian matrices

$$T^i = \begin{vmatrix} \partial\psi/\partial y \\ \partial\varphi/\partial y \end{vmatrix}^i$$

are invertible and satisfy the equations

$$\dot{T}^i = H^i(s^i, \dot{s}^i)T^i \quad (11)$$

where  $H^i$  is the matrix of the curve (surface) geometric parameters.

The primary statement of the spatial control problem is to eliminate the deviation  $e(t)$  (or stabilize the given value  $e = e^*$ ) and provide the desired mode of the longitudinal motion  $s^i(t)$  prescribed by the reference variable  $s = s^*(t)$  or the desired profile of the longitudinal rate  $\dot{s} = \dot{s}_j^*(t)$ .

If the robot is absolutely rigid, its parameters are accurately known and the desired path is exactly determined *a priori*, then to solve the considered problem one can use the conventional tracking techniques [12,15]. The appropriate control system contains a demand generator of the reference coordinates (the interpolator) and a multivariable tracking controller. As a rule, the systems are closed on joint variables measured by internal positional sensors (resolvers, encoders, potentiometers and so on).

The ideal model conditions are usually impracticable and prevented by flexibility of the robot kinematic structure, uncertainty of the robot geometric parameters (especially of those of the robot end-effector), complexity of the prescribed path and the lack of an adequate analytic description of the external objects. A successful solution of the control problem implies employment of sensory systems and the advanced methodologies of path- or surface-following control.

### 3. Sensory systems

The required measurements of the robot absolute and relative position are provided by sensory systems. Usually they involve both simple angular/linear sensors of joint coordinates and complex systems of robotic vision. The latter are based on optical and ultrasonic instruments and accomplish the long-distance location of the environment that is necessary for global world representation, preliminary planning of the robot motion and the description of the external objects. Among basic schemes of external measurements are stationary and gripper-mounted robot vision systems [15] including video-cameras and special processors for calculation of the current end-point Cartesian coordinates  $y_j$  or the environments geometry  $S^i$ . Such systems ensure a wide field of vision but relatively low precision of measurements in the long-distance operational zones. The considerable complexity and over-all dimensions restrict their implementation in the active robotics.

On the other hand, for the constrained locomotion tasks, the most important information is related to contact or indirect interaction of the mechanism with external objects. It cannot be obtained by using only measurements of joint coordinates because of the robot flexibility and the lack of a priori knowledge on environment geometry. Such kind of external information is provided by proximity sensors presented by touching, force/torque and vision based instruments.

Multi-d.o.f. wrist sensors provide force/torque information concerning contact interaction

of the end-effector with external objects. These measurements enable one to estimate the reaction forces used in force-position control. Meanwhile, the measurements of the orthogonal force component  $f$  include the information on the deviation  $e$  from the given hypersurface  $S$  (see Fig. 1) which can be efficiently used for correction of the robot spatial motion.

The specific features of the trajectory control for the motion along the plane light-reflecting path  $S$  make promising proximity sensors with differential photoreceivers (Fig. 2). Simple output signal processing is required to determine the current value of the normal deviation  $e(t)$ .

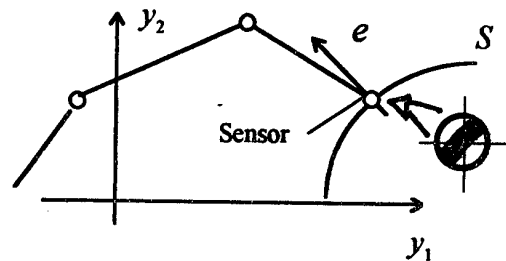


Fig. 2 Optical proximity sensing

The proximate distance locators based on ultrasonic and laser devices are the main part of the scheme for measuring the orthogonal deviation of the robot end-point from spatial objects  $S$  in task of motion along reflecting surfaces (Fig. 3).

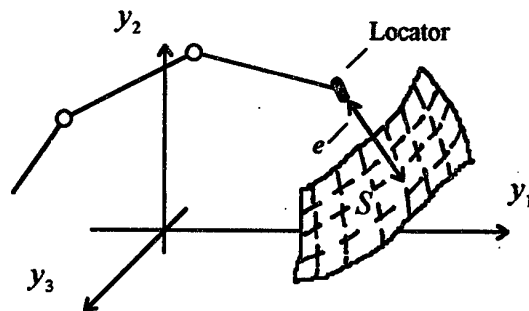


Fig. 3 Distance location

Thus, the schemes of the proximity measurements prove to be the most suitable for current estimation of the robot posture with respect to given objects of the operational space. In comparison with conventional vision systems, they have higher local accuracy and do not require complicated signal processing.

#### 4. Spatial motion control techniques

The choice of the control strategy and corresponding structure of the control system have to obey the requirements for typical locomotion tasks accomplished by active robots and, above all, be compatible with the measurements available. The list of the main control tasks includes [1-5]

- (i) path planning and constructing  $\{S\}^i$  according to a *a priori* world representation and global measurements;
- (ii) controlled motion along the objects given by curves and surfaces  $\{S\}^i$
- (iii) coordinated movement of the bodies of the redundant kinematic mechanisms.

Here we consider path- (surface-) following systems designed on the basis of coordinating control [11-13, 17] paying the main attention to their operation performance and compatibility with low-level external measurements. The design of the coordinating control system is based on the direct statement of the problem of the spatial motion control (see Part 2) where equations (8) establish relationships between the robot output variables  $y_j$  and must be maintained during robot motion.

Using the procedure of coordinate transformation [13, 16, 17] we convert the model (1), (7) to the task-oriented form

$$\begin{vmatrix} \ddot{s} \\ \ddot{e} \end{vmatrix}^i - H^i(s^i, \dot{s}^i) \begin{vmatrix} \dot{s} \\ \dot{e} \end{vmatrix}^i - h^i f = T^i(\dot{C}_q^i \dot{q} + C_q^i A^{-1}(u - b)) \quad (12)$$

where  $h^i = T^i C_q^i A^{-1} C_q^T (\partial \varphi / \partial y)^T$  and introduce the transformed control vector  $\begin{vmatrix} u_s \\ u_e \end{vmatrix}^i$ ,  $u_s^i$  is the longitudinal and  $u_e^i$  is the transversal control variables, such that

$$T^i(\dot{C}_q^i \dot{q} + C_q^i A^{-1}(u - b)) = \begin{vmatrix} u_s \\ u_e \end{vmatrix}^i \quad (13)$$

The controls  $u_s^i$  and  $u_e^i$  are produced by the following local feedback controllers

$$u_s^i = \ddot{s}^* - H_{11}^i(s^i, \dot{s}^i) \dot{s}^i + K_s^i(\Delta s^i) \quad (14)$$

$$u_e^i = -H_{21}^i(s^i, \dot{s}^i) \dot{s}^i + K_e^i(\Delta e^i) \quad (15)$$

where  $\Delta s^i = s^{i*} - s^i$  is the longitudinal error,  $\Delta e^i = e^{i*} - e^i$  is the transversal error,  $K_s^i(\cdot)$  and  $K_e^i(\cdot)$  are the feedback operators that provide the required elimination of the errors and, as a result, the desired mode and, stability of the motion along objects  $S^i$ . The choice of  $K_e^i$  and  $K_s^i$  is accomplished from a class of usual integro-differential operators corresponding to the PID controllers.

The basic system feedforwards are deduced from equation (13) in the form

$$u = b(q, \dot{q}) + A^{-1}(q) \{C_q^i\}^{-1} \left( \{(T^i)^{-1} \begin{vmatrix} u_s \\ u_e \end{vmatrix}^i\} - \{\dot{C}_q^i\} \dot{q} \right) \quad (16)$$

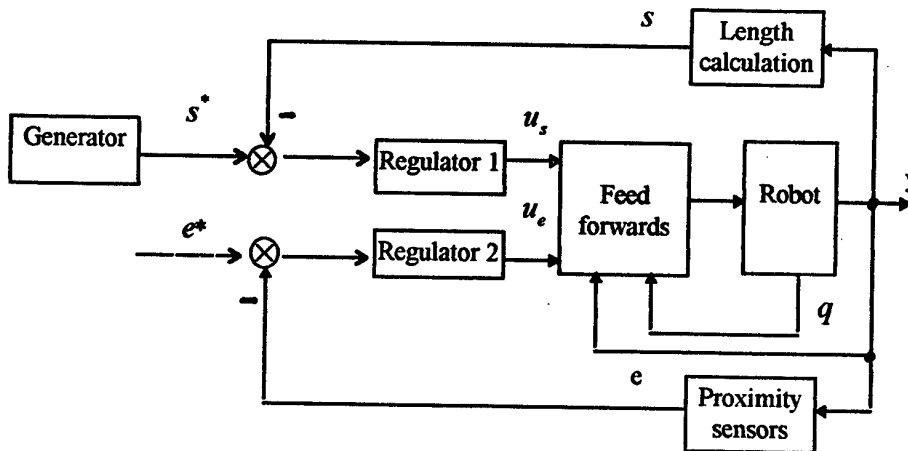


Fig. 4 System of coordinating control

The overall structure of the control system corresponding to equations (14), (15), (16) is presented in Fig. 4 and contains Controllers 1 and 2 providing feedback of the longitudinal displacement  $s$  and deviation measurements  $e$ , respectively, and a feedforward block corresponding to expression (16).

## 5. Conclusion and simulation results

To illustrate efficiency of the approach proposed special cases of motion of planar and spatial robotic mechanisms were analyzed. The control problem was to stabilize the end-point displacement along smooth curves or surfaces, provide a given rate of the longitudinal motion  $\dot{s}^* = \text{const}$  and maintain additional relationships prescribed via trajectories of the mechanism internal points.

*Example 1.* Motion of a planar 14th-link robot along external parabolic object is considered. Additional restrictions are prescribed as motion of the internal points along equidistant curves. The simulation results are given in Fig.5. They demonstrate the asymptotic zeroing of the end-point deviation, well-coordinated behavior and maintaining compact configuration of the robot chain in the course of the required trajectory motion.

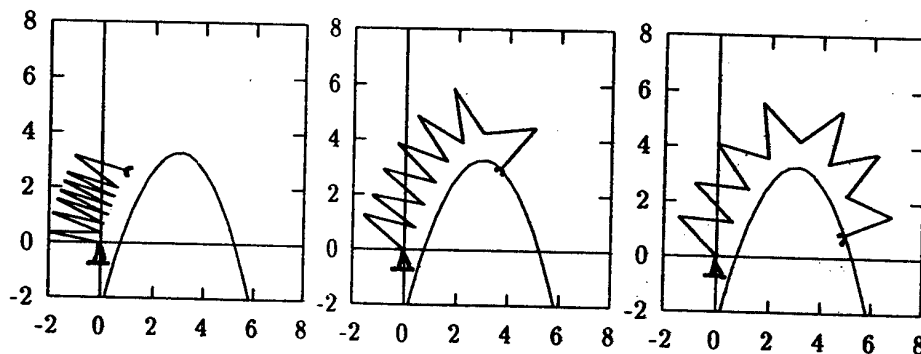


Fig. 5 Motion along parabola

*Example 2.* Motion of a simple spatial 3-link robot along a sphere at longitudinal speeds  $\dot{s}_{1,2}^* = 0.2\text{m/s}$  is shown in Fig.6. The presented results confirm stability of the closed-loop control system and elimination of the deviation of the end-point from the required surface.

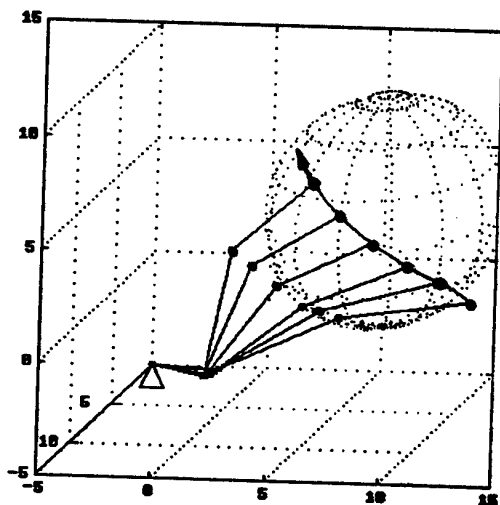


Fig. 6 Motion along sphere

Coordinating control techniques and the proximity measurements give the possibility to reveal all potentials of the sensor-based control and are compatible with the main purposes of the active robotics. Unlike the tracking systems, the proposed control systems are closed on the spatial deviation that provides its better accuracy. The use of the proximity sensors excludes the need to calculate the current value  $\epsilon$  and, therefore, simplifies the control law and allows one to avoid the errors connected with uncertainty of the desired path analytic description. The system does not contain any interpolator and therefore motion along complex curves and surfaces does not cause essential complication of the control law. Moreover, motion along a priori unknown paths can be realized by means of self-learning the presented control system [11].

## References

1. The Second ECPD International Conference on Advanced Robotics, Intelligent and Active Systems (1996), ECPD, Viena
2. Y. Zheng (1993) Recent Trends in Mobile Robots, World Scientific
3. Arimoto, S. (1994) State-of-the-art and future research directions of robot control. 4th IFAC Symp. on Robot Control, v.1, Capri
4. Khatib, O. (1994) Towards integrated robot planning and control. 4th IFAC Symp. on Robot Control, v.3, Capri
5. Proceedings of the Laboratory of Cybernetics and Control Systems, (1996) Miroshnik I.V. and Nikiforov V.O. (Eds.) IFMO, St.Petersburg,
6. Espiau, B., F. Shaumettte, P. Rives (1992) A new approach to visual servoing in robotics. IEEE Trans. on Rob. and Autom., v. 8, no. 3
7. Soatto, S., P. Perona (1994) Structure-independent visual motion control on the essential manifold. 4th IFAC Symp. on Robot Control, v. 3., Capri
8. Narikiyo, T., G. Schmidt and T. Akuta (1994) Theoretical aspects of hybrid position/force control, 4th IFAC Symp. on Robot Control, v.2, Capri
9. Villani, L., C. Canudas de Wit and B. Brogliato (1996) An exponentially stable adaptive force/position control for robot manipulators. 13rd IFAC World Congress, vol.A, San-Francisco
10. Miroshnik, I.V., G.I. Boltunov (1995) Visual sensor-based control of robot spatial motion. The 1st ECPD Int. Conf. on Advanced Robotics and Int. Autom., Athens
11. Miroshnik, I.V., V.O. Nikiforov (1994) Coordinating control and self-learning of robot trajectory motion. 4th IFAC Symp. on Robot Control, v.3, Capri
12. Miroshnik, I.V., V.O. Nikiforov (1995) Coordinating control of robotic manipulator. Int. J. Robotics and Autom., v.11 (3)
13. Miroshnik, I., V. Nikiforov (1996) Trajectory motion control and coordination of multi-link robots. 13rd IFAC World Congress, v. A, San-Francisco
14. Miroshnik, I. V., V. O. Nikiforov and A. V. Lyamin (1996) Trajectory control of mobile manipulators interacting with complex environment. 2nd ECPD Int. Conf. on Advanced Robotics, Intel. and Active Systems, Viena
15. Fu, K.S., R.C. Gonzalez and C.S.G. Lee (1987) Robotics: Control, Sensing, Vision and Exercises, New York, McGraw-Hill Inc.
16. Isidori, A. (1989) Nonlinear Control Systems, New-York: Springer-Verlag.
17. Miroshnik, I.V. (1990) Coordinating Control of Multivariable Systems. Energoatomizdat (Ed.), Leningrad (in Russian)

# A HIERARCHICAL NAVIGATION SYSTEM FOR AN AUTONOMOUS MOBILE ROBOT<sup>1</sup>

Alain Bétourné, Alessandro Fefè, Salvatore Nicosia, Paolo Valigi  
Dipartimento di Informatica, Sistemi e Produzione,  
Università degli Studi di Roma "Tor Vergata"  
Via della Ricerca Scientifica, I-00133 ROMA - ITALY  
e-mail: {nicosia,valigi}@disp.utovrm.it

**Abstract.** In this paper a hierarchical navigation system for an autonomous mobile robot is presented. Specific attention is given to the environment description, to the multi-sensor based positioning system, and to the motion planning algorithm.

A flexible hierarchical data structure is used to model complex, partially known, indoor environments. The motion planning algorithm is based on the concept of configuration space. Exact cell decomposition is used to derive a description of the free space, and graph search tools are then used to compute a possible path. The sensory system is based on the joint use of an encoder based odometer, of a vision system, and of an ultrasonic captor sensory system.

Experimental tests will be presented, showing the good accuracy achieved by the proposed position estimation scheme.

**Keywords.** Mobile robot, autonomous navigation system, motion planning.

**Introduction.** The problem of navigation and motion planning for autonomous mobile robots is a subject of great interest, and over the past years it has received considerable attention (see, among others, [1, 2]). The complete solution of this problem involves several tasks, of different complexity and characterized by quite different time scales. Situations of this type, are best managed by means of a hierarchical approach.

In this paper, a hierarchical navigation system for autonomous mobile robots is presented, and some key features are described in details.

A relevant issue in the design, implementation and operation of a navigation system, is the positioning sub-system. To be fully autonomous, a mobile robot needs the capability to determine with a sufficient degree of accuracy its own position. The odometry (encoder based) sensors usually available on each mobile platform, most often do not have satisfactory performance, for several reasons. Major alternatives are visual feedback, ultrasonic and infrared captors, and inertial navigation equipment. In addition, in order to improve the accuracy of the overall positioning system, sensor data fusion is usually employed.

Another relevant issue is the need for an accurate model of the operational environment, in order to satisfactorily perform motion planning, and obstacle avoidance. Since, in general, the environment cannot be assumed completely known in advance, the sensors used for the positioning task can also be used to update the available knowledge of the environment, usually described by means of a map.

A third issue in the design, implementation and operation of a navigation system is that of motion planning and motion control. Based on the description of the task assigned to the mobile robot, and on the currently available map description of the environment, a motion planning algorithm is used to compute a desired trajectory to be followed by the

---

<sup>1</sup>This work was partly supported by ASI funds. The work of the first author, Dr. Bétourné, has been supported by European Community, in the framework of the ERNET program (CHRX-CT93-0381).

mobile robot, that will be a reference input for a real-time dynamic controller responsible for actual robot guidance.

In this paper, a description of the hierarchical navigation system in use on the LABMATE mobile platform of the Robotics and Industrial Automation Laboratory of the University of Rome "Tor Vergata" will be presented. Attention will be devoted to a flexible data structure modeling the environment, to the motion planning algorithm, and to the visual positioning system. Experimental results will also be presented.

Other navigation systems have been presented in [3, 4, 5, 6, 7]. In [3] standard marks are placed in known locations, in order to allow the adjustment of position information. Each mark is associated a bar code, allowing the system to infer the current position. In [4] the robot navigation system is based on position estimation carried out by using incremental encoder data, together with an inertial navigation system. Sensor data fusion allows to reduce noise level. In [5] the "edge visibility regions" approach is used, to determine robot position. In [6], visual feedback and artificial potential fields are used. The visual feedback is used to follow a tracking line placed on the ground; the artificial potential fields approach is used to plan a desired trajectory, with fictitious charges used to avoid local minima. In [7] a hierarchical navigation system is presented, for applications in real industrial factors, comprising a fuzzy logic module.

**The hierarchical navigation system: An overview.** The hierarchical navigation system for Autonomous Mobile Robot is organized into two main sub-systems: the *sensory sub-system* and the *mobile robot management sub-system*. The sensory sub-system is responsible for determining the robot position with respect to the environment, and for updating the knowledge of the environment. The robot management sub-system is responsible for all the decisions to be taken to allow the mobile robot to successfully achieve the assigned task. The robot management sub-system is organized into three main hierarchical layers: a) map management and updating, and obstacle detection; b) motion planning; and c) motion control. An additional upper layer, not considered here, could also be added to the management sub-system, responsible for proper planning and scheduling of the activities of the robot.

**Robot management sub-system.** The mobile robot management sub-system is responsible for all the decisions allowing the robot to fulfill its duties. In view of the complexity of such a function, and of the different time scales involved, the sub-system is hierarchically organized. The topmost layer considered in the current implementation of the navigation system is in charge for the modeling of the environment. The layer manages the map describing the environment, and interacts with the sensory sub-system in order to update the current robot position. In addition, the interaction with the sensory sub-system is also required for the purpose of map updating and obstacle recognition, in order to cope with a certain level of uncertainties in modeling environment.

The second layer implements the motion planning functionalities. It is based on the currently available environment map, on the actual robot position, and on the assigned robot tasks. The current implementation of the layer is based on the concepts of free space, exact cell decomposition, and graph searching algorithm. A new version of the layer is under investigation and development, based on potential fields.

The motion control layer currently implementation of the navigation system, is based on the primitives, provided by the original robot control software, and will not be described here. An enhanced motion control layer is currently under investigation.

*Map management layer.* The navigation of an AMR requires the availability of a model

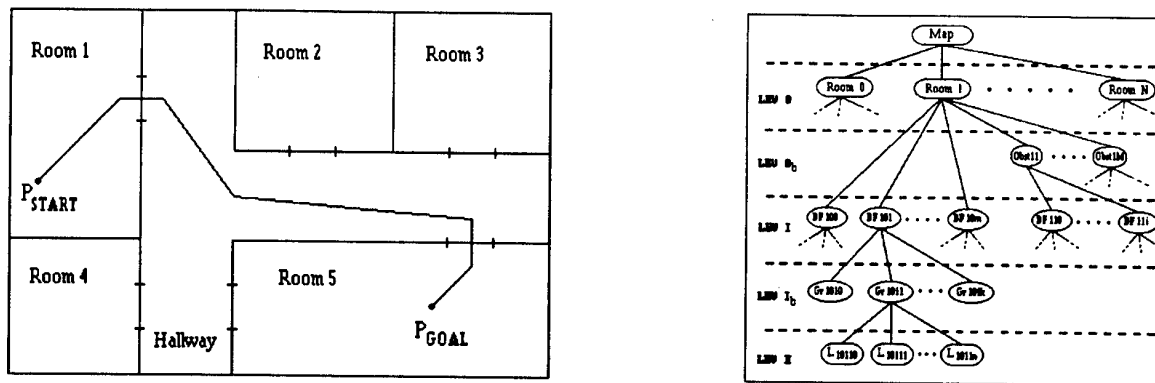


Figure 1: The environment map, and the path (a) The whole hierarchy (b).

of the operation environment. The approach used to model the environment affects the computational efficiency of the whole system. Here, a modified version of the hierarchical model introduced in [8] is considered, which allows to use information in a selective manner. While simple, it contains also all the information that a rendering algorithm would need to construct expectation maps for all the interesting cases.

Fundamental for the considered environment model is the notion of *basic face*: it is a vertical planar entity of unspecified height bounded by two vertical lines. The set of all the basic faces corresponds to the first level of the hierarchy. Such a structure, which is the basic scheme proposed in [8], represents the model of one room alone; then, the map of a larger environment will simply comprise a set of rooms. Taking into account the problems considered in this paper, the above model has been extended, by the introduction of an auxiliary structure, called *group* (of lines): the first group, *group 0*, will describe all the lines that do not belong to any door, whereas the other groups describe the lines corresponding to the doors (the first door in group 1, the second door in group 2, and so on). Hence, the new tree data structure has two more levels, *level 0*, describing rooms, and *level 1<sub>b</sub>*, describing doors through the group structure (see Fig. 1(b)). Obstacles are modeled by one more hierarchical level, *level 0<sub>b</sub>*, in between the room level and the basic face level. A picture of the whole hierarchy is reported in Fig. 1(b).

*Motion planning layer.* The motion planning layer is expected to compute a path, allowing the robot to visit a sequence of desired positions, possibly performing some tasks while in movement. The path planning functionality relies on a sufficiently accurate description of the environment, at least in a neighborhood of the original position. In general, in indoor environment, the path from the origin to the destination can be split into a set of simpler segments (see Fig. 1(a)):

- room crossing (i.e., a door-to-door path);
- door crossing (i.e., a room-to-room path).

Door-to-door planning. Currently, the planning of a door-to-door segment is based on the following three phase procedure (see [2]):

Phase 1. The robot configuration space is constructed, as well as the associated free space, based on the currently available knowledge of the environment.

Phase 2. The free space is exactly decomposed into a proper number of cells by means of the trapezoidal approach, and the connectivity graph is constructed.

Phase 3. A path is determined, by searching the graph. □

The motion planning of a door-to-door segment is restricted to a single room, i.e., all

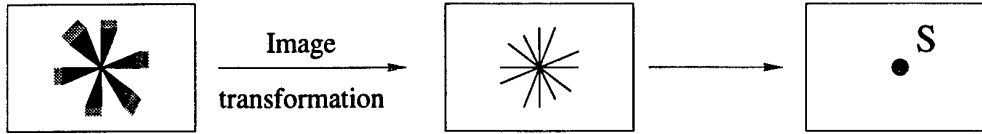


Figure 2: Image transformation for a mark

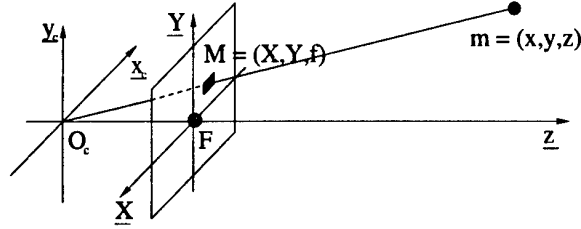


Figure 3: Camera model

the required door-to-door paths are planned independently one from the others. The interaction with the sensory system allows the navigation system to update the map, and then to construct the configuration free space.

**Room-to-room planning.** The planning problem for the room-to-room segment do not pose major problem as such, but it requires accurate information on current robot position. With respect to this problem, in this paper the joint use of ultrasonic captors and vision system is considered.

In order to allow accurate (and successful) navigation of a mobile robot along a room-to-room path segment, a precise knowledge of robot position, at the beginning of the segment, is required. The navigation system presented in this paper is such that just before the execution of the door “crossing” task, it performs a position verification and correction procedure. For this purpose, two landmarks are placed on one side of the door, 60 cm away one from the other, and close to the door. The procedure to travel along a room-to-room path segment can be summarized as follows.

Step 1. The robot, based on a previously planned path, and by only using odometry data, stops in front of the marks, at a given camera-marks distance.

Step 2. The vision sub-system determines the actual robot position.

Step 3. Once the position has been corrected, the door status (i.e., open/close) is determined by using the US captors.

Step 4. If the door is open, the robot goes through the door, otherwise a proper warning routine is executed, e.g., to require operator assistance.  $\square$

**Sensory sub-system. Visual feedback.** The camera-based vision system comprises a camera, installed on the LABMATE mobile robot platform, and an image processing software package, partially developed within the laboratory. The camera allows one to obtain images of the scene with a resolution of  $512 \times 512$  pixels in 256 grey levels, and is attached to the robot so that the camera axis is perpendicular to the robot wheel axis. An ideal model of the camera is considered, with a geometry of perspective projection type. Let  $R_c = (O_c, \underline{x}_c, \underline{y}_c, \underline{z}_c)$  be a reference frame attached to the camera, and let  $\underline{z}_c$  be the camera axis. The focal distance of the camera is denoted by  $f$ . The point  $m = (x, y, z)$ ,  $x, y, z, \in \mathbb{R}$ , on the scene is projected onto the image plane on the point  $M = (X_u, Y_u, f)$ ,

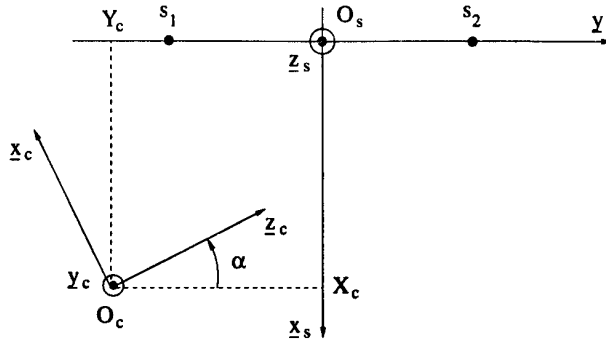


Figure 4: Mark and camera reference frames

$X_u, Y_u, f \in \mathbb{R}$ , and the pixel coordinates  $X$  and  $Y$  of the two points are given by:

$$X = -\frac{f_X}{z}x, \quad Y = \frac{f_Y}{z}y, \quad (1)$$

where  $f_X = fk_X$ , and  $f_Y = fk_Y$  ( $k_X$  and  $k_Y$  positive,  $-k_X$  is used because of the inversion between the image and the scene, see Fig. 3).

Different techniques for image segmentations have been analyzed, and the one based on the two basic steps of image filtering and image segmentation has been chosen.

**Image filtering.** In order to determine obstacle contours or other primitives of the scene, the image is processed by means of a filter based on the Laplace method and on the use of pixel intensity value threshold. The output of such a filter is an image with only two grey levels, where the intensity level is either 0 or 255, the value 255 corresponding to a pixel with an high contrast with the surrounding pixels.

**Segmentation.** As the image resulting by the above filter has not 2D primitives with regular thickness, the Hough transform [9, 10] is used to map the image into an image with regular thickness edges.

**Position Estimation and System Calibration.** To determine the actual position of the mobile robot by using the vision system, *ad hoc* landmarks are used, placed in locations *a priori* known to the navigation system. The use of landmarks for position determination is quite common (see, e.g., [11]). The more complete and realistic case in which the robot position estimation and calibration is carried out by using the “true” environment alone, will be considered in the future. The case considered here can be of interested in a number of real applications, in which landmarks can be easily installed.

It is assumed that pairs of “star” landmarks (see Fig. 2) are fixed at relevant, known, positions along the walls, and used to re-calibrate the robot navigation system. The centers of the two landmarks (of each pair), as projected on the image, will correspond to two points  $S_1$  and  $S_2$ , that can be determined based on the algorithm outlined in the following. Then, by means of the positions (in pixel coordinates) of these points  $S_1$  and  $S_2$ , using the *a priori* knowledge of the (absolute) coordinates of the landmarks  $s_1$  and  $s_2$ , the position of the camera reference frame  $R_c$ , hence the robot position, with respect to the environment, can be determined.

**Determination of the robot position.** Let  $R_s = (O_s, \underline{x}_s, \underline{y}_s, z_s)$  be a reference frame associated to a pair of marks, and placed such that the coordinates of the centers of the two “star” marks, with respect to  $R_s$ , are  $s_1 = (0, -l, z_s)$  and  $s_2 = (0, l, z_s)$ , where  $z_s$  is the elevation of the centers of the two marks with respect to ground (See Fig. 4). The position of the two mark centers  $s_1$  and  $s_2$ , with respect to the reference frame  $R_c$  attached to the

camera (in absolute coordinates), is given by:

$$s_1 = \begin{bmatrix} X_c \cos \alpha + (Y_c + l) \sin \alpha \\ h \\ X_c \sin \alpha - (Y_c + l) \cos \alpha \end{bmatrix}, \quad s_2 = \begin{bmatrix} X_c \cos \alpha + (Y_c - l) \sin \alpha \\ h \\ X_c \sin \alpha - (Y_c - l) \cos \alpha \end{bmatrix}, \quad (2)$$

where  $h$  is the elevation of the marks with respect to the camera reference frame, and  $\alpha$  is the angle between the axes  $z_c$  and  $y_s$ . Based on equations (2), and (1), and by means of the knowledge of the image positions  $S_1 = (X_1, Y_1, f)$  and  $S_2 = (X_2, Y_2, f)$  of the two marks, the robot position can be easily determined. Two cases have to be considered, the case  $Y_1 \approx Y_2$ , and the case  $Y_1 \neq Y_2$ .

1.  $Y_1 \approx Y_2$

Let  $Y$  be the average value of  $Y_1$  and  $Y_2$ . Then, the orientation angle  $\alpha$  is close to  $\pi/2$  ( $\alpha \approx \pi/2$ ). The origin  $O_c$  of the camera reference frame  $R_c$  has the following position with respect to the mark reference frame  $R_s$ :

$$X_c = \begin{cases} \frac{2lf_X}{X_2 - X_1} \\ \frac{f_Y}{Y}h \end{cases}, \quad Y_c = -l \frac{X_1 + X_2}{X_2 - X_1}, \quad (3)$$

where  $x_c$  is determined by any one of the two equations, or by their average values.

2.  $Y_1 \neq Y_2$

The camera (hence the robot) is not along the axis  $x_s$ , and therefore  $\alpha \neq \pi/2$ . The position of the origin  $O_c$  of the camera reference frame, and its orientation angle  $\alpha$ , can be obtained by using the following simple procedure:

- (a) the distance  $z$  of the camera from the vertical plane comprising the two marks is computed as

$$z = z_1 - \frac{X_1}{(X_2 - X_1)}(z_2 - z_1), \quad (4)$$

- (b) the coordinate of the origin  $O_c$  of the camera reference frame is computed as:

$$X_c = z \sin \alpha, \quad Y_c = Y'_c - z \cos \alpha \quad (5)$$

$$\text{where } Y'_c = (-l) - \frac{X_1}{(X_2 - X_1)}2l.$$

Once the position and orientation of the camera reference frame has been determined, the position and orientation of the robot can be easily computed.

**Experimental results.** The procedure for estimating robot position and orientation, based on equations (3) and (5), has been implemented and tested on the LABMATE mobile platform as follow:

- Step 1. the robot has been placed close to the reference marks, in a position such that the marks lie in the camera image;  
 Step 2. an image is captured and processed to locate the two marks, and the two points  $S_1$  and  $S_2$  are computed;  
 Step 3. finally, equations (3) and/or (5) are used to calculate the robot position.  $\square$

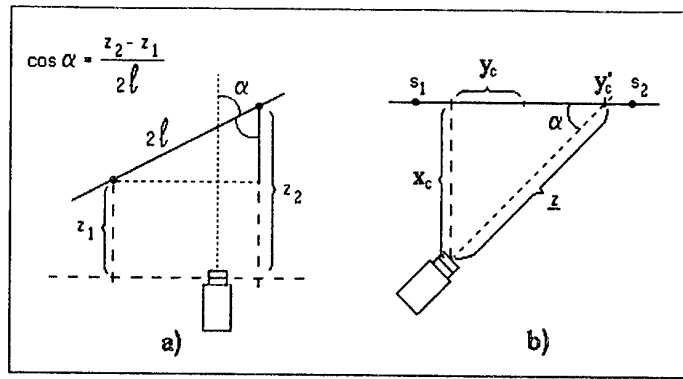


Figure 5: Position estimation

Measured position			Estimated position		
$x_c$	$y_c$	$\alpha$	$\hat{x}_c$	$\hat{y}_c$	$\hat{\alpha}$
103	2	90°	104	1	90°
95	15	90°	95	13	90°
133	-10	90°	135	-11	90°
78	6	90°	80	5	90°
101	-7	90°	99	-7	90°
80	-3	90°	81	-1	90°
158	-19	90°	156	18	90°

Table 1: Position estimation error in the case  $Y_1 \equiv Y_2$

The results for the cases corresponding to robot positions for which  $Y_1 = Y_2$  (i.e., for the cases in which equation (3) is used) are reported in Table 1, whereas the results for the cases  $Y_1 \neq Y_2$  (i.e., for the cases in which equation (5) is used) are shown in Table 2 (In both tables, the  $x$  and  $y$  coordinates are in meters, the angle are in degrees). In both cases, the position estimation error achieved by means of the vision system is smaller than 5 cm. Since the error in position estimation by means the odometry (encoders) is in the order of 1%, the robot position could be corrected by means the vision system about every 10 m (i.e., when an error in the order of 10 cm has been accumulated by means of the odometry system).

## References

- [1] I.J. Cox and G.T. Wilfong. *Autonomous Robot Vehicles*. Springer-Verlag, New-York, NY, 1990.
- [2] J.C. Latombe. *Robot Motion Planning*. Kluwer Academic Publisher, Boston, MA, 1990.
- [3] M.R. Kabuka and A.E. Arenas. Position verification of a mobile robot using standard pattern. *IEEE J. of Robotics and Automation*, 11:505-516, 1987.

Measured position			Estimated position		
$x_c$	$y_c$	$\alpha$	$\hat{x}_c$	$\hat{y}_c$	$\hat{\alpha}$
113	98	130°	109	96	131°
90	26	108°	92	27	109°
98	55	119°	98	51	118°
91	86	125°	90	84	125°
126	115	129°	130	113	127°
92	11	101°	95	13	100°
107	85	131°	111	88	133°
88	81	131°	89	81	131°

Table 2: Position estimation error in the case  $Y_1 \neq Y_2$

- [4] D.N. Green, J.Z. Sasiadek, and G.S. Vukovich. Path tracking, obstacle avoidance and position estimation by an autonomous, wheeled planetary rover. In *IEEE Int. Conference on Robotics and Automation*, pages 1300–1305, San Diego, CA, 1994.
- [5] R. Talluri and J.K. Aggarwal. Mobile robot self-location using model-image feature correspondence. *IEEE Trans. on Robotics and Automation*, 20(1):63–77, 1996.
- [6] F. Serradilla and D. Maravall. A navigation system for mobile robots using visual feedback and artificial potential fields. In *Int. Conf. on Robotics*, pages xx–yy, Wien, 1996.
- [7] J. Vandorpe, H. Van Brussel, and H. Xu. LiAS: A reflexive navigation architecture for an intelligent mobile robot system. *IEEE Trans. on Industrial Electronics*, 43(3):432–440, June 1996.
- [8] A. Kosaka and C. Kak. Fast vision-guided mobile robot navigation using model-based reasoning and prediction of uncertainties. *CVGIP: Image Understanding*, 56(3):271–329, 1992.
- [9] J. Princen, J. Illingworth, and J. Kittler. A formal definition of the Hough transformation: Properties and relationship. *J. Math. Imaging Vision*, 1:153–168, 1992.
- [10] A. Bétourné, S. Nicosia, and P. Valigi. A vision based obstacle detection system for mobile robots. In C. Bonivento, C. Melchiorri, and H. Tolle, editors, *Advances in Robotics. The ERNET Perspective. Proc. of the Research Workshop of ERNET, European Robotics Network, Darmstadt, Germany, 9-10 September 1996*, pages 173–182. World Scientific Publishing Co. Ltd., Singapore, 1996.
- [11] T. Kambara, J. Miurae, and Y. Shirai. Selection of efficient landmarks for an autonomous vehicle. In *Proceedings of the 1993 IEEE/RSJ Int. Conference on Intelligent Robots and Systems*, pages 1332–1338, 1993.

# ROBUST HYBRID FORCE/POSITION CONTROL OF ROBOTS IN CONDITIONS OF UNCERTAINTY

Sergey F. Burdakov and Leonid V. Paramonov  
St. Petersburg State Technical University  
Department of Mechanics and Control Processes  
29, Politechnicheskaya str., St. Petersburg, 195251, RUSSIA  
E-mail: controll@citadel.stu.neva.ru

**Abstract.** Method of the compensatory type, allowing to synthesize control for want of parametrical uncertainty of the mathematical model of elastic robot and incomplete measurements of a state vector is offered. High-gain feedback is used for the proposed control law.

**Keywords.** Flexible robot, robust hybrid force/position control, parametrical uncertainty, high-gain feedback.

**Introduction.** The tasks, where force factors are considered as controlled variables, arise for want of automatic control of assembling processes, processing surfaces and outlines of preparations, mounting and unmount of constructions, including under extreme conditions. Control of such processes on the basis of feedback provides the robot has, besides main sensors of feedbacks, information on coordinates and velocities, additional force sensors, describing conditions of force interaction in places of contact of the robot to the other objects. The contact is usually carried out through a end effector of the robot, therefore the force sensor is installed near the end effector. In most cases the principle of work of force sensors is based on the use of elastic sensitive elements [1]. Availability of force sensors and contact interactions adds to the robot new properties, namely common rigidity of the construction is reduced and closed kinematic circuits are emerged. It results in specificity of hybrid force/position control tasks and appearance of new problems [1-4]. The most actual from theoretical and practical points of view problems are:

- development of mathematical models of robots, as objects of control with allowance for elasticity of the elements and external force factors, being by controlled variables;
- synthesis of algorithms of hybrid force/position control under incomplete measurements of a state vector with allowance for interconnected dynamics of the robot;
- use of methods of active blanking of elastic oscillations for want of force/position control.

In the present paper for want of solution of these problems within the condition of a *priori* parametrical uncertainty of the mathematical model of the robot is additionally introduced.

**Problem formulation.** Contact interaction, distinctive for surface handling processes, is considered when it is required to fulfil surface movement with given force of contact among end effector and surface. The calculated scheme, explaining the given process with use of a double-link robot, is indicated on Fig.1. The end effector by the mass  $m_0$  through the passive joint P is connected to the robot link, their interaction is characterized by force of response  $F$  and elastic force  $R = ky$ ,  $y = l_1 \sin q_1 + l_2 \sin(q_1 + q_2)$ , where  $k$  - stiffness of the elastic element. Interaction of the end effector with the surface is characterized by force of normal response  $N = m_0 g - R$  and force of dry friction  $F_f = -\mu N \dot{x}/|\dot{x}|$ ,  $\dot{x} = -l_1 \dot{q}_1 \sin q_1 - l_2(\dot{q}_1 + \dot{q}_2) \sin(q_1 + q_2)$ , where  $\mu$  - coefficient of friction.

In a common case, the equation describing robot dynamics for want of considered contact interactions with allowance for elasticity in active joints leads to

$$J\ddot{q}_m + C(q_m - q) = u$$

$$A(q)\ddot{q} + b(q, \dot{q}) + g(q) + L^\top(q) \begin{bmatrix} -F_f \\ R \end{bmatrix} + C(q - q_m) = 0, \quad (1)$$

where  $q \in R^n$  and  $q_m \in R^n$  - vectors of generalized coordinates of links and curls of motors accordingly;  $u \in R^n$  - vector of control;  $A(q)$  - matrix of inertia of links,  $J$  - diagonal matrix of inertia of curls of drives;  $C$  - diagonal matrix of stiffnesses of transmissions, linking curls of motors to links;  $b(q, \dot{q})$  - vector of centrifugal and coriolis forces;  $g(q)$  - vector of gravity;  $L(q)$  - Jakoby matrix.

It is supposed that:

- 1) in the mathematical model (1) only matrix  $J$  is known with an adequate accuracy;
- 2) coordinates of links  $q$  and velocities of curls of motors  $\dot{q}_m$  are measured;
- 3) the programme trajectory  $q_d(t)$  is given at the finite time interval  $t \in [0, T]$ . It has first  $\dot{q}_d(t)$  and second  $\ddot{q}_d(t)$  limited derivatives, thus the velocity of movement along the trajectory is sufficient for the absence of conditions of discontinuous movement, stipulated by availability dry friction [5];
- 4) the separation of the end effector from the surface does not occur, i.e.  $|N - N_d| < N_d$ , where  $N_d$  - programme force of contact.

It is required to construct feedback control  $u$  ensuring:

- 1) asymptotic stability of any fixed configuration  $q_d^* = \text{const}$ ,  $N_d = \text{const}$

$$\|q(t) - q_d^*\|_{t \rightarrow \infty} = 0, \quad \|N - N_d\|_{t \rightarrow \infty} = 0; \quad (2)$$

- 2) that, since some moment  $t_0$ , the following conditions held:

$$\|q(t) - q_d(t)\|_{t \in [t_0, T]} \leq \delta, \quad \|N(t) - N_d\|_{t \in [t_0, T]} \leq \varepsilon, \quad (3)$$

where  $\delta > 0$  and  $\varepsilon > 0$  - given exactitudes of improvement of the programme trajectory.

**The method of indirect compensation.** Method of the compensatory type, allowing to synthesize control for want of parametrical uncertainty of the mathematical model of elastic robot and incomplete measurement of the state vector is offered. We shall define vector of new variables  $z$ , which are the outcome conversions with the help of realizabled differentiating filter of the first order of the linear combination of measured variables  $q$  and  $\dot{q}_m$

$$\begin{aligned} \nu \dot{x} &= -x + (\alpha - \nu^{-1}E)(\dot{e}_m - \xi e) \\ z &= x + \nu^{-1}(\dot{e}_m - \xi e), \end{aligned} \quad (4)$$

where  $e = q - q_d$  and  $e_m = q_m - q_d$  - tracking error for links and curls of motors;  $\nu$  - small constant of time of the filter;  $\alpha$  and  $\xi$  - diagonal matrix of coefficients;  $E$  - unit matrix.

In functional form the equation (4) can be rewritten as

$$z = (\nu p + 1)^{-1}(Ep + \alpha)(\dot{e}_m - \xi e), \quad (5)$$

where  $p$  - operator of derivation.

We shall denote one more diagonal matrix of coefficients  $\eta$  and we shall consider linear combination  $z + \eta e$ . With allowance for (5) we shall receive

$$z + \eta e = (\nu p + 1)^{-1}(\ddot{e}_m + \alpha \dot{e}_m - \theta \dot{e} + \beta e), \quad (6)$$

where  $\theta = \xi - \nu\eta$ ,  $\beta = \eta - \alpha\xi$ .

We shall consider expression in brackets in the right part of (6). Substituting there the expression  $\dot{e}_m$  and  $\ddot{e}_m$ , found from the second equation (1), we shall obtain

$$\ddot{e}_m + \alpha \dot{e}_m - \theta \dot{e} + \beta e = p(Ep + \alpha)\{C^{-1}[A(q)\ddot{q} + b'(q, \dot{q}) + g(q)] + e\} - \theta \dot{e} + \beta e, \quad (7)$$

where  $b'(q, \dot{q}) = b(q, \dot{q}) + L^T(q) \begin{bmatrix} -F_f \\ R \end{bmatrix}$ .

Multiplying the first equation from (1) at the left by  $J^{-1}$ , we shall lead it in the aspect

$$\ddot{e}_m + \alpha \dot{e}_m - \theta \dot{e} + \beta e = v - J^{-1}C(e_m - e) + \alpha \dot{e}_m - \theta \dot{e} + \beta e \quad (8)$$

The following relation is used here

$$u = J(v + \ddot{q}_d) \quad (9)$$

Taking into account, that the  $J$  and  $\ddot{q}_d$  are known, we shall consider  $v$  as a new control. If the matrix of stiffnesses  $C$  was known and except measurements  $q$  and  $\dot{q}_m$ , additional measurements of  $q_m$  and  $\dot{q}$  were available, then in correspondence with the method of compensation it would be possible to construct control law

$$v_c = J^{-1}C(e_m - e) - \alpha \dot{e}_m + \theta \dot{e} - \beta e. \quad (10)$$

With this control equation (8) can be rewritten as

$$\ddot{e}_m + \alpha \dot{e}_m - \theta \dot{e} + \beta e = 0,$$

so according to (7)

$$p(Ep + \alpha)\{C^{-1}[A(q)\ddot{q} + b'(q, \dot{q}) + g(q)] + e\} - \theta \dot{e} + \beta e = 0 \quad (11)$$

The equation (11) describes the closed-loop system (1), (9), (10). Let  $\|C\| \gg c$ , where  $c > 0$  and such, that

$$\left\| \frac{\partial [b'(q, 0) + g(q)]}{\partial q} \right\| \leq c, \quad \forall q \in R^n.$$

Then it is possible to show, that the closed-loop system (11) will be asymptotically stable in the sense of (2) for want of realization of sufficient conditions

$$\alpha_i > 0, \quad 0 < \theta_i/\alpha_i < 1, \quad 0 < \beta_i < \underline{\omega}^2(1 - \theta_i/\alpha_i)\theta_i/\alpha_i, \quad i = \overline{1, n}, \quad (12)$$

where  $\underline{\omega}$  - minimum value from the spectrum of frequencies of the robot for want of fixed curls of motors  $\det[-\omega^2 A(q_d^*) + C] = 0$ .

Uniting (10) and (11) we shall obtain the functional equation

$$v_c = H(p)e, \quad (13)$$

where  $H(p) = p^{-1}[(J^{-1}C + Ep^2)(Ep + \alpha)^{-1}(\theta p - \beta) - J^{-1}Cp]$ .

Assuming the control constraints  $|v_{ic}| \leq v_{imax}$  ( $i = \overline{1, n}$ ) from (13) it is possible to obtain an upper bound of the value  $\delta$  from (3)

$$\|e\| \leq \delta, \quad \delta = \|H^{-1}(j\omega)\|_{\infty} \|v_{max}\| \quad (14)$$

In expression (14) the norm of a vector is induced by a norm of a matrix

$$\|e\| = \left[ \sum_{i=1}^n e_i^2 \right]^{1/2}, \quad \|H^{-1}(j\omega)\|_{\infty} = \left\{ \max_{0 \leq \omega < \infty} \text{eig}[H^{-1\top}(j\omega)H^{-1}(j\omega)] \right\}^{1/2}$$

The vector  $r \in R^m$  of coordinates of the joint P (Fig.1) is connected kinematically by relation  $r = f(q)$  with the vector of coordinates of links  $q$ . In deviations from programme trajectory we obtain

$$\Delta r = L(q_d)e, \quad L(q) = \left\{ \frac{\partial r_j}{\partial q_i} \right\}, \quad j = \overline{1, m}, i = \overline{1, n} \quad (15)$$

With allowance for (14) and (15) it is possible to obtain a handset of an exactitude in cartesian coordinates, as well as  $\varepsilon$  value from (3).

For definition of realizable control  $v$  as (10) we shall construct the procedure of gradient type. We shall enter an measure of closeness of control  $v$  to control  $v_c$

$$I(t) = \frac{1}{2}(v - v_c)^{\top}(v - v_c) \quad (16)$$

The absolute minimum of the measure (14)  $I(t) = 0$  is reached for want of control (10). Control  $v$ , ensuring  $\lim_{t \rightarrow \infty} I(t) = 0$ , has an aspect [6]

$$\dot{v} = -\gamma \text{grad}_v I, \quad v(0) = v_0, \quad (17)$$

where  $\gamma > 0$  - positive defined diagonal matrix of coefficients;  $v_0$  - initial value of  $v_c$ .

Involving (8) and (10) the expression (17) can be rewritten as

$$\dot{v} = -\gamma(\ddot{e}_m + \alpha \dot{e}_m - \theta \dot{e} + \beta e) \quad (18)$$

With increasing  $\gamma$  the control law (9), (18) aspires to control law (9), (10), found by the method of compensation. It does not contain the unknown matrix of stiffnesses  $C$  any more, however it is also not realizable, as requiring additional measurements  $\ddot{e}_m$  and  $\dot{e}$ . With allowance for relations (4) - (6) we shall note realizable control law as (9), (18)

$$\begin{aligned} u &= J(\ddot{q}_d + v), \quad \dot{v} = -\gamma(z + \eta e) \\ \nu \dot{x} &= -x + (\alpha - \nu^{-1}E)(\dot{e}_m - \xi e) \\ z &= x + \nu^{-1}(\dot{e}_m - \xi e) \end{aligned} \quad (19)$$

Uniting (1) and (9), we shall obtain the functional equation of the closed-loop system

$$p[\gamma^{-1}(\nu p + 1)(Ep^2 + J^{-1}C) + Ep + \alpha]\{C^{-1}[A(q)\ddot{q} + b'(q, \dot{q}) + g(q)] + e\} - \theta \dot{e} + \beta e = 0 \quad (20)$$

the order of which is equal to  $6n$ . Noting the state variables equation (20) it is possible to obtain the system of the differential equations with small parameters for want of partial derivatives. For want of conditions

$$\nu^{-1} > \|\alpha\|, \quad \|\gamma\| > \gamma^* \quad (21)$$

the main assertion about separation of movements is fulfilled in singular perturbed systems [7]. It means, that for want of rather large  $\|\gamma\|$  for an arbitrarily small period  $t_0$  a movement of closed-loop systems (20) will be as close to a slow movements (11) as wished.

**Simulation results.** A robot, with calculated scheme is depicted in Fig.1, is considered for want of the following input data  $l_1 = l_2 = 1$  m,  $m_1 = m_2 = 200$  N,  $m_0 = 50$  N,  $\mu = 0.57$ ,  $k = 10^4$  N/m. The stiffness  $k$  of the elastic sensor is choosed, that maximum modal frequency  $\bar{\omega}_0 = 6$  Hz is less then minimal modal frequency  $\underline{\omega} = 10$  Hz ( $\bar{\omega}_0$  is maximum modal frequency for various configurations of the robot, in the supposition of an absolute rigidity of joints;  $\underline{\omega}$  is minimal modal frequency found with allowance for flexibility of joints).

The schedule of the programme velocity  $\dot{x}_d(t)$  is indicated on Fig.2. For it  $t_1 = T - t_2 = 0.2$  s,  $T = 5.2$  s,  $\dot{x}_{dmax} = 0.2$  m/s,  $x_0 = 0.2$  m,  $x_T = 1.2$  m. Programme force of contact between end effector and surface  $N_d = 25$  N. A required accuracy of improvement of the program trajectory:  $\max_t |x - x_d| \leq 0.5$  mm,  $\max_t |\dot{x} - \dot{x}_d| \leq 0.01$  m/s,  $\max_t |N - N_d| \leq 1$  N.

Graphics of closed-loop system processes for want of  $\alpha_i = 40$  1/s,  $\theta_i = 20$  1/s,  $\beta_i = 400$  1/s<sup>2</sup>,  $\nu = 0.02$  s,  $\gamma_i = 150$  1/s ( $i = 1, 2$ ) are shown at the Fig.3. The graphics of the process  $N(t)$  corresponded to  $\gamma_i = 150$  1/s is marked by 2. To Fig.1 there corresponds value  $\gamma = 75$  1/s. For this value of  $\gamma_i$  a handset exactitudes  $\max_t |N - N_d|$  is more then required one. The simulation was performed for want of the entry conditions certainly exceeding real for practice value. Nevertheless, for want of both values exit on movement (11), obtained by a method of indirect compensations was ensured, during  $t_0$  near the  $\gamma_i^{-1}$ . On the graphics of the process  $e(t)$  the error at the stage of movement with constant velocity  $\dot{x}_{dmax}$  is visible. It is the result of the following: for want of movement (11) the closed-loop system has the first order astatism property only.

**Conclusion.** Simulation has shown, that the control law (20) can be used in the case of pretty high level of uncertainty of the mathematical model of the robot (1). It is actually required to know only ranges inertial and stiffness characteristics. However to reach least limiting dynamic indexes, appropriate to the equation (10)  $\gamma$  is required to be increased. But increasing of  $\gamma$  reduce to significant influence of noise of sensors feedbacks. Therefore it is recommended to choose  $\nu$  and  $\gamma$  so, only to supply realization of conditions (21). Improving dynamic indexes in this case it is possible to supply at the expense of introduction of additional feedbacks on force of interaction  $R$  [3]<sup>1</sup>.

<sup>1</sup>This research is conducted within the research grant No. 97-01-01159 from RFBR and No. 94-965 from INTAS

## References

1. Gornievskij D.M., A.M.Formalskij, A.U.Shneider (1994). Control by systems of manipulators on the basis the information on gains (in Russian). Fizmatlit, Moskow.
2. Mason M.T. (1981). Compliance and forse control for computer Controlled manipulators. IEEE Trans. System, Man., Cybernet., vol.11, N6, pp.418-432.
3. Wang D., Y.C.Soh, C.C.Cheah (1995). Robust motion and forse control of constrained manipulators by learning. Automatica, vol.31, N2, pp.257-262.
4. Burdakov S.F. (1989). About synthesis of the control algorithms by movement of the robot in conditions of uncertainty of a trajectory (in Russian). Izv. AN USSR A mechanics of a rigid body, N2, pp.54-59.
5. Burdakov S.F., A.L.Volgarev, V.Ja.Katkovnik, V.A.Polianskij (1988). Mathimatical model and exactitude of the robot TUR - 10 for want of movement on low velocities (in Russian). Design and reliability of machines (transactiones LPI), N425, pp.84-88.
6. Pshenichnij B.N., U.M.Danilin (1975). The numerical methods in extremum problems(in Russian). Science, Moskow.
7. Vasileva A.B., V.F.Butuzov (1973). Asymptotic expansions of solutions is singular of the perturbed equations (in Russian). Science, Moskow.

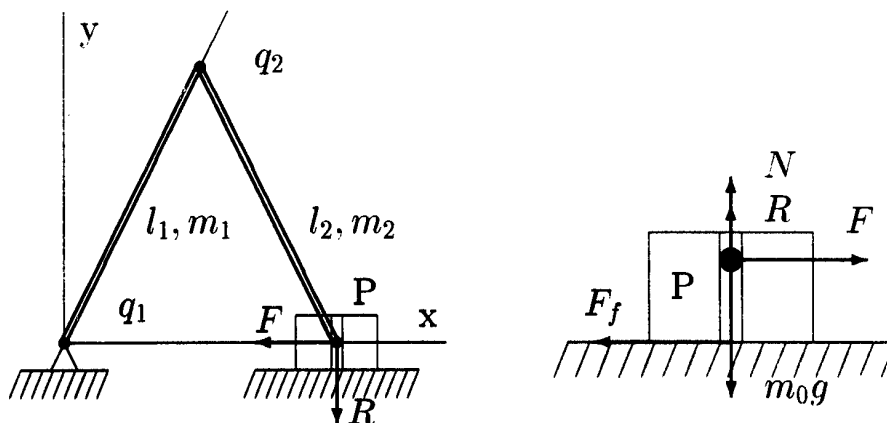


Fig.1. The calculation sheme of a double-link robot

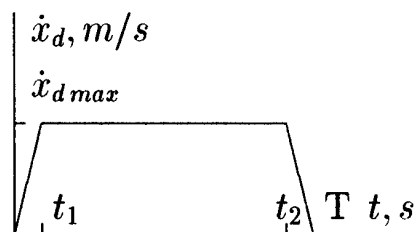


Fig.2. The programme velocity  $\dot{x}_d$

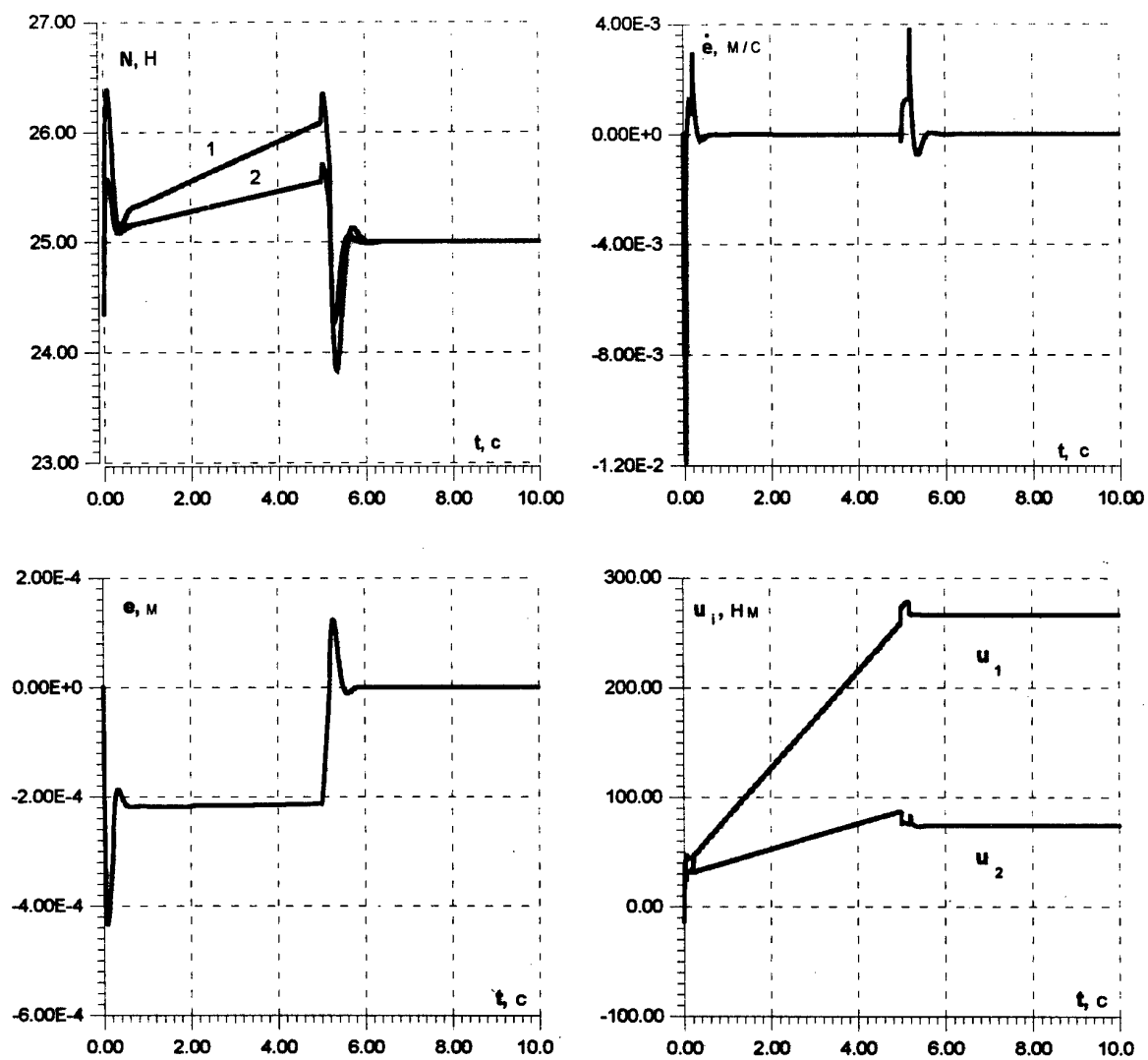


Fig.3. Simulation results

# ADAPTIVE CONTROL WITH PRECISE IDENTIFICATION OF UNKNOWN PARAMETERS OF DYNAMIC MODEL FOR ROBOT MANIPULATORS

Andrey G. Altbregin,  
Vladimir A. Korolev  
St. Petersburg State Technical University  
Mechanisms and Machinery Department  
Chair of Flexible Automated Complexes  
18 Boulevard Novatorov apt. 27,  
198216 St. Petersburg, RUSSIA

**Abstract.** General problem of achieving certain motion quality characteristics (precision, operating speed etc.) obtains specific aspects when a robot manipulator is exposed to unstable or indeterminate functioning conditions, which occur rather often when a manipulator is intended for unknown or changing environment. Under such conditions natural uncertainty of the parameters of dynamic model decreases motion quality considerably. This research is dedicated to an adaptive control with precise identification of unknown parameters of dynamic model, which not only allows to ensure required motion quality, but also provides control system with precise data on object and environment parameters, that may be applied for objects identification and dynamic motion programming.

**Keywords.** Robot manipulator, adaptive control, uncertain environment, identification of indeterminate parameters, trajectory programming.

The most apparent example of uncertain conditions is the work of a manipulator in extreme environment and indeterminate landscape. Especially it concerns various kinds of research manipulators, intended to be exploited underwater, in fire etc. High level of uncertainty is also typical for computer integrated manufacturing. First of all, a manipulator is supposed to operate a range of objects (parts), and this range may be rather big. Re-adjustment of manipulator for operating an object different from current one (mechanical - changing the tool etc., as well as programmed - changing control parameters, processing mode) takes certain time that decreases the efficiency of functioning of the robot complex as a whole, especially if frequent or even arbitrary change of objects is presupposed. It is desirable that the programmed path is executed with given precision for any object (out of the possible range) without additional time spent for the equipment re-adjustment. Beside the uncertainty of the object parameters mentioned above, uncertainty of the situation in working area and unstable production conditions are very typical for the work of a robot complex. Thus, the necessity of introduction of special algorithms that would adapt the control system to uncertainty of object and environment parameters appears very clearly.

It must be mentioned here that adaptive control does not necessarily imply precise identification of unknown parameters. Many adaptive algorithms consist in self-adjustment of parameters of the control law based on certain qualitative criteria, thus ensuring required

quality of manipulator performance, but not delivering exact values of indeterminate parameters of the object and environment. Such an approach [1] gives satisfactory results in many cases. At the same time, precise identification of unknown parameters gives additional advantages for control automation. Namely, let us assume that weight and dimensions of the object are functions of a set of parameters, which can vary arbitrarily within a pre-defined area. Precise identification of those parameters provides control system with possibilities to:

- 1) identify and classify the object of manipulation;
- 2) perform instant motion programming in case technological process is dependant on identified parameters.

The essence of identification approach [1] to adaptive control consist in application of certain "test" signals to the manipulator servo-system and subsequent measurement of the motion caused by these "test" signals. It results in a system of equations containing the vector of unknown parameters  $\xi$ . Solution of these equations delivers identified parameters. It is expedient to combine identification with control itself. Thus, on the one hand, control is utilised to study the dynamics of the robot system by current identification of its parameters, on the other hand, the control law with identified parameters provides for the required system response while performing the pre-programmed motion.

The general scheme of an adaptive control system is shown in the Fig.1. **Programming unit** plans target (ideal) trajectory  $x(t)$  based on data received from the informational system of the robotic complex. Adaptation quality criteria is defined as an **estimating** equation:

$$\varphi(\xi_{est}, t) \equiv \delta - \|u(t) - U(x, \dot{x}, \xi_{est})\| > 0, \quad (1)$$

where  $u(t)$  - certain control law;

$U(x, \dot{x}, \xi)$  - an operator defining dynamic model of the system:

$$u = U(x, \dot{x}, \xi) = G(x, \dot{x}) \cdot \tau(\xi) \quad (2)$$

$\tau(\xi)$  - vector-function transforming dynamic model of the system to a form linear in unknown parameters;

$\xi_{est}$  - current estimation of the vector of parameters  $\xi$ .

Based on the value of estimating function  $\varphi(\xi_{est}, t)$ , **adapting unit** calculates a current estimation  $\xi_{est}$  of the vector of parameters  $\xi$  according to the chosen algorithm of adaptation.

As far as control system is digital it is reasonable to utilise one of discrete recursive algorithms of adaptation, that have certain advantages compared to continuous ones. Namely,

- recursive algorithms of adaptation deliver precise estimation of the unknown vector of parameters  $\xi$ , that is in general impossible for continuous algorithms [1];
- recursive algorithms of adaptation secure finite time of adaptation, allowing to estimate not only the total period of adaptation, but the moment of the end of transition process as well, that is also impossible in case of continuous algorithms [4].

The most appropriate for adaptation with identification are recursive algorithms of the gradient type:

$$\tau_{k+1} = \gamma_k \tau_k + \lambda_k \nabla_{\tau} \varphi (\tau_k, t_k') \quad (3)$$

where  $\gamma_k, \lambda_k$  - parameters of the algorithm;

$\nabla_{\tau} \varphi (\tau_k, t_k')$  - gradient of the function  $\varphi(\tau, t)$  with respect to  $\tau$  taken at the point  $\tau = \tau_k, t = t_k'$ .

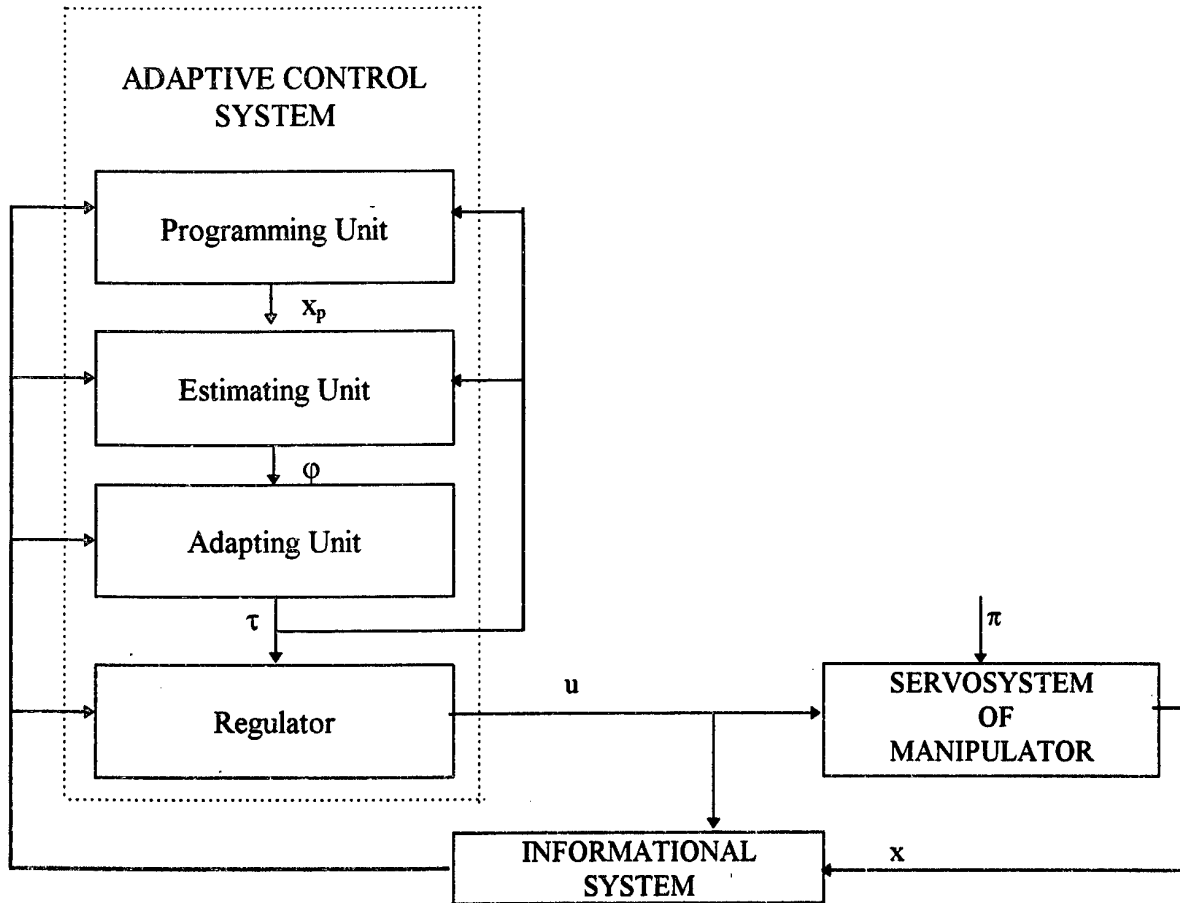


Fig. 1 General scheme of an adaptive control system.

The choice of the parameters  $\gamma_k$  and  $\lambda_k$  is subject to a chosen criteria of the quality of adaptation (precise estimation of the unknown parameters, maximum estimation speed etc.) It must be taken into account that most efficient (in regard of estimation speed) algorithms require as a rule more complicated calculations. That is why it is essential to find a reasonable compromise between efficiency of adaptive algorithm and capability of the processor utilised in the control system.

Let us study an adaptive algorithm of the mentioned type applied to a robot manipulator with servo-drives based on DC electrical motors. Dynamics model of a robot manipulator is described by the following vector equation:

$$U(\vartheta, \dot{\vartheta}, \ddot{\vartheta}, \xi) = A(\vartheta, \dot{\vartheta}, \xi) \ddot{\vartheta} + B(\vartheta, \dot{\vartheta}, \xi) \quad (4)$$

where  $\mathbf{U}$  - vector of control voltages applied to servo-motors;  
 $\mathbf{q}$  - vector of generalised co-ordinates of the manipulator [3].

We shall introduce a system state vector

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix} \quad (5)$$

Then dynamics model of the manipulator can be transformed to the form (2) for most of manipulators and robot complexes [1].

Stabilising control will be constructed as follows:

$$\mathbf{u}(t, \mathbf{x}) = \mathbf{G}(\mathbf{x}, \dot{\mathbf{x}}_p + \Gamma(\mathbf{x} - \mathbf{x}_p)) \cdot \tau_{est} \quad (6)$$

where  $\Gamma = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \Gamma_1 & \Gamma_2 \end{bmatrix}$  - a stable matrix of feedback coefficients;

$\Gamma_1$  - sub-matrix of proportional feedback coefficients;

$\Gamma_2$  - sub-matrix of differential feedback coefficients;

$\tau_{est} = \tau(\xi_{est})$  - a current estimation of the vector of unknown parameters of dynamics model;

$\mathbf{x}_p$  - target trajectory.

For identification of the vector of unknown parameters  $\xi$  we shall utilise a recursive adaptive algorithm suggested in [2]. The solution of the estimating equation is delivered by a recursive algorithm of the following type:

$$\tau_{k+1} = \tau_k + \lambda_k \cdot \nabla_{\tau} \varphi(\tau_k, t'_k), \quad (7)$$

where  $\tau_{k+1}$  - next estimation of the vector  $\tau(\mathbf{J})$ ;

$t'_k$  - a moment at which the estimating equation is violated;

$\nabla_{\tau} \varphi$  - gradient of the function  $\varphi(\tau, t)$  with respect to  $\tau$ ;

$$\lambda_k = \frac{(\tau(\xi) - \tau_k) \cdot \nabla_{\tau} \varphi(\tau_k, t'_k)}{\|\nabla_{\tau} \varphi(\tau_k, t'_k)\|^2} \quad (8)$$

- parameter of the algorithm.

It must be emphasised here that in Eq. (8) the product of scalar multiplication  $\tau(\xi) \cdot \nabla_{\tau} \varphi(\tau_k, t'_k)$

can be expressed in terms of the variables whose values are definite at each step of the adaptive algorithm:

$$\tau(\xi) \cdot \nabla_{\tau} \varphi(\tau_k, t'_k) = \frac{[u(t, x) - B(\vartheta, \dot{\vartheta}, \xi_{est})] \cdot [u(t, x) - U(x, \dot{x}, \xi_{est})]}{\|U(x, \dot{x}, \xi_{est}) - B(\vartheta, \dot{\vartheta}, \xi_{est})\|} \quad (9)$$

There was a computer simulation performed for the dynamics of the manipulator PUMA-560 controlled with application of the adaptive stabilising strategy described above. The plots of system response versus time for the first joint of the manipulator are shown in fig.2: curve 1 corresponds to system response with adaptive algorithm activated, curve 2 represents the response of the system with the same initial state and feedback coefficients but without adaptation. The results of system simulation allow to make the following conclusions:

- 1) Comparison of response of the system with and without adaptation demonstrates the efficiency and expediency of adaptive control itself. In particular, simulation of the robot motion in case only one parameter (the moment of inertia  $J$  of the arm) is identified gives the following result: if discrepancy between initial estimation of the parameter  $J$  and its actual value is 33%, due to adaptation transitional period is reduced by approximately 5% (when required precision is 0.001 rad), and over-regulation is reduced by approximately 23%.

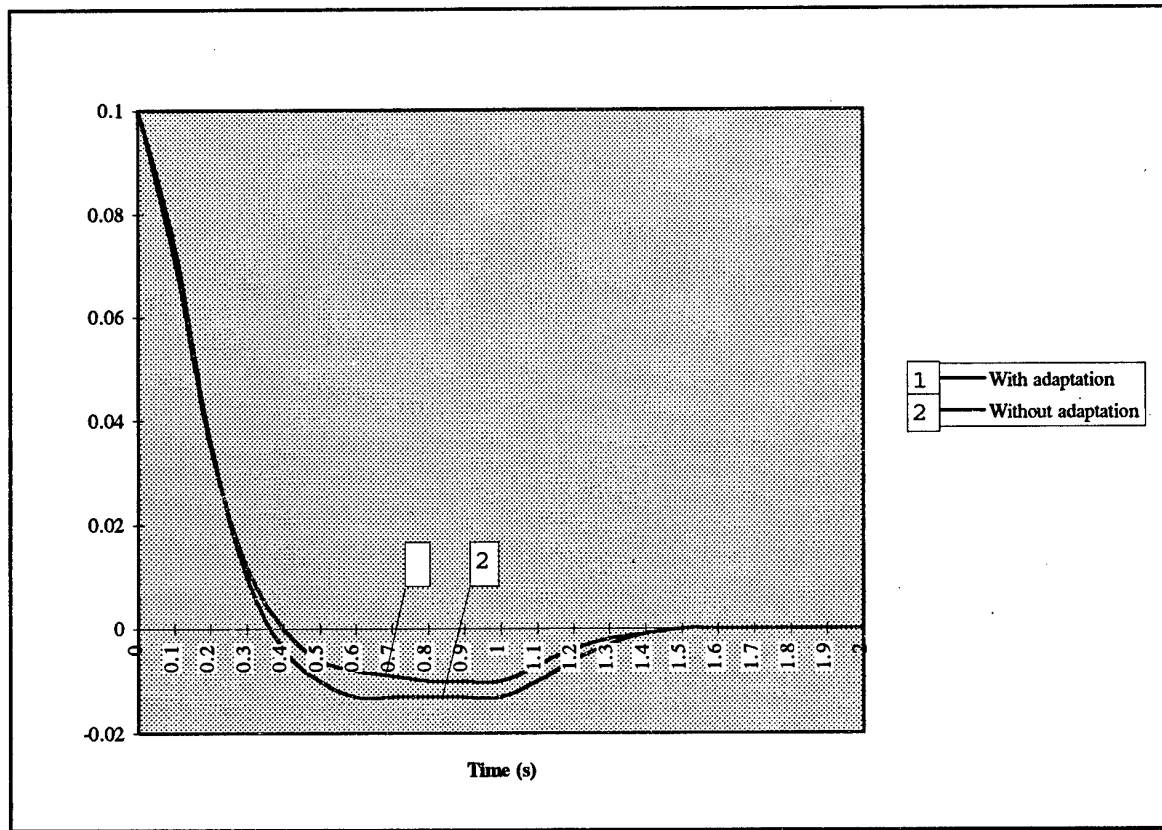


Fig.2 System response with and without adaptation.

- 2) System simulation with consideration of errors of the sensors of speed and acceleration shows, that the precision of identification of unknown parameters depends considerably on precision of the values of speed and acceleration delivered to control system of manipulator by its informational system. For instance, if the speed in the first joint of manipulator is measured with precision 0.5% and acceleration - with precision 5%, and we suppose that sensors errors have a distribution low close to normal one, the average squared error of estimation of the moment of inertia makes approximately 9%. Figure 3 (curve 1) represents the relation between sensor precision and average squared estimation error. Estimation precision may be increased if the sensors data are statistically averaged. In particular, in case sensors error is governed by normal distribution low, local linear approximation of the sensors output may be applied. As simulation results show ( Fig. 3, curve 2 ), such a method allows to reduce estimation error by 2-3 times.
- 3) Modelling results show that the identified parameters are estimated with acceptable (that is, attainable in principle) precision in rather short period of time. For instance, in case only one parameter is identified, its estimation with acceptable precision is achieved already in the first step of identification, that is, the period of identification is finite and rather small (practically equivalent to the time of computation of one step of the algorithm of identification). Thus, a theoretical conclusion about finiteness of adaptation time of the suggested here algorithm [1] may be applied to stochastic model of the system. Consequently mentioned algorithm may be utilised for current identification unstable parameters when these parameters change slow enough.

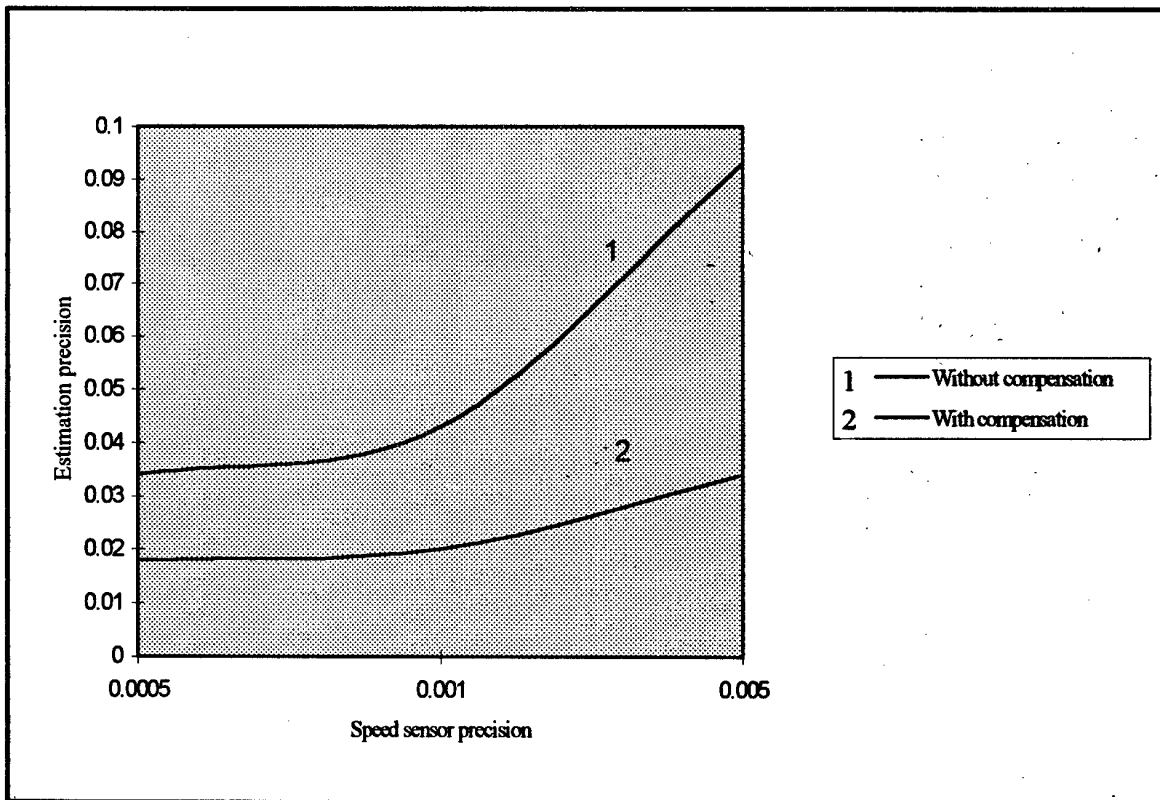


Fig. 3 Relation between speed sensor precision and estimation precision.

In conclusion, we would like to emphasise that the adaptive algorithm of identification described above may be applied not only to robot manipulators, but to any particular component of robot complexes. Control system may be adapted not only to unknown dynamic parameters of the equipment, but to all kinds of external disturbing forces, unstable coercions etc. as well. It not only gives the possibility to efficiently solve the problem of system stabilisation, but also considerably enlarges the control system capability in regard of environment recognition and control intelligence.

*References:*

1. Timofeev, A. V. (1988) Adaptive robotic complexes. Mashinostroenie, Leningrad.
2. Timofeev, A. V. (1980) Construction of adaptive control systems. Energia, Leningrad.
3. Fu, K. S., R. C. Gonzalez, C. S. G. Lee (1987) Robotics: Control, Sensing, Vision, and Intelligence. McGraw - Hill Book Company.
4. Sragovitch, V. G (1981) Adaptive control. Nauka, Moscow.

# TIME - MINIMAL ROBOT CONTROL ALONG KNOWN PATH

Felix M. Kulakov and Victor B. Naumov

Russian Academy of Science

St. Petersburg Institute for Informatics and Automation, Robotics Laboratory

14th line, 39

St. Petersburg 199178, RUSSIA

e-mail: kul@mail.iias.spb.su

**Abstract.** The paper presents new effective methodology of optimal robot control generation with minimal time expenditure for robot motion along known path. This path is generated usually by robot path planning algorithm on base of the known initial and final position of robot and position of obstacles. Another possibility is path planning by teaching with help of sensor glove, put on hand of man-operator. The proposed method of optimal robot control is based on the "direct" and "reverse" integration of special equations, received from the robot dynamic equations. This paper is a result of research in project N 94-2785 frame, sponsored by INTAS and project N 96-01260, sponsored by RFFR.

**Keywords.** Robot, optimal control, Pontrjagin maximal principle, planning for teaching, sensor glove.

**Introduction.** At present there are a great number of methods for optimal robot control generation, which provide minimal robot motion time along known path [1-3]. This paper describes the new method which has a certain advantage in comparison with known methods. The key idea of this method is to determine the sign change of the drive control as the moment, according to the intersection point of two phase curves, which is generated by numerical integration of  $n$  differential equations in "forward" and "backward" directions (from moment of planned path beginning in direction time increase and from moment of planned path ending in direction time decrease). In the course of the integration it is necessary to take into consideration that the joint co-ordinates of the robot are connected by  $(n-1)$  algebraic equations which describe the planned path.

The statement of problem is as follows. *The path of robot is known.* It is defined as a simple curve in Jordan sense, which in parametrical representation has the following form:

$$g = f(l), \quad (1)$$

where  $g = (g_1, g_2, \dots, g_n)$  -  $n$ -dimensional vector of joint coordinates of a robot;

$f(l)$  -  $n$ -dimensional vector function of scalar parameter  $l$ ;

$g_b = f(l_b)$ ,  $g_f = f(l_f)$  - the initial and final points of a path.

This path is generated usually by robot path planning algorithm based on the known initial and final position of robot and position of obstacles. Another possibility is path planning by teaching. In particular it is possible to use the new perspective approach to the robot learning. It consists of the operation of spatial movement of special sensate glove put on operator hand and equipped with the position sensor. During this operation in database of robot control system the information which is applicable to complete forming of set of the glove position vectors is entered. Basing on these data in result of interpolation path of glove position is generated.

The differential equations of manipulator dynamics are known too:

$$A(g)g + g^T B(g)g + Kg + C(g) = Q,$$

they connect the vector of controls for the drives  $Q = (Q_1, Q_2, \dots, Q_n)$ , and the vector of joint coordinates  $g$ ;

$A(g)$ ,  $K(g)$  -  $(n \times n)$ -matrixes;

$B(g)$  -  $(n \times n \times n)$ -matrix;

$C(g)$  -  $n$ -vector.

The replacement of variable  $g$  in equations its expression (1) provides the generation of  $n$  following equations:

$$M(l)l + N(l)l^2 + P(l)l + R(l) = Q, \quad (2)$$

where  $M(l) = A(f(l)) \partial f / \partial l$ ,  $N(l) = A(f(l)) \partial^2 f / \partial l^2 + (\partial f / \partial l)^T B(f(l)) \partial f / \partial l$ ,  $P(l) = K \partial f / \partial g$ ,  $R(l) = C(f(l))$  -  $(n \times 1)$ -matrixes.

Another equivalent description of equations (2) is

$$M_j(l)l + N_j(l)l^2 + P_j(l)l + R_j(l) = Q_j, \quad j = 1, 2, \dots, n, \quad (2a)$$

$N_j$ ,  $P_j$ ,  $R_j$ ,  $M_j$  - the scalar functions of the variable  $l$ .

$$l + a_j(l)l^2 + b_j(l)l + c_j(l) = d_j(l)Q_j, \quad (2b)$$

where  $a_j(l) = M_j^{-1}(l)N(l)$ ,  $b_j(l) = M_j^{-1}(l)P(l)$ ,  $c_j(l) = M_j^{-1}(l)R(l)$ ,  $d_j(l) = M_j^{-1}(l)$ .

The following limitations for controls of drives are known:

$$|Q| \leq Q_{\max} \quad (3)$$

It is necessary to find the law of time variation for vector of controls for drives  $Q_j(t)$  which transfers the vector  $g$ , subordinated to the Eq. (1) from the given initial position  $g(t_b)$ ,  $g(t_b) = 0$  to the given final position  $g(t_f)$ ,  $g(t_f) = 0$  and provides minimal time expenditure, i.e. the minimal value of criteria

$$\int_{t_b}^{t_f} \Phi_0 \cdot dt = \int_{t_b}^{t_f} l \cdot dt, \quad (4)$$

where the vector of controls can change in the definite limits (Eq. (3)).

Obviously the optimal law of time variation of controls for drives will provide the transference of the variable  $l$ , subordinated to Eq. (2) from the given initial position  $l(t_b)$ ,  $l(t_b) = 0$  to the given final position  $l(t_f)$ ,  $l(t_f) = 0$  with the minimal time expenditure. Therefore, it is possible to use the second statement of the problem, which is one-dimensional optimal problem. It has the following difference from the classical one-dimensional problem: there are in this case  $n$

differential equations connecting the variable  $l$  and  $n$  limitations for the control signals instead of one equation and one limitation for control signal in case of the classical one-dimensional optimal problem.

**Approach to the problem solution.** The approach to the problem solution is based on the correct fact, that there is always among of  $n$  limitations (3) for controls one so called main limitation. If this main limitation is satisfied to the optimal process  $l(t) = l^{opt}(t)$  in certain moment then the rest  $(n-1)$  limitations are satisfied in this moment too. The main limitations are replaced one by another during the optimal process in general case.

Then the following method of problem decision is convenient. The optimal process  $l(t)$  is divided on  $m$  temporal intervals by following manner: the boundaries of these intervals are the time moments of the replacement of one main limitation by another main limitation. In accordance with Bellman principle any part of the optimal process is the optimal process. Therefore, the general optimal problem it is possible to divide on  $m$  one-dimensional optimal sub-problems. Each of them is sub-problem of the definition of optimal process  $l^k(t)$  and one optimal control  $Q_j^k(t)$  for drive with the number  $j$  corresponding to the main limitation for time interval with the number  $k=1,2,...,m$ . The rest  $(n-1)$  joint forces  $Q_i^k$  ( $i \neq j$ ) are calculated with the help of rest  $(n-1)$  equations (Eq. (2)) by the substitution of the defined optimal process  $l(t)$  in the left parts of these equations.

The course of the decision for each sub-problem of definition of optimal process  $l^k(t)$  and one optimal control  $Q_j^k(t)$  for drive with number  $j$  is based on use of Pontrjagin maximum principle. The number  $j$  corresponds to the main limitation for time interval with number  $k$ .

In accordance with this principle Hamilton function which connects vector of state  $(l^k, \dot{l}^k)$  and the additional variables  $p_0^k, p_1^k, p_2^k$  for each time interval with number  $k$  must have maximum value, if the control of drive with the number  $j$  is the optimal value. Hamilton function in this case is

$$H^k = p_0^k \Phi_0^k + p_1^k \Phi_1^k + p_2^k \Phi_2^k, \quad (5)$$

where  $\Phi_0^k = 1$ ,  $\Phi_1^k = \dot{l}^k$ ,  $\Phi_2^k = -a_1(l^k)(\dot{l}^k)^2 - b_j(l^k)\dot{l}^k - c_j(l^k) + d_j(l^k)Q_j^k$  are received on base of Eq. (2b).

Hamilton function is linear concerning the unknown control  $Q_j^k$  and reaches the maximum if

$$Q_j^k = \text{sign}(p_2^k d(l^k) Q_{j\max}) \quad (6)$$

Also it is true:

$$\text{sign}(d_j(l^k) Q_j) = \text{sign}(p_2^k), \quad (6a)$$

i.e. the sign of the right part for differential equation (2b) and the sign of variable  $p_2^k$  coincide. The value  $l$  is known if the Eq. (2b) are known and for the calculation of the auxiliary variables  $p_1, p_2$  the following adjoint equations are used:

$$p_1^k = - \frac{\partial H^k}{\partial \dot{l}} = p_2^k \omega^k \quad (7)$$

$$p_2^k = - \frac{\partial H^k}{\partial l} = - p_1^k + p_2^k h^k$$

where:

$$\omega^k = \frac{\partial a(l^k)}{\partial l} (l^k)^2 + \frac{\partial b(l^k)}{\partial l} l^k + \frac{\partial c(l^k)}{\partial l} - \frac{\partial d(l^k)}{\partial l} \text{sign}(p_2^k d(l^k)) Q_{j\max}^k$$

$$h^k = 2a(l^k) l^k + b^k(l^k), \quad k=1,2,\dots,m.$$

Apparently, that the equations (7) are the linear differential equations on the temporary sections, where the sign of  $p_2^k d(l^k)$  does not change.

Really, the coefficients of this equation  $h^k$  and  $\omega^k$  are the functions of  $l^k$ ,  $l^k$  of the time variation of which are defined by the Eq. (2b), and as far as on specified sections of the time in equation (2b) do not enter  $p_1^k$  and  $p_2^k$ , then the processes of the time variation for  $l^k$  and  $l^k$  can be defined independently from the Eq. (7). Thus,  $h^k$  and  $\omega^k$  in the Eq. (7) are known determined from Eq. (2b) functions of the time, other coefficients  $p_1^k$ ,  $p_2^k$  are constant and equal to units, therefore, they are linear.

At first, the situation will be considered when the coefficients  $h^k$  and  $\omega^k$  does not depend from the time, i.e. are constant, and  $(h^k)^2/2 \geq \omega^k$ . In this case the roots of the characteristic equation of the system (7) are real and equal  $\alpha_{1,2}^k = -h^k/2 \pm (h^k)^2/4 - \omega^k)^{1/2}$ , and  $p_2^k$  is the following function of the time:

$$p_2^k = C_1^k e^{\alpha_1^k t} + C_2^k e^{\alpha_2^k t}, \quad k = 1,2,\dots,m. \quad (8)$$

where:  $C_1^k = (\alpha_1^k \alpha_2^k p_1^k(t_{k-1}) - \alpha_1^k p_2^k(t_{k-1})) / (\alpha_2^k - \alpha_1^k)$ ,  
 $C_2^k = (\alpha_2^k p_2^k(t_{k-1}) - \alpha_1^k \alpha_2^k p_1^k(t_{k-1})) / (\alpha_2^k - \alpha_1^k)$ ,  
 $\alpha_{1,2}^k = -h^k/2 \pm (h^k)^2/4 - \omega^k)^{1/2}$ ,  
 $t_{k-1}, t_k$  - the initial and final time moment of the interval  $k$ .

Each of  $p_2^k(t)$ ,  $k=1,2,\dots,m$  is the part of resulting continuous process  $p_2(t)$  on the temporary interval  $k$  at  $t_{k-1} < t \leq t_k$ . It concerns and to  $p_1^k(t)$ , and also processes  $l^k$ ,  $l^k$ , which are the parts of resulting continuous process. Therefore, on contiguous by the friend to the friend temporary intervals should take place the condition of the concurrence of boundary values  $p_1^k(t_{k-1}) = p_1^{k-1}(t_{k-1})$  and  $p_2^k(t_{k-1}) = p_2^{k-1}(t_{k-1})$ .

With the help each of  $m$  dependencies (8) for  $p_2^k(t)$  it is possible to receive the time moment  $\tau_0$  of reducing  $p_2^k(t)$  in zero, i.e. possible changing of sign  $p_2^k(t)$  on the interval  $k$ :

$$\tau_0 = \frac{1}{\alpha_2^k - \alpha_1^k} \ln \frac{1 - (p_2^k(t_{k-1})/p_1^k(t_{k-1})) \cdot 1/\alpha_2^k}{1 - (p_2^k(t_{k-1})/p_1^k(t_{k-1})) \cdot 1/\alpha_1^k} \quad (9)$$

The analysis of this expression has shown, that the unitary conversion in zero of the variable  $p_2^k(t)$  through the final time always takes place at such initial significance  $p_2, p_1$  on any  $k$  interval, when  $p_2^k(t_{k-1})/p_1^k(t_{k-1}) > 0$  and the following conditions are executed:

- in case of positive signs of both roots  $\alpha_1^k$  and  $\alpha_2^k$ :

$$\left| \frac{1-p_2^k(t_{k-1})}{1-p_1^k(t_{k-1})} \frac{1}{\alpha_1} \right| < 1; \quad \left| \frac{1-p_2^k(t_{k-1})}{1-p_1^k(t_{k-1})} \frac{1}{\alpha_2} \right| < 1;$$

- in case of different signs of roots it is enough fulfillment of any previous conditions;  
- in case of negative signs of roots the additional conditions are away.

Obviously, all other possible combinations of the significances of entry conditions correspond to the ratio  $p_2^k(t_{k-1})/p_1^k(t_{k-1}) < 0$ . For them the Eq. (9) gives either negative time, or under negative sign for  $\ln$ ; it means, that the reference in zero  $p_2^k(t)$  it is impossible in this case, excepting the case of the negative roots at fulfillment of the conditions

$$\left| \frac{1-p_2^k(t_{k-1})}{1-p_1^k(t_{k-1})} \frac{1}{\alpha_1} \right| > 1; \quad \left| \frac{1-p_2^k(t_{k-1})}{1-p_1^k(t_{k-1})} \frac{1}{\alpha_2} \right| > 1;$$

From these statements follows:

1. If in the moment  $\tau_0$  on  $k$  interval  $t_{k-1} < \tau_0 \leq t_k$  change of a sign of process  $p_2^k(t)$ , has taken place, it is possible only, if the initial significances of this interval satisfied to the inequality  $p_2^k(t_{k-1})/p_1^k(t_{k-1}) > 0$ .
2. After change of the sign  $p_2^k(t)$  always the ratio  $p_2^k(t_{k-1})/p_1^k(t_{k-1}) < 0$ , takes place, and it means, that at any  $t > \tau_0$  it is impossible the secondary reference in zero  $p_2^k(t)$ .

In case positively of both roots or roots different on sign it is obviously.

In case, when both root are negative, secondary reference, basically, probably at the fulfillment of above-stated conditions

$$\left| \frac{1-p_2^k(t_{k-1})}{1-p_1^k(t_{k-1})} \frac{1}{\alpha_1} \right| > 1; \quad \left| \frac{1-p_2^k(t_{k-1})}{1-p_1^k(t_{k-1})} \frac{1}{\alpha_2} \right| > 1;$$

However, at the moment of transition and at once after the transition  $p_2^k(t)$  through zero specified inequalities are not executed because of small value  $p_2^k(t)$ , therefore after the shift of mark  $p_2^k(t)$  the initial significances for following interval with number  $(k+1)$  appear in the area, from which is impossible the secondary shift of the sign of the variable  $p_2^k(t)$ .

Thus, taking into consideration, that sign of the right part  $d_j(l^k)Q_j$  of differential equations (2b) and sign of  $p_2^k$  coincides for achievement of optimal process on time  $l(t)$  it is necessary to change the sign of  $d_j(l^k)Q_j$  not more once. And, it is switched (changes the sign) just that right part with number  $j$ , which in the given moment corresponds to the main limitation.

Now the situation will be considered, if to remove the assumption about the constancy of the coefficients  $h^k$  and  $\omega^k$  in systems of adjoint Eq. (7) for additional variable  $p_2^k(t)$ ,  $p_1^k(t)$ .

It automatically goes to that are removed limitations about constancy of factors in equations (2b), for the optimized process  $l(t)$ . At the same time, freedom of variations of coefficients is limited to the condition, that linearized in any point  $(p_1^k, p_2^k)$  phase space the system of equations (7), for additional variables  $p_1^k, p_2^k$  has real roots. Obviously, and linearize of an equation (2b) in this case in any point of the phase space will have real roots.

For finding out of character of change of the control whole interval of the time duration of the processes  $p_2(t)$ ,  $p_1(t)$  is divided into sequence no matter how small temporary intervals. It is possible to assert, that the changes  $p_2^k(t)$ ,  $p_1^k(t)$  on each  $k$  from these intervals with no matter how high accuracy are described by systems of equations of the form (7). Then all reasons, which are took into consideration earlier for the proof of uniqueness of the reference in zero of the process  $p_2(t)$ , described by these equations if they have real roots, are true. And, hence, pursuant to (6a) during the whole process of change of variables  $p_2(t)$  and  $p_1(t)$  sign of right part of differential equations (2b)  $d(l^k)Q_j$  should change not more once.

As to number of switches of the control for drives  $Q_j$ , as far as pursuant to Eq. (6)  $\text{sign}Q_j = \text{sign}p_2^k d(l^k)$ , this number equally to the sum of number of changes of the sign additional variable  $p_2$ , equal to one, and number of changes of the sign of  $d(l)$ . Last it is always possible to calculate for current  $l$ , since the equations (2b) are known. Hence to define moment of switching sign for  $Q_j$  called change  $d(l)$ , does not present problems.

To define the moment of switching of the control for drive  $Q_j^k$  on  $k$  interval, called by change of sign  $p_2^k(t)$ , more difficulty. For this purpose it is possible to use the below-mentioned ideas, based on fact of only one of replacement for sign of right part of differential equations (2b) during optimal process  $l(t)$ .

1. It is carried out by numerical method "direct" integration of one from equations (2b) with number  $j$  which correspond to main limitation for first temporal interval. Initial point of integration is  $t=t_b$ ,  $l(t_b)=l_b$ ,  $l(t_b)=0$ , final point  $t=t_r^1$  corresponds to moment of replacement of main limitation with number  $j$  by other limitation with number  $k$ . Then it is carried out integration of equation with number  $i$  for second interval from  $t_r^1$  to  $t_r^2$  etc. up to final interval with number  $m$ , where  $l$  will reach final value  $l_f$ . Sign of right part of equations during integration must not change, in accordance abovementioned conclusions therefore if sign of coefficient  $d_j^k(l)$  is replaced during integration, this replacement must be compensated by replacement of sign of  $Q_j^k(t)$  force.

2. Simultaneously with "direct" integrating it is necessary to carry out "reverse" integration of equations of system (2b) initial point for parameter  $l$  is known point which corresponds final value of parameter  $l=l_f$ . Number of equation correspond to number of main limitation on final interval of optimal process. Then it is necessary to continue integration for next interval etc. up to moment, where  $l$  will reach value  $l_b$  corresponding beginning of optimal process  $l^{opt}(t)$ .

Obviously processes  $l(t)$ ,  $l(t)$  calculated by "direct" integration will be correspond to optimal processes from time moment  $t=t_b$  until sign of right part of equation is replaced on opposite sign. As it was substantiate this replacement it is possible only one times.

Processes  $l(t)$ ,  $\dot{l}(t)$  found by "reverse" integration will correspond to optimal processes from final time moment to time moment of replacement of sign of right part of equation. This moment is only one. Obviously this moment is the same moment that is time moment of the sign replacement of right part of equation during of "direct" integration. Therefore for definition of this moment it is possible to use phase curves  $l' = f(l)$  calculated on base date received in process of "direct" in "reverse" integration. Intersection of these curves will correspond to time moment  $t_0$  of replacement for sign of right part. Direct integration from  $t = t_0$  to time moment of the replacement sign of right part of equation generates first part of this process. Reverse integration generates rest part of optimal process. Result of these direct and reverse integrations is optimal process  $l(t)$  and optimal law of time variation control for drives.

**Algorithm and software for generation of optimal control.** Algorithm of generation of optimal control was developed on base to above stated approach is following:

1. Generation path as continues function  $g = f(l)$  of parameter  $l$  on base of known points  $g_1, g_2, \dots, g_r$ .
2. Generation system of differential equations for  $l$ , by substitution of vector of joint coordinate  $g = f(l)$  in the equations by its expression.
3. Definition from  $n$  of equations of the system (2b) numbers of equations, which must be integrated directly from the moment  $t = t_b$  and reversly from the moment  $t = t_f$ . With this purpose calculation of values of the second derivatives of the parameter  $l$  in initial  $l(t_b)$  and final  $l(t_f)$  moment of the time is made on expressions

$$l(t_f) = d_j(l_f) Q_{j\max} - c_j(l_f), \quad l(t_b) = d_j(l_b) Q_{j\max} - c_j(l_b), \quad j = 1, 2, \dots, n \quad (10)$$

Minimal absolute values from all these calculated values correspond the numbers of equations of the system (2b), which should be integrated in direct (index  $j$ ) and reverse (index  $k$ ) directions, accordingly for the initial conditions  $l(t_b) = l_b$ ,  $\dot{l}(t_b) = 0$  for the final conditions  $l(t_f) = l_f$ ,  $\dot{l}(t_f) = 0$ .

4. Definition of signs of the  $d_j(l_b)Q_j$  and  $d_k(l_f)Q_k$  in initial and final moment of the time for generation of right parts equations (2b) with number  $j$  and  $k$ .

As  $d_j(l)Q_{j\max} > c_j(l)$  in any time, besides sign  $l(t_b) = \text{sign}(l_f - l_b)$ , therefore in accordance (10)  $\text{sign} d_j(l_b)Q_{j\max} = \text{sign}(l_f - l_b)$  and  $\text{sign} d_k(l_f)Q_{k\max} = -\text{sign}(l_f - l_b)$ .

5. Realization of direct and reverse integration on intervals of change of parameter  $l$  from  $l_b$  to  $l_f$  and from  $l_f$  to  $l_b$ , accordingly. The integration begins with equations  $j$  (direct integrations) and with equation  $k$  (reverse integration).

During integration are executed:

- check of observance  $(n-1)$  inequalities  $Q_i \leq |Q_{j\max}|$  (the exception makes the inequality, appropriate to main limitation), and in case of breakdown of some inequality  $i$  needs to be executed shift of the equation for integration;
- check of the sign of the coefficient  $d_j(l)$  and in case of shift of the sign in certain time moment is changed on opposite the sign  $Q_j$ .

As the result of integration are generated:

- direct and reverse branch of phase trajectories  $l=\varphi(l)$ ,
- direct and reverse the transients  $l=f_1(t)$ ,  $l=f_2(t)$ ,
- moments of replacement of main limitations and indexes  $j$ , appropriate their numbers are stored,
- lows of time variation for controls  $Q_j$  not achieved of main limitations are generated.

6. Point of crossing of direct and reverse branches of phase trajectories  $l=\varphi(l)$  is searched; this point correspond the time moment of change for sign of  $p_2$  or right part  $d_j(l)Q_j$  of equation (2b) with number  $j$ .

The software for generation of optimal control has two parts.

First part has provides adaptation of functioning of second part to concrete robot manipulator. It is described in [4]. This part of software provides generation of dynamic equations of any concrete robot manipulator with kinematic scheme, which is the particular case of tree-like structure generated by bodies connected by means of point with angular or linear displacement. Input for this part is dynamic parameters and parameters of construction of concrete manipulator. Obviously this part is used only in case of change one manipulator by another.

Second part of software executes algorithm of generation of control for robot drives.

Inputs for this part are:

- sequence of vectors of joint coordinates, corresponding path for robot manipulator, avoiding obstacles;
- data, corresponding to dynamic equations of concrete manipulator, generated by first part of software.

**Computer experiment.** Computer experiments were carried out for test and validations of developed method for generation optimal robot control.

Conditions of one experiment are following.

Conditions of experiment:

linear path:            initial point = (900, 50, 800, 180, 0, 0) [mm, grad]  
                             end point = (-600, 50, 800, 180, 0, 0) [mm, grad]

limitation for robot drive control signals:

$$U_{\min} = (-335, -600, -206, -50.5, -40.3, -20.8) \text{ [Nm]}$$
$$U_{\max} = (335, 600, 206, 50.5, 40.3, 20.8) \text{ [Nm]}$$

limitation for velocity of joint coordinates:

$$g_{\min} = (-180, -90, -180, -110, -110, -180) \text{ [grad/s]}$$
$$g_{\max} = (180, 90, 180, 110, 110, 180) \text{ [grad/s]}$$

Dynamic parameters are parameters for robot type PUMA (carrying capacity 50 N).

Results of experiment are shown on Fig.1.

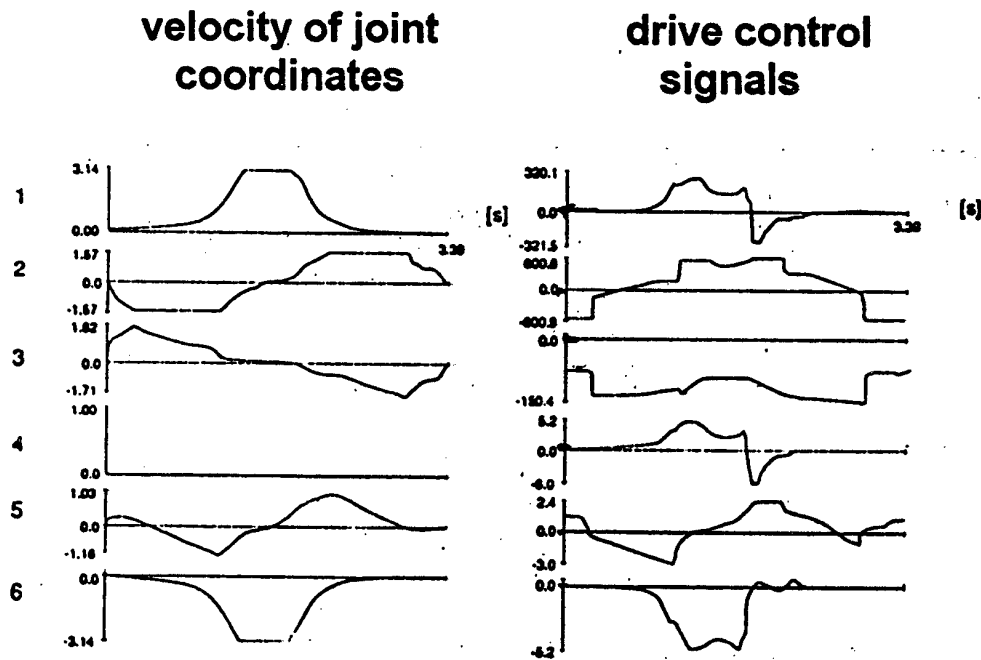


Fig.1 Optimal joint velocities and controls for joint drives

### References

1. Bobrow, J. E., S. Dubowsky, J. S. Gibson (1985). Time-Optimal Control of Robotic Manipulator. Intern. Journal of Robotics Res., Vol. 4, No. 3, pp.3-17.
2. Dlabka, M., J. Held, Y. Zhang (1988). A Practical Approach for Planning and Realization of Optimal Trajectories for Industrial Robots. Proc. of the IFAC Intern. Symp. on Robot Control, Karlsruhe, FRG, pp.517-522.
3. Duelen, G., H. Munch, D. Surdilovic (1990) Advanced Robot Control Systems. The Int. Conf. on Manufacturing Systems and Environment, Looking Toward the 21st Century. Japan Society of Mechanical Engineers, Tokyo, May 29 - June 1.
4. Kulakov, F. M., H. Loose, G. V. Makarova (1986). Computer Aided Engineering of Industrial Robots (CAEIR). Preprints 13 and 14, Leningrad Research Computer Centre. (In Russian).

## Cooperation in Human-Robot-Teams

Thomas Laengle, Thomas Hoeniger, Ulrich Rembold, Heinz Woern

Institute for Real-Time Computer Systems and Robotics

University of Karlsruhe, D-76128 Karlsruhe, Germany

E-mail: {rembold / laengle}@ira.uka.de

**Abstract.** In this paper, the concept of human-robot-team is presented. Because of its high flexibility and adaptability, the human-robot cooperation is expected to have a wide range of applications in uncertain environments, not only in future construction and manufacturing industries but also in service branches. A multi-agent control architecture gives an appropriate frame for the flexibility of the human-robot-team. Robots are considered as intelligent autonomous assistants of humans which can mutually interact on a symbolic level and a physical level. This interaction is achieved through the exchange of information between humans and robots, the interpretation of the transmitted information, the coordination of the activities and the cooperation between independent system components. Equipped with sensing modalities for the perception of the environment, the robot system KAMRO (Karlsruhe Autonomous Mobile Robot) is introduced to demonstrate the principles of the cooperation among humans and robot agents. Finally, emphasis is placed on future studies of human-robot-cooperation.

**Keywords.** Human-Robot team, Man-machine cooperation, Autonomous robot, Communication level, Distributed control

## 1 Motivation

In the past, the main goal of automation was the construction of systems excluding the involvement of humans. Man has been considered as the weak point in the operation of automatized systems: he is subject to errors, suffers from fatigue and often has an unreliable memory. In the best cases humans have been regarded as obstacles in an automatized manufacturing rather than as active factors. In fact, to guarantee the safety of humans within the working area of a robot, additional sensors and equipment are required, which would increase the cost of an automatic system. Therefore many efforts have been made to separate the working area of robot systems from man.

But today the demands for high flexible robotic systems, which must have the capabilities of adapting themselves to an uncertain environment, are rapidly increasing. This is the case not only in manufacturing, where customer requirements have to be met, new technologies should be integrated, products and propositions of each customer must be identified and individualized, but also in the services, where robots have to move in unstructured and changing environments.

Because of these requirements it is unrealistic to consider a fully automated factory or service facility. Furthermore the trend consists of integrating the man in the production process. In this context, the human-robot-team has an important role to play in this context. The main idea is to combine the complementary capabilities of robots and humans. The robot is quicker and more reliable but relatively inflexible, while the man is flexible but often unreliable. The human-robot-cooperation can be considered as one of the solutions for meeting the future requirements in the industry and services.

In this paper a new concept of the human-robot-team is discussed, which allows to fulfil the high adaptability and flexibility required by new applications of robotics. In Section 2, after introducing the principle of the human-robot-team, the state of the art is presented. In Section 3, the analysis of the interactions between the robot and the operator is described. Section 4 is concerned with our previous work and emphasizes on future studies on human-robot-cooperation. Finally we present a short conclusion in Section 5.

## 2 State of the Art

Before presenting the state of the art about man-robot-cooperation, it is necessary to define what has to be understood under this concept. First, cooperation implies a certain degree of autonomy of partners. The decision making in the team will be shared between the operator and

the robot. Therefore a simple Master-Slave architecture, usually used for teleoperated robots, is in our case not sufficient and will no longer be considered here. Second, in this paper we focus only on the human-robot-cooperation with *natural* man-machine-interface such as language, gesture and touch. We are interested in providing an intuitive and universal access to the robot for manufacturing people without any specific knowledge. So human-machine-interface like virtual reality or graphical interface ([Suzuki *et al.* 1995]) are not discussed here.

The arm-manipulator cooperation has already been presented in [1], but only one degree of freedom was considered. In [2] the coordinated task execution by a man and a mobile manipulator is simulated for very simple cases: the manipulated object has to be maintained in the same attitude. In this publication the problem of loading/unloading was not considered. The control system proposed in [3] for the Man-Machine-Environment Interactions is also applied to a manipulator with only one degree of freedom.

A system understanding the human behavior, not by monitoring the motion of the man, but by monitoring the motion of the objects is presented in [4]. Thus this system requires a minimal knowledge about its environment. In [5] conscious and unconscious behaviors have been considered. In [6] an action recognizer based on real time visual recognition of human pick and place action sequences is presented.

Furthermore, the very important aspect of human safety is mostly neglected. In [7] two safety systems have been presented: one using an accelerometer and the other using cameras. However the man-robot-interface is limited to a simple speech recognition. The operator just supports the robot if necessary.

Some key issues for human-robot-team have not yet been considered. The communication at the physical level between the robot and the man is at the beginning: it is possible only in very simple cases (limited degree of freedom, simplified environments), it suffers on lack of sensors and it does not integrate multi-modal sensory information. There are also no metrics and standard benchmark problems to be used for evaluating the human-robot-cooperation. Besides safety requirements for humans working in the same working area with the robot are almost inexistant.

### 3 Analysis of the problem

The performance of any complex system depends on the interaction between its components. To make robots capable of working together with men in different domains like transportation, assembly, maintenance and other services, it is necessary to understand the nature of interactions between the human and machine components. Robots are considered as intelligent

autonomous assistant of humans, which can mutually interact on a symbolic (exchange of digital messages on a network) and a physical level (visual, tactile, acoustic). Interaction can be achieved with three concepts [8]:

### 3.1 Communication

The communication between robots and humans comprises the transmission of the information at the physical level and the interpretation of the information.

#### *Transmission of the information at the physical level*

The transmission of information from one partner to another occurs through predefined interaction patterns or protocols, which allow the receiver to understand the intention of the transmitter. Communication transmission of digital data, which enables robots to communicate with each other in normal cases, is however, no longer sufficient in the case of the human-robot-teams (Fig. 1.a and 1.b). Within the team, the communication at a physical level, such as contact forces, must also be involved. The sensors should be used not only for the task execution, but also to recognise situations of possible interaction with other agents and to receive the transmitted information as well.

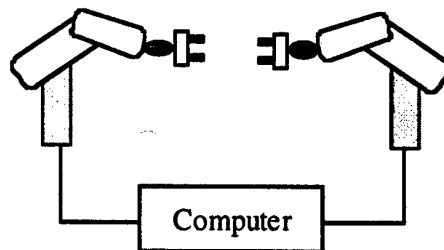


Fig. 1.a Communication at the symbolic level in the Robot-Robot Team

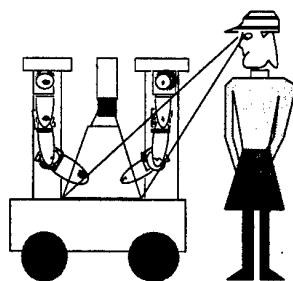


Fig. 1.b Communication at the physical level in the Human-Robot Team

#### *Interpretation of the transmitted information*

The interpretation makes sense out of the transmitted information (Fig. 2). Particularly for the human-robot-cooperation, it is very important to avoid interpretation errors to prevent the human from danger. The transmitter should make sure that his intentions are clear enough to the receiver. For the human the confirmation of the good interpretation of the intention is reached through analysing the implicit behaviour of the partner as defined in [4]. But this is

possible only if the behaviour of the partner is not ambiguous. Particularly, the robot has to perform human-like motions [9] and to exhibit reflexes similar to those observed in humans: like retracting an arm when it hits something.

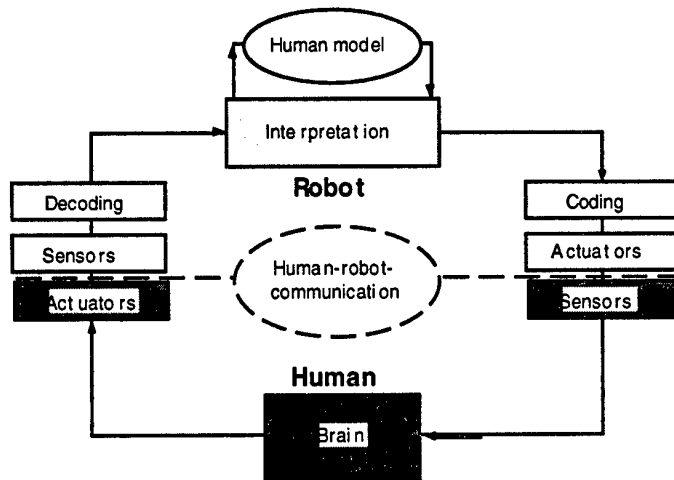


Fig. 2 Interpretation of the information received by the robot

### 3.2 Coordination

Several agents can have overlapping working spaces. In order to avoid mutual disturbance and conflict between two working agents and prevent the system from dead-lock, it is necessary to plan, to order or to coordinate the activities of these agents during an action.

### 3.3 Cooperation

Many problems cannot be solved by single agents working in isolation, because they do not have all the necessary expertise, resources or information. On the other hand, it may be impractical to assign permanently all necessary tasks to a single agent, because of historical and physical constraints [10]. This explains the necessity of temporary alliances through cooperation where each partner contributes with his specific capabilities to the accomplishment of the final goal.

The question of human safety becomes increasingly important as future robots will be required to share their working area with human operator. The following issues have to be considered: What are the requirements on the hardware and on the software? What are the possible dangers for the human? How can the robot recognize the danger? How should the robot react in dangerous situations?

Standard benchmark problems and algorithms to evaluate the cooperation between the human and the robot have to be defined. Formal metrics to evaluate the progress made have to be developed.

#### 4 Experiments on previous work

In our Institute several experiments have been made with the mobile two-arm robot system KAMRO (Karlsruhe Autonomous Mobile Robot). The robot consists of a mobile platform with an omnidirectional drive system using MECANUM wheels and two PUMA manipulators with 6 degrees of freedom mounted in a hanging configuration. It is also equipped with a multitude of sensors: one over-head camera, two hand-cameras, force/torque sensors on each manipulator and ultrasonic sensors on the mobile platform. (Fig. 3).

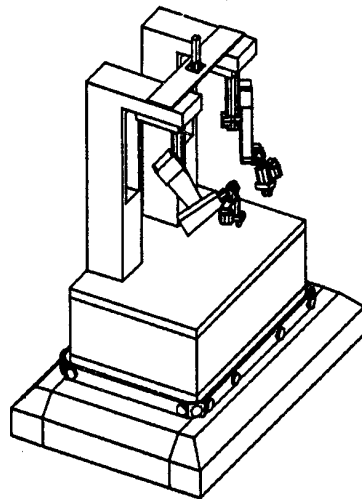


Fig. 3 KAMRO (Karlsruhe Autonomous Mobile Robot)

The experiments made with the mobile manipulations demonstrate the principles of the communication between human and robots at the physical level. An operator guides the two manipulators of KAMRO by moving manually the endeffectors with his arms. The operator has just to concentrate on positioning the endeffectors, whereas the robot takes care of the collision avoidance between the two manipulators. Besides, the mobile platform must ensure that the two manipulators are always in a region of optimal configuration [11].

The experiments made with KAMARA (Karlsruhe Multi-Agent Robot Architecture) (Fig. 4) demonstrate the validity of the developed multi-agent model and the high flexibility of this architecture, which allows dynamical reconfiguration of the agent-team for cooperation and coordination [12]. Furthermore, the communication protocol on the symbolic level and the negotiation between the autonomous agents are adapted for the human-robot-communication.

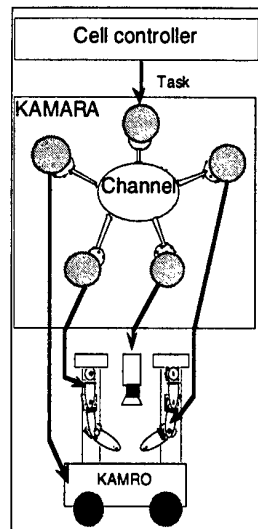


Fig. 4 Distributed control architecture KAMARA

Based on these works, further studies must be done to demonstrate the human-robot-cooperation within the multi-agent system:

- Extending the communication model to the physical level and extending the perception methods through the fusion of various sensors, for example by combining the force/torque sensors data with the vision sensors data.
- Analysing and modelling of the human behaviour to create a knowledge base for the agents. This knowledge base will be used for implicit communication, so that the human can control the robots without any additional information about the system and can thus concentrate himself only on the important aspects of the executing task.
- Interpreting the intention of sensor data with the help of the created knowledge base. To guarantee the reliability of the interpretation, it is important for the agents to have abilities for error detecting and self-readjusting.
- Developing a human agent and integrating it into the multi-agent control architecture KAMARA by constituting the human-robot-team.
- Analysing the safety requirements and presenting corresponding methods to meet them.

## 5 Conclusions

In this paper, the concept of human-robot-teams has been presented, which is expected to have a wide range of application in industry and services due to its high flexibility and adaptability. To implement the concept, four issues towards the problem of human-robot-interaction should

be analysed: communication, interpretation of the transmitted data, coordination and cooperation. Besides, special measures have to be taken into account to guarantee the security of the human during his interaction with the robots. Previous work and experiments with the autonomous mobile robot KAMRO have demonstrated promising results for approaching the subject of human-robot-team. Based on these achievement, additional research will be conducted to create an environment for man-machine-cooperation in manufacturing and services.

## 6 Acknowledgements

The research work was performed at the Institute for Real-Time Computer Systems and Robotics (IPR), Prof. Dr.-Ing. U. Rembold, Prof. Dr.-Ing H. Wörn and Prof. Dr.-Ing. R. Dillmann, Faculty for Computer Science, University of Karlsruhe. The research was partially funded by the Nato Contract CRG 951002, with support of the German Research Foundation (Graduiertenkolleg "Controllability of Complex Systems", DFG).

## 7 References

1. Al-Jarrah, O.M.; Y.F. Zheng (1996): *Arm-manipulator coordination for load sharing using compliant control*. Int. Conf. on Robotics and Automation, vol.2, pp. 100-1005, 1996
2. Yamamoto, Y.; H. Eda; X. Yun (1996): *Coordinated Task Execution of a Human and a Mobile Manipulator*. Int. Conf. on Robotics and Automation, Vol.2, pp. 1006-1011, 1996
3. Kazuhiro, K.; Y. Fujisawa; T. Fukuda (1993): *Mechanical System Control with Man-Machine-Environment Interactions*. Int. Conf. on Robotics and Automation, Vol. 1, pp. 239-244, Atlanta, Georgia, May 2-6, 1993
4. Mizoguchi, H.; T. Sato; T. Ishikawa (1996): *Robotic-Office to Support Office Work by Human Behavior Understanding Function with Networked Machines*. IEEE Int. Conf. on Robotics and Automation, pp. 2968-2975, Minneapolis, Minnesota, Apr. 22-28, 1996
5. Sato, T.; Y. Nishida; J. Ichikawa; Y. Hatamura; H. Mizoguchi (1994): *Active Understanding of Human Intention by Robot through Monitoring of Human Behavior*. Int. Conf. on Intelligent Robots and Systems, vol. 1, pp. 405-414, Germany, 1994

6. Kuniyoshi, Y; M. Inaba; H. Inoue (1992): *Seeing, Understanding and Doing Human Task*. Int. Conf. on Robotics and Automation, Vol. 1, pp. 2-9, Nice, France, May 12-14, 1992
7. Vischer, D. (1992): *Cooperating Robot with Visual and Tactile Skills*. Int. Conf. on Robotics and Automation, Vol. 3, pp. 2018-2025, Nice, France, May 12-14, 1992
8. Wettstein, H. (1993): *Systemsrchitektur*. Hanser-Verlag, München, Germany
9. Kajikawa, S.; T. Okino; K. Ohba; H. Inooka (1995): *Motion Planning for Hand-Over between Human and Robot*. Int. Conf. on Intelligent Robots and Systems, vol. 1, pp. 193-199, 1995
10. Jennings, N. (1993): *Cooperation in industrial multi-agent system*. World Scientific Series in Computer Science, Vol. 43, 1993
11. Nassal, U. (1996): *Fuzzy Control for Mobile Manipulation*. IEEE Int. Conf. on Robotics and Automation, pp. 2264-2269, 1996
12. Laengle, T.; U. Rembold (1996): *A Distributed Control Architecture for intelligent Systems*. SPIE's Int. Symp. on intelligent Systems and advanced Manufacturing, Boston, USA, Nov. 18-22, 1996. In: SPIE Proc. of the Int. Conf. on Advanced Sensor and Control-System Interface, Vol. 2911, pp. 52-61
13. Suzuki, T.; K. Yokota; H. Asama; H. Kaetsu; I. Endo (1995): *Cooperation between the Human Operator and the Multi-Agent Robotic System: Evaluation of Agent Monitoring Methods for the Human Interface System*. Int. Conf. on Intelligent Robots and Systems, vol. 1, pp. 206-211, 1995

# UNIFIED SOFTWARE INTENDED FOR ROBOT WITH TECHNICAL VISION SYSTEM AND ARTIFICIAL INTELLIGENCE ELEMENTS \*

Vladimir D. Arkhipov, Vladimir L. Vartanov and Vladimir P. Makarychev

State Scientific Center -

Central R&D Institute of Robotic and Technical Cybernetics

Tikhoretsky ave 21 St. Petersburg, 194064 Russia

Phone: +7-812-552-47-64 Fax: +7-812-552-47-10

e-mail: makar@mail.rtc.neva.ru

**Abstract.** One of the most important quality of robot control system is unification. In case of multiple intellectual robots the importance of this factor is strong increasing. The authors suggest the conception and compact universal structure of its organization as well as a compact set of program modules (elements of this structure). There are several important parts of the system based on original and effective algorithms: optimal trajectory construction, trajectory construction in surroundings with obstacles, servosystem with dynamic's calculation in real time, etc. The technical vision system is one of the most multiple part of robot. This work presents the system based on co-ordinate method [4] consisting in methodical utilization co-ordinate the various objects points in process of segmentation, pattern recognition and position definition. There are several real special and laboratory robots make good use suggested systems: among them the cosmic manipulator for Russian Space Shuttle (BURAN). The results of experimental investigations take place.

**Keywords.** Robot, control, robot software, unification, time-sharing mode, technological operation, dynamics, obstacles, simulation, vision system.

**Introduction.** Robotic is a popular and rapidly developing branch of science and technology in the West as in research laboratories as in industry. Now a next leap is coming to a head consisting in a wide application of robotic systems with the artificial intelligence elements.

A unified software of the robot control systems is intended for the realization of the adaptive and intelligent control under different conditions: in manufacture to fulfill delicate (including assembling) operations, in extra-ordinary situations with a limited human participation.

To achieve a software unification multipurposeness it is necessary to construct a compact universal structure of its organizations as well as a compact set of program modules (elements of this structure) assuring the whole range of possible applications. Besides them a realization of the adaptive properties puts very strict requirements on the software organization.

**Control system structure.** The authors together with his colleagues in the Laboratory for Robot Control and Simulation of State Scientific Center - Central R&D Institute of Robotic and Technical Cybernetics (CRDI RTC) tries to realize this unification concept during several years already [1,2,6]: a basic unified software for robot control and simulation (manipulators, mobile robots) has been developed that is used in several real laboratory and special robots. The structural unification is based on two important principles (Fig.1):

---

\* This research is conducted within the research grant No. 94 2785 from INTAS.

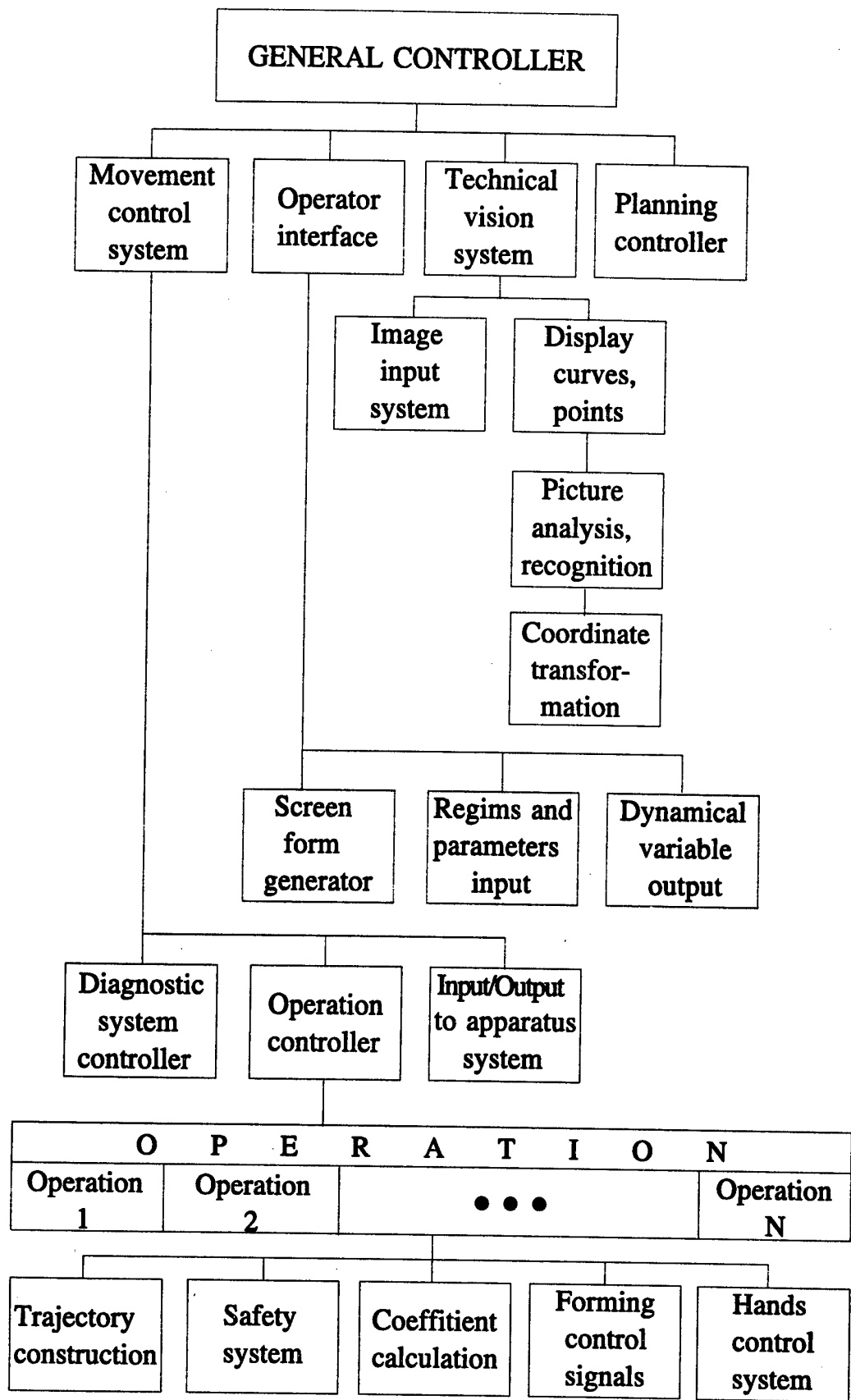


Fig. 1. Structural schem unified program software of robot.

- ♦ a formation of the main functional modules as program processes executed in a time-sharing mode;
- ♦ making good use of object-oriented methods and languages as C++;
- ♦ a formation of the extremely important intermediate (framework) level of technological operation (TOP).

The a time-sharing mode permit simultaneously to execute the control drives processes with high frequency and complicated intellectual but not quickly processes like pattern recognition and forming world model by bit in background regim.

The object-oriented methods permit good compact structure and provide for easily accompany and improvements.

TOP module realizes a logic of robot functioning under conditions of an environment proceeding from the given technological operations (movement to the point, grasping of the object, inspection, etc.). In has a changeable input point depending on TOP stage, fulfils logical operators only as a rule, chooses a necessary branch of the process depending on signals about robot and environment state, and runs necessary mathematical calculating processes (modules).

Apart from the main unified TOP module, the software includes the following main unified modules:

- trajectory construction that in its turn consists of many unified submodules: co-ordinate system transformation, direct and inverse kinematic tasks, etc. [3];
- trajectory following that realizes various control algorithms including those with robot dynamics consideration [5];
- elastic properties consideration for various robot structures that is equivalent to the finite elements method;
- friendly user interface developed in the unified programming environment Visual C++;
- software interface with hardware unified on a logical level (different types of sending from the unified set);
- save obstacles detour (while given beforehand only) [3];
- application of cartographic system during automatic driving (initial stage of development);
- technical vision system [4].

**Control algorithm.** Effective, allowing dynamics algorithm looks so [1,2] :

$$v_{q,p} = v_p / c_q + q ,$$

$$v_p = ( M_p / c_m i_{red} + c_1 i_{red} q' ) / k_y ,$$

$$M_p = A(q) [ q'' + G_1(q' - q_p') + G_0(q - q_p) ] + b(q, q') ,$$

where

$M_p, v_p, v_{q,p}$  - controls moments, velocities and positions, accordingly,

$c_m, c_1, i_{red}, k_y, c_q$  - drives parameters,

$$b(q, q') = b_1(q, q') + b_2(q) + b_3(q') + b_4 ,$$

$$A(q) = \sum_{j>i; j,i=1}^n a_{0i}^i q'^i, \quad i$$

$$b_1(q, q') = \sum_{j>i; j,i=1}^n a_{1i}^{ij} q'^i q'_j - \text{centrifugals and coriolises forces,}$$

$b_2(q)$  - vector gravity forces ,

$b_3(q')$  - friction forces ,

$b_4$  - disturbances forces, among them principle unknown,

$$a_{0i}^i = \sum_{l=\max(i,k)}^n \text{tr}(B_l^i H_l B_l^{k*}) = \sum_{l=\max(i,k)}^n h_l^{i,k} ,$$

$$a_{1i}^{ij} = \sum_{l=\max(i,j,k)}^n \text{tr}(B_l^{ij} H_l B_l^{k*}) = \sum_{l=\max(i,j,k)}^n h_l^{ij,k} .$$

There matrix  $B_l^i$  ,  $B_l^{ij}$  and  $H_l$  depend on geometrical and mass-inertia parameter manipulator, accordingly,  $B_{H,l}^{i,k} = B_l^i H_l B_l^{k*}$  .

The calculation of coefficients  $a_{0i}^i$  ,  $a_{1i}^{ij}$  for speed make use of correlation [5]:

$$h_l^{i,k} = h_l^{k,i} , \quad i, k = 1, l; \quad l=1, n; \quad a_{0k}^i = a_{0i}^k , \quad i, k = 1, n;$$

$$h_l^{ij,k} = h_l^{ji,k} = -h_l^{ik,j} ; \quad i, j, k = 1, l; \quad l=1, n; \quad a_{1i}^{ij} = -a_{1j}^{ik} , \quad i, j, k = 1, n;$$

$$|h_l^{ij,k}| = |h_l^{ji,k}| = |-h_l^{ik,j}| \leq \bar{h}_l^{j,k} = \sqrt{B_{H,l}^{j,k} B_{H,l}^{j,k*}} ; \quad i \leq j \leq k; \quad i, j, k, l = 1, n;$$

Force operations algorithms base on kinematic task target with necessary dynamics parameters (velocities and moments) and subsequent realization this target with the help of considered effective servosystem algorithm. The aim is to create vectors force  $f_x$  and moment  $m_x$  of manipulator grip in configuration  $q^0$ . Let  $q^p$  is terminal point of operation, disposed at the line of action force and moment, and  $q^T$  is the mirror point for  $q^0$  relatively  $q^p$ . The line and angle distances between points  $q^0$  and  $q^p$  for force and moment are  $r_f$  ,  $r_m$  .

Let the operators right and inverse kinematics tasks are

$$z = (p, f, g)^* = F(q) \quad \text{and} \quad q = F^{-1}(z, q^0) .$$

Then for target point  $q^T = F^{-1}(z^T, q^0)$  its co-ordinates are:

$$p^T = p + 2f_x / |f_x| r_f, \quad f^T = f + 2W_x f, \quad g^T = g + 2W_x g ,$$

$$W_x = \begin{bmatrix} 0 & \omega_{x1} & \omega_{x2} \\ -\omega_{x1} & 0 & \omega_{x3} \\ -\omega_{x2} & -\omega_{x3} & 0 \end{bmatrix} ; \quad \omega = (\omega_{x1}, \omega_{x2}, \omega_{x3})^* = m_x / |m_x| r_m .$$

For determinate  $|f_x|$  and  $|m_x|$  vector  $M = F_M^{-1}(f_x, m_x)$  , where  $F_M^{-1}$  - is operator inverse kinematic task for forces and moments.

The very useful modification algorithm is the vibration algorithm:

$$\dot{q}_p(t) = \dot{q}_p(t) + a \sin(\omega t) .$$

Two direction of vibration especially important :

- $a = \Delta q = q^0 - q^T$  - along force  $f_x$  axis;
- $a = \Delta q$  - perpendicular force  $f_x$  axis;  $\Delta q$  is find from equation

$$\langle f_x, \Delta x \rangle = \langle f_x, \partial F / \partial q \Delta q \rangle = \langle [\partial F / \partial q]^T f_x, \Delta q \rangle = 0 .$$

There are infinitely amount decisions assumed this parametrisation.

**Technical vision system.** The technical vision system is the most important part of robot software providing adaptive and intellectual properties.

For the decision of a problem of a manipulator's grip conclusion, supplied by one or two television cameras, in a determined position concerning an object, it is needed to know a complete set of coordinates at least three points of object, not lying on one direct. The output of a procedure of an object characteristic points recognition with the television camera's help can be submitted in a kind of angular spherical coordinates of these points in co-ordinate system of television camera.

Thus, for reception of a complete set of spherical co-ordinates included in a initial data structure of the mentioned problem, it is needed to decide a problem of distances determination for three points of an object relatively the co-ordinate system of the television camera at the task of their angular spherical co-ordinates in two cases:

1. *Co-ordinate system of one television camera.* It can be submitted by following statement of a problem: angular coordinates  $(\phi_i, \psi_i)$  of points  $C_i$ ,  $i = 1, 2, 3$ , and distance  $d_1 = |C_1 C_2|$ ,  $d_2 = |C_1 C_3|$ ,  $d_3 = |C_2 C_3|$ , between them are given, it is necessary to find out distances  $r_i = |OC_i|$ ,  $i = 1, 2, 3$ , from the center O of co-ordinate system up to given points. The analytical decision of the given problem is found. It is reduced to the decision of an equation of 4 degrees. Research of the given problem shows, that the quantity of the decisions can vary from one up to four, and the unique decision is rather exception, than rule. For the unequivocal definition of object position with the help of one camera it is necessary to use additional conditions or to involve into consideration fourth point of the object.
2. *Coordinate system of two cameras.* This case is almost trivial.

We shall consider a moving of manipulator's grip supplied with television camera, to the determined griped position concerning the job's object. In pithy aspect the statement of a problem is those:

- position of an object (including its orientation) relatively of coordinate system, connected with television camera is known,
- desired position of grip concerning an object is given,
- condition of manipulator (i.e. mutual arrangement of its links) and position of coordinate system of television camera concerning a link are known.

It is necessary to determine a final position of grip concerning root coordinate system.

The position of any object in the space can be unequivocally determined by its three points not lying on one line. For each separate object the three "registration point" choice is individual. They would be characteristic three points of an object those can be easily detected by television camera and those can enable to get orientation of an object in the space with enough precisely. For bodies of the symmetric form that can be points being determined from the geometrical form of an object. In a general kind we will consider that the position of an object is set by spherical coordinates  $(\phi_i, \psi_i, r_i)$  of three points  $C_i$ ,  $i = 1, 2, 3$ , of its three points in camera coordinate system.

**Image analysis.** The recognition and fixing of bench-mark points (in absence of markers) requires the complex analysis of object's image and the similar analysis of all images is inexpedient. That results in necessity of the preliminary procedure of object's zone allocation in the image. Space digital averaging of the whole image lies in the base of the given procedure. It enables to execute simultaneously low-frequency filtration and compression of the information. Further contour detection of areas similar on intensity is made. Then search of contours adequate on form and size for the working object and allocation of an appropriate zone of the image is implemented.

Rough spatial discretization results in areas similar on intensity with spasmodic by change of intensity on borders that permits to make curving of these areas not resorting to high-capacious on number of operations to procedures of space differentiation as the operator Sobel and making thinning of a contour. In this case there is sufficient a simple procedure of fixing of boundary points of the specified areas. It should mean that spatial discretization enables to simplify image processing, at the same time makes hardened the image and lowers the accuracy of object's characteristic points determination. However for the purposes of detection and determination of an object zone this lowering of the accuracy has not essential significance and as a whole the spatial discretization here has a positive not a negative character.

Over the allocated object zone of vision cadre, that have the image of given object (working object), it is made multiplane computer processing with the purpose of recognition and almost exact determination of coordinates of object's registration points. The specified processing contains a following sequence of procedures [7]:

- low-frequency space filtration, in particular, with the help of a mask

$$H = 1/10 \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix};$$

- nonlinear operation of 2-dimension discrete differentiation with the help of the operator of Sobel with a window by dimension  $3 \times 3$

$$\begin{bmatrix} A_0 & A_1 & A_2 \\ A_7 & F_{ij} & A_3 \\ A_6 & A_5 & A_4 \end{bmatrix}, \quad F_{ij} = |A_0 + 2A_7 + A_6 - A_2 - 2A_3 - A_4|;$$

- making thinning of contours; determination of external and internal contours of an object;

- formation of function of curvature of a contour;
- the analysis of the form of a contour and determination of its special points;
- determination registration points of an object and calculation of their spherical angular coordinates in a system of coordinates of television camera.

**Contour curvature.** The function of curvature of a contour is a convenient means of the mathematical analysis of the form of a contour and determination of its special points. Clearly, that to rectilinear sites of a contour there should correspond significance close to zero of function of curvature, to arches of a circle there should correspond constant significance of function of curvature, backcircle proportional to radius; to the tops of a contour in a kind polygon should correspond spade's images bursts of significances of function of curvature. In practice the specified properties of function of curvature can be fulfilled with a various degree of accuracy and the reception of good result in many respects depends on selection of parameters of processing, such, as number of basic points of a contour, threshold significances of function of curvature and valuation of its derivative for acceptance that or other decision and etc..

We shall present for a example a problem of definition (determination) of spherical coordinate corners of 4th bench-mark points of cube, located in the tops of its top side, provided that cube lies on a table. As a result of described higher procedures external and internal contours of cube are defined and under determined conditions, imposed on possible foreshortened of cube relatively of television camera, contour of the top side is allocated. For example, if cub is looked round with television camera predominance from above, the contour of the top side will be internal contour heaviest on area on the cub's image. Thus the "central point" of the area, limited by external contour, will be, as a rule, to be inside a contour of the top side of cube and, if it will be chosen as a initial point (  $i_0, j_0$  ) in algorithm of definition (determination) of a contour in a mode of functioning, adequate fixing of an internal contour, the given algorithm will allocate a contour of the top side of cube. The function of curvature of this four-coal contour, received with the help submitted higher a procedure, will have spade's images bursts in points, appropriate to tops of quadrangle. These points can be allocated at the certain (determined) selection threshold  $\gamma_p$  of significance of function of curvature in such a manner that if  $\gamma(k) > \gamma_p$ , to  $k$  - number of a basic point, appropriate to top of contour. As follows from procedure determination of co-ordinates bench-mark's points of an object in algorithm of definition of a target rule of grab should be submitted in a certain(determined) sequence. The identification functions allocated with help of curvature of a contour of cube's bench-mark's points under the numbers can be made on certain logic pursuant to conditions to final foreshortening of cube on vision cadre and conditions of its grab with grab of a robot.

The indicated example permits to present general ideology of identification of object's bench-mark's points. An output of this process will be four pairs (  $i_0(k_l), j_0(k_l)$  ),  $l=1,2,3,4$ , of pixel's coordinates of object's bench-mark's points on visioncadre. Further procedures of definition of spherical co-ordinate corners of these points and complete set of their spherical co-ordinates in television system co-ordinate, definition of a target rule of grab of a robot are the same, as in case of a marker system of technical vision. These procedures were described higher.

**Realization.** Representing unified software with visual system researched into two physics stands:

- ♦ model of self-stepped cosmicrobot "CIRCUL" in State Scientific Center - Central R&D Institute of Robotic and Technical Cybernetics;

- ♦ robot PUMA by team-works with colleagues in St. Petersburg Institute for Information and Automation of RAS.

There are PC 486, standard television system and frame-graber as a controlled computer and apparatus, correspondingly.

Strategy combined input image applied for raise the system reaction velocity. The image input procedure is varied in accordance with stage of robot functioning and stage of image processing. The all frames or the continues parts or the discharge parts of them (part of lines and columns) are inputted.

The operations of searching, escorting and taking objects of special or regular form are investigated in experiments. The manipulations' procedures by beforehand setting or getting in teaching patterns are applied. The operation of contact and force interaction by object are successfully investigated.

Creation unified software permit to execute in real robots number of simple intellectual operations.

1. The robot by automatic spiral movements finds the object in working zone, then defines it decart co-ordinates and moves to it direction, in process of moving corrects co-ordinates and takes object at the end.
2. The man in hand regim of control teaches robot to take object. The robot remembers the trajectory by its inner gauges of position and by technical vision system. Then the object changes its position in working zone and the robot repeats the taking of object by trajectory like teaching according to geometrical form and dynamics characteristics.
3. The man by special glove with television camera executes typical movement ore taking of object. For more precise definition co-ordinates of glove the bench-mark's points in working zone can be used. Then robot repeats the movement of glove exactly or necessary displacement.

Besides the software controlling the robot directly, it is necessary to draw attention to the regular and rather powerful mathematical model [3] of the robot developed according to an ideology of the object-oriented programming with C++ language: i.e. a creation of the "virtual object" ROBOT consisting of the data structures and methods (operations) connected with them.

The developed software is used in several robots: the on-board manipulator of the Russian Space Shuttle BURAN, laboratory manipulators equipped with a technical vision systems as well as for robot investigations and simulations with research aims (the model includes a complete set of the real software!). Besides a real-time simulation mode exists that is necessary for training equipment.

**Conclusions.** The authors would presume that the above proposed research can successfully conform to various robots tasks, especially with intellectually elements \*.

## References

1. Динамика управления роботами.-М.: Наука. Главная редакция физико-математической литературы. 1984.-336 с.

2. Макарычев В.П., Половко С.А., Ступин К.Н. Математическое обеспечение системы управления манипуляторов космического корабля "Буран". Робототехника и техническая кибернетика. Сборник научных трудов. - Санкт-Петербург: Санкт-Петербургский государственный технический университет, 1994. с. 75-80.
3. Макарычев В.П., Архипов В.Д., Байбус И.Г., Майрансаев Е.А., Сукачев М.А. Математическое моделирование системы бортовых манипуляторов корабля "Буран" в реальном времени. Робототехника и техническая кибернетика. Сборник научных трудов. - Санкт-Петербург: Санкт-Петербургский государственный технический университет, 1994. с. 80-85.
4. Вартанов В.Л., Макарычев В.П. Математическое обеспечение управления роботом с системой технического зрения. 2-я Всероссийская с участием стран СНГ конференция "Распознавание образов и анализ изображений: новые информационные технологии" (РОАИ-2-95). Тезисы докладов. ч.4, 1995, с. 45-47.
5. Макарычев В.П. Алгоритмы контурного и силового управления манипулятором. VI Научно-техническая конференция "Робототехника для экстремальных условий". Материалы конференции. - Санкт-Петербург: Санкт-Петербургский государственный технический университет, 1996, с. 207-217.
6. Makarychev Vladimir P. Unified software intended for robot with technical vision system and artificial intelligence elements. International Conference Instrumentation in Ecology and Human Safety (IEMS): Proceedings/ St. Petersburg State Academy of Aerospace Instrumentation, 1996, 224 p.
7. William K. Pratt. Digital image processing. A Wiley - Interscience publication John Wiley and Sons, New-York/Chichester/Brisbane/Toronto, 1978.

# THE MATHEMATICAL MODEL AND THE NUMERICAL ROBOT SIMULATION IN CAD PACKAGE CAEIR

A.E. Solovjev  
Russian Academy of Science  
St. Petersburg Institute for Information and Automation  
Robotics Laboratory  
39, 14th line, St.-Petersburg, 199178, Russia  
e-mail: sol@mail.iias.spb.su

**Abstract.** In article the rational mathematical model for numeric modeling with real-time robot simulation is described. In the mathematical model the set of equations of rigid body's motion in Lagrange's form and set of Appel's equations taking into consideration holonomic and non-holonomic connections are used. Also in present article the methods and algorithms of dynamic modeling of a system of rigid bodies for robotics task and brief description of the package Computer Aided Engineering for Industrial Robots (CAEIR), based on considered algorithms are considered. So far as, in researching of robots the dynamic tasks (direct and inverse) are more interesting than another tasks, authors pay attention just on these problems. This researching is sponsored by INTAS program, project 94-2785.

**Keywords.** Mathematical model, numerical methods, robot, simulation, CAD system.

The dynamics of movement of the system of rigid bodies, connected by the hinges of rotations and sliding, is described by Lagrange's and Appel's equations. The generalized form of equations of movement for the body  $G_j$  with  $n$  degrees of freedom has the form:

$$A_{ik}^j(q) \ddot{q}_k + B_i^j(q, \dot{q}) = Q_i^j(t), \quad (1)$$

if the kinetic energy of the body is submitted in the kind:

$$T_{kin}^j = (1/2) \dot{q}^T A^j(q) \dot{q} = (1/2) \dot{q}_i A_{ik}^j(q) \dot{q}_k,$$

where  $q_i = q_i(t)$  ( $i = 1, n$ ) - the general coordinates,  
 $\dot{q}_i$  - the general speeds,  
 $\ddot{q}_i$  - the general accelerations,  
 $Q_i^j(t)$  - the general forces,  
 $A_{ik}^j(q)$  - the matrix of kinetic energy of the body  $G_j$ ,  
 $T$  - the transposing index,  
 $j = 1, m$  - the number of the rigid bodies,  
 $i = 1, n$  - the index of the general coordinates.

The matrix  $A_{ik}^j(q)$  is positively determined and is defined from kinematic equations of the system of bodies:

$$A_{ik}^j(q) = m_j \frac{\partial V_j^T}{\partial \dot{q}_i} \frac{\partial V_j}{\partial \dot{q}_k} + \frac{\partial \omega_j^T}{\partial \dot{q}_i} J_j \frac{\partial \omega_j}{\partial \dot{q}_k}, \quad (2)$$

where  $m_j$  - the weight of the body,  
 $V_j$  - the speed of the body,  
 $\omega_j$  - the angle speed of the body,  
 $J_j$  - the inertia tensor of the body.

The task of dynamics of movement of bodies (the inverse task of dynamic) has the following statement: to find the law of movement of general coordinates  $q_i(t)$  in the considered frame reference on the given general forces  $Q_i(t)$  and the initial condition of the body. The decision of this problem requires to integrate a system of implicit inhomogeneous equations of the second order (Eq. (1)) on the given time interval  $0 \leq t \leq T$ . Usually the implicit method of integration on the interval  $\Delta t = t_i + h$ , where  $h$  - step of integration, received in view of the linear approximation general acceleration  $\ddot{q}_i(t)$  on this interval and on the iterative method of the decision of set of linear algebraic equations are offered. We shall consider these ideas.

If on the interval  $h$  general acceleration  $\ddot{q}_i(t)$  is approximated by linear function, the speed  $\dot{q}_i(t)$  is approximated by a square-law function and the general coordinates - by a cubic function from the time  $t$  :

$$\begin{aligned} q(t) &= at^3 + bt^2 + ct + d, \\ \dot{q}(t) &= 3at^2 + 2bt + c, \\ \ddot{q}(t) &= 6at + 2b \end{aligned} \quad (3)$$

If  $\Delta t \in [t_i, t_{i+1}]$ , on the border of the interval we have:

$$\begin{aligned} q(t_i) &= at_i^3 + bt_i^2 + ct_i + d, & q(t_{i+1}) &= at_{i+1}^3 + bt_{i+1}^2 + ct_{i+1} + d, \\ \dot{q}(t_i) &= 3at_i^2 + 2bt_i + c, & \dot{q}(t_{i+1}) &= 3at_{i+1}^2 + 2bt_{i+1} + c, \\ \ddot{q}(t_i) &= 6at_i + 2b, & \ddot{q}(t_{i+1}) &= 6at_{i+1} + 2b \end{aligned} \quad (4)$$

We shall consider

$$\begin{aligned} q(t_{i+1}) &= a(t_i^3 + 3t_i^2h + 3t_ih^2 + h^3) + b(t_i^2 + 2t_ih + h^2) + c(t_i + h) + d = \\ &= (at_i^3 + bt_i^2 + ct_i) + (3at_i^2 + 2bt_i + c)h + (3at_i + b)h^2 + ah^3 = \\ &= q(t_i) + h\dot{q}(t_i) + h^2[\ddot{q}(t_i) + 2ah]/2 \end{aligned} \quad (5)$$

$$\ddot{q}(t_{i+1}) = (6at_i + 2b) + 6ah = \ddot{q}(t_i) + 6ah \quad (6)$$

From Eq. (6) we shall receive

$$2ah = [\ddot{q}(t_{i+1}) - \ddot{q}(t_i)]/3 \quad (7)$$

Having substituted Eq. (7) in Eq. (5), we shall receive

$$\begin{aligned} q(t_{i+1}) &= q(t_i) + h\dot{q}(t_i) + (h^2/2)[\ddot{q}(t_i) + \ddot{q}(t_{i+1})/3 - \ddot{q}(t_i)/3] = \\ &= q(t_i) + h\dot{q}(t_i) + (h^2/6)[2\ddot{q}(t_i) + \ddot{q}(t_{i+1})] \end{aligned} \quad (8)$$

Having conducted analytic transformation, we shall receive

$$\dot{q}(t_{i+1}) = 3a(t_i^2 + 2t_ih + h^2) + 2b(t_i + h) + c =$$

$$\begin{aligned}
&= \dot{q}(t_i) + h[\ddot{q}(t_i) + 3ah] = \\
&= \dot{q}(t_i) + (h/2)[\ddot{q}(t_i) + \ddot{q}(t_{i+1})]
\end{aligned} \tag{9}$$

So, we have received equations for the general coordinates  $q(t_{i+1})$  and the speeds  $\dot{q}(t_{i+1})$  through their significance on the previous step  $(t_i)$  and accelerations  $\ddot{q}(t_i)$  and  $\ddot{q}(t_{i+1})$ .

We shall return to a set of implicit differential equations of the second order for rigid body  $j$ , which we shall record in the form:

$$H_i^j(t, q, \dot{q}, \ddot{q}) = A_{ik}^j(q) \ddot{q}_k + B_i^j(q, \dot{q}) - Q_i^j(t) = 0, \tag{10}$$

where  $H_i^j(t, q, \dot{q}, \ddot{q})$  - the residual, and  $A_{ik}^j(q)$  - the matrix of kinetic energy of the system. The residual and the matrix of kinetic energy are defined using system parameters and the current conditions  $q(t), \dot{q}(t), \ddot{q}(t)$ . In Eq. (10) senior derivatives  $(\ddot{q}_k)$  are entered linearly. The index  $j$ , describing quantity of rigid bodies, we shall not consider.

As far as the Eq. (10) describes movement of the mechanical system, the matrix  $A_{ik}^j$  is symmetric and positively determined. From the mechanics Gauss function of compulsion is known:

$$F(\ddot{q}) = (1/2) \sum_{i,k=1}^n A_{ik} \ddot{q}_i \ddot{q}_k + \sum_{i=1}^n B_i(t) \ddot{q}_i \tag{11}$$

Gauss principle approves, that the movement of the mechanical system in each moment of time  $t$  minimizes Gauss function of compulsion.

$$\delta F(\ddot{q}) = \sum_{i=1}^n (\partial F(\ddot{q}) / \partial \ddot{q}_i) \delta \ddot{q}_i \tag{12}$$

As far as variations  $\delta \ddot{q}_i$  are independent,

$$\partial F(\ddot{q}) / \partial \ddot{q}_i = H_i^j(t, q, \dot{q}, \ddot{q}) = 0, (i=1, n) \tag{13}$$

If  $q_k(t)$  and  $\dot{q}_k(t)$  at the moment of a time  $t=t_i$  are known, it is possible to find unknown values  $\ddot{q}_k(t)$  from Eq. (10) using known algorithms for the system of linear equations. Therefore, the decision of Eq. (10) needs to be conducted in two stages:

1. To extrapolate values  $q_k(t)$  and  $\dot{q}_k(t)$  in  $t=t_i$ , proceeding from known significance in  $t = t_{i-1} = t_i - h$  under the Eq. (8), (9).

2. To decide the system of linear equations relatively  $\ddot{q}_k$ , using the Eq. (10).

The set of linear algebraic equations (Eq. (10)) we shall solve using Newton's method. The iterative circuit for systems of linear equations has the form:

$$\ddot{q}_j^{[l+1]} = \ddot{q}_j^{[l]} - A_{jk}^{-1} H_k(\ddot{q}_j^{[l]}), \tag{14}$$

where the index  $l$  - the number of iteration of Newton's method and  $A_{jk}^{-1} = \text{inv } A_{jk}$

If in the diagonal elements in the matrix  $A_{jk}$  are positive and are prevailing over the nondiagonal, the iterative formula acquires the kind:

$$\ddot{q}_j^{[l+1]} = \ddot{q}_j^{[l]} - A_{jj}^{-1} H_j(\ddot{q}_j^{[l]}) \quad (15)$$

The algorithms of calculation in this case have the kind:

1. We choose the step of integration  $h$  and step of calculation of the matrix  $A_{jk} - h_M$  and number of iterations  $IT$  on  $l$ :

$$h = (t_1 - t_0)/M, \quad h_M = \alpha h, \quad \alpha > 1, \quad t_i = t_0 + ih, \quad (i=1, J)$$

2. We calculate the matrix  $A_{ik}$  and  $A_{ik}^{-1}$ .

3. We choose the initial approximation  $\ddot{q}^{[0]}(t+t_i) = \ddot{q}(t)$ .

4. We execute the iteration for  $l=(1, L)$ .

4.1. We extrapolate  $\ddot{q}^{[l]}(t+t_i)$  and  $\dot{q}^{[l]}(t+t_i)$  under the Eq. (8), (9) with

$$\ddot{q}(t+t_i) = \ddot{q}^{[l-1]}(t+t_i).$$

4.2. We calculate  $\ddot{q}^{[l]}(t+t_i)$  under the iterative formula in Eq. (14).

5.  $T = T + hI$ ,  $I = I + 1$ .

6. If  $T > T_E$  then go to end, otherwise: if  $i=0$ , go to item 2, otherwise to 3.

At  $T=0$  it should execute steps 3 and 4 with  $hI=0$  and  $\ddot{q}^{[0]}(t_0)=0$ .

We shall consider the proposed algorithm of the decision of the system (Eq. (10)) with the help of the obvious and implicit circuits of integration of Runge-Kutta and Adams types. The algorithm of calculation will have the following kind:

1. We choose the step of integration  $h$ , step of recalculation of a matrix  $A_{ik} - h_M$  and the number of iterations  $IT$  on  $l$ :

$$h = (t_1 - t_0)/M, \quad h_M = \alpha h, \quad \alpha > 1, \quad t_i = t_0 + ih, \quad (i=1, J).$$

2. We calculate the general forces  $Q_i(t)$ , matrixes  $A_{ik}$  and  $A_{ik}^{-1}$ .

3. We choose the initial approximation  $\ddot{q}^{[0]}(t+t_i) = \ddot{q}(t)$ .

4. We execute the iteration for  $l=(1, L)$   $\ddot{q}^{[l]}(t+t_i)$  under the iterative formula in Eq. (14).

5. We calculate the residual  $H_i^j(t, q, \dot{q}, \ddot{q})$  for  $t+t_{i-1}$ .

6. We calculate  $B_i(q, \dot{q})$ :  $B_i(q, \dot{q}) = H_i - A_{ik} \ddot{q}_k + Q_i(t)$

7. We record the system of equations in the Cauchy form of the first order:

$$\begin{aligned} \dot{q}_{ik} &= (-B_i + Q_i) A_{ik}^{-1}, \\ \dot{q}_k &= q_{1k} \end{aligned}$$

8. We integrate equations on one step  $h$  with the help of the explicit or implicit circuits.

9.  $T = T + hI$ ,  $I = I + 1$ .

10. If  $T > T_E$  then go to end, otherwise: if  $i=0$ , go to 2, otherwise to 3.

At  $T=0$  it should execute steps 3 and 4 with  $hI=0$  and  $\ddot{q}^{[0]}(t_0)=0$ .

On the basis of offered methods and algorithms the program of robots modeling with the imposing holonomic and non-holonomic bonds is made.

CAEIR package helps to design the wide range of the controlled mechanisms (robots, means of transportation, tools) and building constructions, formed of bodies system by using computer simulation. The typical computer simulations are the numerical solutions of direct and inverse dynamic, kinematic and geometric tasks for mechanism and buildings.

The package allows in dialog to do the following: to form computer models (dynamic, kinematics, geometric) of a wide range of kinematics construction with the scheme, which consists of the tree-like structure formed by bodies connected by means of joints with angular or linear displacement. There is the possibility to wide the range of modeled mechanisms at the expense of imposing on the mechanisms holonomic and non-holonomic links; to form models of information and control systems of these controlled mechanisms; to make the different machine experiments with the modeled mechanisms for the purpose of research; to present current and full output results of experiments in 3D graphics.

CAEIR package is written using FORTRAN and C++ languages for MS DOS. Using CAEIR we may simulate mechanisms with the fifteen degrees of freedom (fifteen main differential equations) and fifteen constraints (algebraic equations of constraints).

# MULTILINK MECHANISM AS A CONSTRUCTIONAL BASE FOR INFORMATION SUPPORT OF A MAN-MACHINE ASSEMBLY COMPLEX

Stanislav Daurov

Saratov State Technical University  
Politechnicheskaya 77, 410016 Saratov, Russia  
e-mail: robsii@office.sstu.saratov.su

Eugene Ovsyannikov

St. Petersburg Institute for Informatics and Automation of the Russian  
Academy of Sciences  
14th Line, 39 199178 St. Petersburg, Russia  
e-mail: eko@iiias.spb.su

Gerald Ströbel

Bremen Institute of Industrial Technology and Applied Work Science  
at the University of Bremen  
Hochschulring 20, D-28359 Bremen, Germany  
e-mail: str@biba.uni-bremen.de

**Abstract.** The report is devoted to the questions of use of man-machine complex of multilink mechanism as an information system. The purpose and main functions of the information system are determined. The ways of its realization are considered and the choice of one of them is justified. The principles of construction of kinematics diagram of a anthropomorphic measuring gripper are formulated. The biquaternion technique is used for development of mathematical model of the measuring gripper and methods of computation complexity reduction are offered. The simulation model is developed and the first results of its application for accomplishing the simple assembly operations are investigated. The directions of further researches are defined.

**Keywords.** Robot industrial assembly, multilink mechanism, anthropomorphic measuring gripper.

## *Part 1. The Ways of Realization of an Information System*

The man-machine assembly complex can be represented as a set of three systems, namely: information, processing and executive. The information system collects all the data about any movement made by an operator when the model (training) assemble of an arbitrary object is performed. The processing system transforms the output data of the first system into the control information of the executive one. An industrial assembly robot replicating all the motions of the operator can be used as the executive system. Thus, the main applications of the information system are: the acquisition of measuring information from transducers, the preliminary processing of the information in order to reduce its volume, and the storage of output data.

Taking into account these main applications of the information system one can suggest the following approaches of its implementation: a multilink mechanism (anthropomorphic measuring gripper), a combination of multilink mechanism with a system of technical vision,

a mechanism basing on a system of technical vision and finally a mechanism basing on the new principles of sensitive elements of transducers.

The first approach implies a creation of the simplified models of the hand having the minimal necessary number of fingers (three) and the links (shoulder, elbow, arm, wrist) connecting the hand with the base reference system. The common principles of design of all parts of the system as well as relatively wide spectrum of mathematical methods suitable for description and analysis of measuring gripper model can be rated to the advantages of this method. Otherwise, the reciprocal relationship between the accuracy of the system and its complexity (operation possibility) is the main drawback of the approach discussed.

The second method is based on creation of the hand in the form of the multilink mechanism just like in the previous approach and its connection to the base reference system by means of the technical vision devices. Significant merits of this method are a facilitation of operator's work during the accomplishment of assembly operations and reduction of the multilink mechanism complexity. However, this slight decrease in complexity (about 25% is achieved at expense of introduction of rather sophisticated and expensive system of technical vision including at least two cameras.

The main point of the third approach is observation of all the components of measuring gripper using the system of technical vision. In this case there is no necessity in the gripper itself since it can be substituted by the operator's hand. The further analysis of the given method, the particular problems to be solved by the information system being taken into account, leads to the conclusion that the system of technical vision should perform tracking of the location and orientation in the space of manipulating object rather than motions of operator's hand. Under considered conditions the hand of operator becomes an interference which requires to use the ramified network of camera in order to reduce its influence. Although in fact the considered system provides for operator the ideal assembly conditions its creation is characterized by utmost complexity from the technical and algorithmic point of view.

The last way of system realization (hypothetical one in the meantime) assumes the creation of sensitive glove of manufactured 'skin', each cell of which being respond to touch and bending (compression and tension) with the signal of electrical nature for example. The magnitudes of the whole set of cells could determine in the unique manner the form of the glove itself.

- The comparative examination of the suggested approaches performed at the present stage of investigation allows one to make choice in favour of the first of them. Alongside with the merits mentioned above the representation of measuring gripper in the form of multilink mechanism permits the relatively simple computer simulation to be performed. The main goal of this simulation is to evaluate:

- - the intensity of information flows from various units of measuring gripper,
- - the necessary accuracy and speed of acquisition of measuring information,
- - possible ways of processing of the measuring information.

## ***Part 2 The Mathematical Model of a Measuring Gripper***

Thus, it is necessary to construct and exam a mathematical model of the information system or rather an anthropomorphic measuring gripper in order to obtain this information. Let us take a configuration of human hand as a prototype for design of mathematical model of measuring gripper. Taking into account the extremely complexity of human hand kinematics (about 30 degrees of freedom) the strategy 'from simple to complex' was accepted.

In fact it means that at the first stage the most simple model should be chosen and in further it will be complicated. During the analysis each model is estimated rated on two properties:

- - kinematic complexity (number of degrees of freedom)
- - the convenience level of carrying on the assembly operations by the operator.

The first property is evaluated by number of degrees of freedom of a model and points out the mathematical complexity of a model. The evaluation of second property may be only qualitative (low, medium or high) so it is possible to estimate the level of convenience of assembly operations execution by an operator in the couple 'human hand-anthropomorphic gripper'. Without physical simulation a level of comfort may be rated only indirectly. It is obviously that the level of comfort of the model will be raised in accordance with the increase in its redundancy (or kinetic complexity).

Thus, the consequence analysis of various models allows one to choose the model of the required complexity as the subject of examination and hence the necessary level of comfort for operator. A kinematic model of an informational anthropomorphic gripper consists of an arm and a tradactilous hand (index, middle fingers and a thumb). In fact, the kinematic diagram of an arm is the replica of the first four links of the industrial robot 'Puma-260'.

A kinematic diagram of the index finger is a extension of the arm and consists of four links oriented in the same direction. The first link is essentially a palm on which the first joints of the thumb and middle finger are fixed. The characteristic feature of configuration of the middle finger including its four links is the ability of motion in two orthogonal planes that enables to it to adapt for various objects of different shape. The thumb consists of five links and its kinematics is the most complex because of joints mobile in three planes.

The reference coordinate systems and parameters of all kinamatic links of informational anthropomorphic gripper has been revealed (specified).

The most commonly used technique for description of multilink mechanical mechanisms is the method of homogeneous transformations [1]. Since recently the quaternion technique, helical calculus [2] and biquaternion technique [3] have been finding wide use. The biquaternion technique unites two previous ones. The circumstance has predetermined the author's choice.

Single biquaternion (dual quaternion) has the form;

$$\Lambda = \Lambda_0 + i_1 \cdot \Lambda_1 + i_2 \cdot \Lambda_2 + i_3 \cdot \Lambda_3 \quad (1)$$

where:  $\Lambda_0 = \cos(\Phi/2)$ ,  $\Lambda_i = \sin(\Phi/2) \cdot \cos(\Gamma_i)$ ,  $i=1,2,3$

The components  $\Lambda_0, \Lambda_i$ , of a biquaternion are the dual analogues of the components  $\lambda_0, \lambda_i$ , of a single quaternion  $\lambda$  and represent the functions dual of variables:  $\Lambda_0$  is the function of one dual variable  $\Phi = \varphi + \&\varphi$ , rather than  $\Lambda_i$  is the functions of two dual variables  $\Phi$  and  $\Gamma_i = \gamma_i + \&\gamma_i^0$  ( $\&$  - Klifford's symbol of complexity).

The biquaternion from expression (1) can be represented as follows:

$$\Lambda = \cos(\Phi/2) + (i_1 \cos\Gamma_1 + i_2 \cos\Gamma_2 + i_3 \cos\Gamma_3) \sin(\Phi/2) \quad (2)$$

The last expression clearly shows that from the physical point of view the dual angle  $\Gamma_i$  characterizes the location and orientation of the axis of screw movement with respect to the base reference system and dual angle  $\Phi$  defines the rotation of the body about this axis by an angle  $\varphi$  as well as displacement of the body within the distance  $\varphi^0$  in the direction of the axis. Alongside with biquaternion technique the biquaternion matrices are used for the description of spatial movement of the body:

$$M(\Lambda) = \begin{vmatrix} \Lambda_0 & -\Lambda_1 & -\Lambda_2 & -\Lambda_3 \\ \Lambda_1 & \Lambda_0 & -\Lambda_3 & -\Lambda_2 \\ \Lambda_2 & \Lambda_3 & \Lambda_0 & -\Lambda_1 \\ \Lambda_3 & -\Lambda_2 & \Lambda_1 & \Lambda_0 \end{vmatrix} \quad (3)$$

The biquaternion matrix  $M$  is the dual analogue of the corresponding  $m$  and it can be written as follows:

$$M = m + \&m^0 = m \exp(m^T m^0) = m(E + \& m^T m^0) = m [E + \&m(r_y)/2].$$

$$\text{Finally: } M = m [E + \&m(r_y)/2]. \quad (4)$$

where:  $m$  is the quaternion matrix of real variables defining the orientation of the body with respect to the base coordinate system;  $m(r_y)$  is the matrix of radius vector  $r$ , linking the origin of coordinate system connected with a kinetic link and the base reference coordinate system.

$$m = \begin{vmatrix} \lambda_0 & -\lambda_1 & -\lambda_2 & -\lambda_3 \\ \lambda_1 & \lambda_0 & -\lambda_3 & -\lambda_2 \\ \lambda_2 & \lambda_3 & \lambda_0 & -\lambda_1 \\ \lambda_3 & -\lambda_2 & \lambda_1 & \lambda_0 \end{vmatrix}, m(r_y) = \begin{vmatrix} 0 & -y_1 & -y_2 & -y_3 \\ y_1 & 0 & -y_3 & -y_2 \\ y_2 & y_3 & 0 & -y_1 \\ y_3 & -y_2 & y_1 & 0 \end{vmatrix} \quad (5)$$

Coming back to the measuring gripper one can describe its  $i$ -th kinematic link in terms of biquaternion technique:

$$M_i = m_i [E + \&m_i(r_y)/2]. \quad (6)$$

where:  $m_i$  and  $(m_i(r_y))$  - determine orientation and situation of the  $i$ -th kinematic link relatively  $(i-1)$ -th of a coordinate system connected with the  $(i-1)$ -th link respectively.

After the conventional designations one can obtain the resulting biquaternion:  $M_i ==> {}^{i-1}M_i$ ,  $m_i ==> {}^{i-1}m_i$ ,  $m_i(r_y) ==> {}^{i-1}r_i$  and we shall determine resulting biquaternion:

$${}^0M_k = {}^0M_1 \cdot {}^1M_2 \dots {}^{i-1}M_i \dots {}^{k-1}M_k.$$

which in turn can be split into two iterative expressions:

$${}^im_k = {}^im_{i+1} \cdot {}^{i+1}m_k, \quad {}^ir_k = {}^ir_{i+1} + {}^im_{i+1} \cdot {}^{i+1}r_k \cdot ({}^im_{i+1})^T,$$

$$k = \text{const}, \quad i = k-1, k-2, \dots, 1, 0. \quad (7)$$

The biquaternion technique is characterized by the convenient short form of notation but it is rather sophisticated from the computational point of view. However, the analysis of matrices in the main expressions (5,6) shows that they are all skew-symmetric and some of them are sparse. This characteristic feature enable to reduce drastically the volume of computations (about 3 times) in comparison with homogeneous transformations.

### ***Part 3 A Simulation Model and Results of its Study***

The simulation model anthropomorphic measuring gripper is developed on the basis of algorithm language of a high level and consists of a set of direct and reverse problems for all main parts of the measuring gripper. In order to carry on the program-simulated movements of the gripper as well as executive the procedure of grappling of an object it was necessary to solve the inverse problem for the measuring gripper. Since this gripper consists of four parts, one of them being common to the others, it seems reasonable to solve the inverse problem initially for the hand. The location of an assembling object (for instance a cylindrical bar) in the base reference system is known. To provide the normal gripping of the object the manipulator's hand should take the specified position with respect to it that makes it possible to determine the coordinates and orientation of the hand. Thus, the initial data required for solution of inverse problem for the hand is obtained. The next stage consists in determination of the coordinates of the points where the fingers tips of the measuring gripper are to contact the object surface. All the inverse problems for the measuring gripper have been solved by a geometrical method.

The route of a program-simulated movement of the measuring gripper consists of certain places corresponding to the characteristic positions of the gripper and linear segments between these places, namely: initial position (the hand lies on the assembly table), segment of hand movement to the point of gripping under the object, procedure of object gripping, vertical lift of the object, horizontal movement of the hand to the point under the goal, vertical drop (unclasp of the object), return to the reference position. Basing on the data obtained the curves of generalized coordinates' changes have been calculated that confirmed the initial assumptions.

The nowadays stage of investigations is closely connected with the development of mathematical and simulation model of the anthropomorphic measuring gripper and check of its qualitative fitness. In further research work it is planned to examine the accuracy of simulation of the given trajectory by the model as well as , to design the algorithm of gripping for the objects of an arbitrary shape and transformation algorithm of generalized coordinates of measuring gripper into the generalized coordinates of industrial robot. The practical usefulness of the research work performed consist in the result that one can predict and solve the majority of problems appeared without the creation of the physical spectrum of the information system.

The present stage of researche is connected to development of mathematical and program model anthropomorphic measuring gripper and check of its qualitative adequacy. Is hereinafter planned to investigate accuracy of realization by model of a given trajectory of movement, to develop algorithm of grab of objects of any form, to develop algorithm of transformation of generalized coordinates measuring gripper in generalized coordinates of a manipulator of a industrial robot. The practical value of work consists in a volume, that before creation of physical model of a information system can be predicted and many problems are resolved.

### ***References***

- 1.Фу К., Гонсалес Р., Ли К. Робототехника: Пер. с англ. М.: Мир, 1989.
- 2.Воробьев Е. И., Дименберг Ф.М. Пространственные шарнирные механизмы. Замкнутые и открытые кинематические цепи. - М.: Наука, Гл. Ред. Физ.-мат. лит., 1991- 264 с.
- 3.Перков Н.Ф., Челноков Ю.Н. Применение бикватернионных матриц в кинематике пространственных механизмов/ Машиностроение - 1981.- N4,с.60-66.

## **LATE PAPERS**

# THE BASIC INFORMATIONAL CONTENTS LEVELS FOR SATELLITE ECOLOGICAL INVESTIGATIONS

Oleg I.Smokty  
Institute for Informatics and Automation  
of Russian Academy of Sciences,  
14 Line, 39, Post Box 199178, St.Petersburg,  
e-mail: soi@iias.spb.su

**Abstract.** The key problems and concept of basic informational contents levels for satellite ecological investigations are discussed. Mentioned problems are being modelled on the basis of two unified and conciliated mathematic approaches showing the necessity and the importance of informational contents analysis for data received from space observation ecological systems and appropriate theoretical models of environment radiation fields.

**Keywords.** Space ecological monitoring, informational contents, radiation field, direct and inverse operators, optimal planning, optimization, response function, optical model, atmosphere-surface system.

**Introduction.** Since global and regional ecological problems all over the world become ever more actual, the necessity for their priority solving lays the new claims to the proper complex and system investigations. Well known that today these investigations are based on the whole complex of methods for receiving, interpretation, analysis and distribution of remote sensing and contact measurements of the environmental state parameters data. Thus, it is necessary to add the imitative mathematic modeling, informational contents, study and optimal planning of the space observation systems. However now we use only the fragmentary methods for the global geospherical processes to be united with the remote sensing data optimization, solving of geoinformatics problems, analysis of the temporary or regular observations, biospherical parameterization in the global climate models and the problems of risk and forecasting of global environmental changes. It is natural that for giving the complete picture of all methods mentioned above it is necessary to identify the connection between social, economical, demographical processes and public health state with the ecological and natural resource problems.

The methods pointed out are forming the knowledge data base for estimation, control, forecasting and management of geospherical and biospherical state parameters which evolve under the influence of natural and anthropogeneous factors. In full measure the realization of mentioned research trends is possible only in the future. However currently we could give the illustration of these methods on the examples of solving some actual problems. The informational contents of the environment optical data having been retrieved using multispectral satellite information is modelled on the basis of two unified and conciliated mathematical approaches. The kernel of the

approaches consist of joint mathematical description of satellite sensors working and measurement data trends with one side and the operators set of direct-inverse problems solutions and input optical models of the "atmosphere-surface" system with the other side. The representative complex imitative model consist of three basic independent but jointly considered multitudes of submodels:

1. multitude of  $G$  are input optical parameters submodels of unified "atmosphere-surface" system;
2. multitude of  $L(G)$  are radiation fields submodels for used input optical parameters submodels  $G$ , here  $L$  are the operators set of direct problems solutions;
3. multitude of  $L^{-1}\{L(G)\} = G^*$  are output retrieved optical parameters submodels of unified "atmosphere-surface" system, here  $L^{-1}$  are the operators set of inverse problems solutions.

In this paper for the estimation of modeling data informational contents on the basis of an initially selected grid points  $g$  and calculation errors  $\varepsilon$  the Nyquist-Kotelnikov's theorem is used [1].

### 1. The Conception and Key Problems of Global Ecological Investigations.

It seems to be necessary first of all to base the conception of the global ecological investigations on the following statements:

1. The environment forming a part of geosphere and sphere of human inhabitation (biosphere) are the open dynamical system existing thanks to the Sun energy and the force of gravitation. Stability of such biospherical and geospherical systems is provided by internal closed cycles and substantive and energetical circulation. Also this stability is provided by forming of natural structures of various time-spatial scale. These structures and their components exist in constant interaction, experiencing synergetical changes and cycle fluctuations.
2. From the system point of view geosphere and biosphere elaborate the coming streams of energy, mass and information. With that every component of the geospherical -biospherical system is the statistical time-spatial dynamic and informational filter for the consequence of other components (subsystems).
3. From the ecological point of view in the environment the anthropogeneous loads are increasing constantly the action zone of natural factors is diminishing and at the same time increases the breach of the energetic balance and global substances circulation. All these circumstances come to the biospherical evolution which is mostly unfavorable for the life processes. The evolution is going to the diminishing of biospherical productivity and stability.

**2. Space Monitoring, Informational Capacity and Optimal Planning of Measurements.** Solving of the problems mentioned above requires a creating of the global systems of observation using both usual and satellite means of observations. Taking into account the global scale of receiving of ecological information there is no

doubt about the priority of space monitoring systems including appropriate systems of informational providing. To solve the mentioned problem is necessary to use models of theoretical or empirical correlations connecting in different technical, natural and physical conditions of space survey the characteristics of radiative fields which are informative both to the Earth's conditions parameters and its atmosphere including their spatial optical non-uniformities.

Let us represent the spectral characteristics of measured from the space brightness of natural objects and backgrounds like as a coefficient of reflection  $\rho$  which is depended on many factors [2]:

$$\rho = F\{[A]_1, [B]_2, [C]_3, [D]_4, [E]_5\} \quad (1)$$

The first group  $A$  in (1) is spectral one. The second group  $B$  is atmospheric optical one. The third group  $C$  is device geometrical one. The fourth group  $D$  characterizes the natural conditions of space survey determining the state of investigated natural objects. The fifth group  $E$  unites technical parameters of the spectral photometrical data receiving. During the optimal planning of spectral optical measurements it is necessary to indicate those parameters and variables values at every group which could provide existence of the maximum informational contents of the environment remote sensing from space. Beyond that during the solving of the space optical monitoring optimization problem and determining its informational contents it is necessary to consider the natural object, atmosphere and spectral optical device not separately but jointly with whole informational measuring optical channel of remote sensing like as a linear optical system (filter).

Let us consider conception of informational capacity of satellite measuring system making use of example of spectral optical system with three spatial free degrees and couple of independent components of linear polarization degree for the cases of coherent and non-coherent radiation. This conception is based on the informational capacity invariance theorem [3]. According to this theorem for the optical system is fixed not a time-spatial wideness of the band frequency but a certain number of free degrees of this optical system is meant the level of optical signal discretization (number of separate measurements) by the system which is required for its complete retrieval. Using the results [3] we get the following expressions for the informational capacity of space optical system (filter) for the non-coherent case:

$$N = L_x L_y L_z L_{\Delta\lambda} L_T \log_2 \left( 1 + \frac{\bar{\rho}}{\Delta\bar{\rho}} \right)^{\frac{1}{2}} \quad (2)$$

and

$$N = L_x L_y L_z L_{\Delta\lambda} L_T \log_2 \left( 1 + \frac{\bar{\rho}}{\Delta\bar{\rho}} \right) \quad (3)$$

for the coherent case. In the expressions (2)-(3) the following designations are used:  $\bar{\rho}$  is average power of optical signal,  $\Delta\bar{\rho}$  is additional noise power,  $\frac{\bar{\rho}}{\Delta\bar{\rho}}$  is number of permissible discretization levels (relation "signal-noise");  $L_x, L_y, L_z$  are spatial frequency (size of spatial element of signal discretization) according to the coordinates  $x, y, z$ ;  $L_{\Delta\lambda}$  is spectral element of signal discretization;  $L_T$  is temporary frequency of

signal discretization. Given relations demonstrate the logarithmic development of the informational capacity of the space optical system during the increasing of the relation "signal-noise" -  $\frac{\rho}{\Delta\rho}$ . Except this the coherent optical system the informational capacity twice more then in the case of non-coherent system. It is easy to explain taking into account the fact that when all other parameters are equal in the coherent case every measurement has the information in the amplitude and in the phase of registering signal where as in the non-coherent case - in the amplitude only. By analogy with this increase of informational capacity of remote optical measurements has to be observed in the case of linear-polarized radiation case in comparison with the scalar case. The special significance pointed circumstances has for the planning and carrying out of spectral polarization measurements of the environmental brightness on the board of orbital manned space station "Mir".

**3. The Modeling of Informational Contents Levels Related to Applied Radiative Transfer Problems.** For the realization of referred above research trends input optical multilayer models of the Earth's atmosphere and natural underlying surfaces first of all have to be combined with the environment radiative forward models adopting the analytical, semianalytical and numerical solutions of the radiative transfer theory. If we represent forward radiative transfer models in mathematical terms of precise modeling operators  $L$  and input multitude of exact optical parameter  $G$  as well as the approximated related field operators  $\tilde{L}$  and multitudes  $\tilde{G}$  we have the following possible functional combination:

$$\{L[G] \oplus L[\tilde{G}]\} \oplus \{\tilde{L}[G] \oplus \tilde{L}[\tilde{G}]\} \quad (4)$$

The precise and approximative radiative modeling schemes concerning the multitude of unknown intensities  $\{I\}$  can be described as follows:

$$\{I\}_L = \{L[G] \oplus L[\tilde{G}]\} \oplus \{I\}_{\tilde{L}} = \{\tilde{L}[G] \oplus \tilde{L}[\tilde{G}]\} \quad (5)$$

The same scheme can be adopted in the frame of the inverse problems solutions in order to evaluate the multitude of retrieved exact and approximative optical parameters  $G_*$  and  $\tilde{G}_*$  making use of appropriate modeling of exact and approximative inverse field operators  $L^{-1}$  respectively:

$$G_{L^{-1}}^* = [L^{-1}\{L[G]\} \oplus L^{-1}\{L[\tilde{G}]\}] \oplus \tilde{G}_{\tilde{L}^{-1}}^* = [\tilde{L}^{-1}\{\tilde{L}[\tilde{G}]\} \oplus \tilde{L}^{-1}\{\tilde{L}[G]\}] \quad (6)$$

Informational contents of input optical "atmosphere-surface" models will be entirely determined by chosen discretization levels  $N[G \oplus \tilde{G}]$  and appropriate errors  $\varepsilon_N$  for representation of multitudes  $G$  and  $\tilde{G}$  in each point of used grid  $g$ . The same conclusion is related to informational contents of direct problems solutions:

$$\tilde{N}\{G \oplus \tilde{G}\}, \tilde{\varepsilon}\{[L \oplus \tilde{L}] \oplus [G \oplus \tilde{G}]\}. \quad (7)$$

Likewise we have for informational contents of inverse problems solutions:

$$\delta\{[L^{-1}] \oplus [L(G) \oplus L(\tilde{G})], \quad (8)$$

$$\delta[\tilde{L}^{-1}] \oplus [\tilde{L}(\tilde{G}) \oplus \tilde{L}(G)], \quad (9)$$

$$M\{G_* \oplus \tilde{G}_*\} = M[L^{-1}\{L(G) \oplus L(\tilde{G})\} \oplus \tilde{L}^{-1}\{\tilde{L}(G) \oplus \tilde{L}(\tilde{G})\}]. \quad (10)$$

Finally besides the levels of discretization  $N$  and errors  $\varepsilon_N$  for input multitude of optical parameters and output multitude of environment radiation fields remarked above informational contents will be determined by chosen structures of direct and inverse field operators  $L$  and  $L^{-1}$  also.

**4. Input Optical Models for the "Atmosphere-Surface" System.** In order to analyze the level of informational contents it is needed to define detailly the set of input optical data concerning to the multilayer vertically nonuniform atmosphere and natural reflecting surfaces. For general point of view this ones can be described by the multitude  $G$  already defined in foregoing consideration. Such multitude  $G$  contains input information first of all about the spectral atmospheric optical thickness  $\tau_0(\lambda)$ , the spectral atmospheric single scattering albedo  $\Lambda(\lambda, \tau)$  and spectral atmospheric scattering phase function  $x\{\gamma, \tau\}$  needed for carrying out of radiation field modeling in dependence on wavelength  $\lambda$ . Hereby the multitude  $G$  can be realized partially following as  $G\{\tau_0(\lambda), \Lambda(\tau, \lambda), x(\gamma, \tau, \lambda)\}$ . Since the Earth's atmosphere is coupled with underlying surface we need to add the appropriate optical parameters as like as the surface phase function  $\chi_{sur}(\gamma, \lambda)$  and the surface single reflecting albedo  $\Lambda_{sur}(\lambda)$ . Thus the final realization of multitude  $G$  will be given follows as  $G\{\tau_0(\lambda), \Lambda_{atm}(\tau, \lambda), x_{atm}(\gamma, \tau, \lambda), \Lambda_{sur}(\lambda), \chi_{sur}(\gamma, \lambda)\}$ . Then the informational contents of input optical parameters multitude  $G$  will be defined by chosen grid point  $g$  of current scattering angles  $\gamma$ , considered in terms of solar angles  $\arccos \zeta$  and vision angles  $\arccos \eta$ , azimuth's angles  $\varphi$  and likewise chosen discretization levels of the wavelength  $\lambda$ , optical parameters representation in each current point of used grid point  $g$ . Consequently the multitude  $\{I\}$  representation will be depended from their initial informational levels which is determined by used grid point  $g$  and errors  $\varepsilon$  above adopted in the multitude  $G$  of input optical parameters. The second level of informational contents for modeling optical values will be determined by their Fourier transforms making use of well known classical approach and taking into account the grid point  $g$ , errors levels  $\varepsilon$  and the number of moments  $p_{atm}^m(\tau, \eta, \zeta, \lambda)$  received according to chosen atmospheric  $x_{atm}(\gamma, \tau, \lambda)$  and surface  $x_{sur}(\gamma, \tau, \lambda)$  phase functions [4]:

$$p_{atm}^m(\tau, \eta, \zeta, \lambda) = \frac{1}{2\pi} \int_0^{2\pi} x_{atm}(\tau, \eta, \zeta, \lambda) \cos m\varphi, \quad m = 0, 1, \dots, M_1, \quad (11)$$

$$x_{atm}(\tau, \eta, \zeta, \varphi, \lambda) = p_{atm}^0 + 2 \sum_{m=1}^{M_1} p_{atm}^m(\tau, \eta, \zeta, \lambda) \cos m\varphi, \quad (12)$$

$$p_{sur}^m(\tau, \eta, \zeta, \lambda) = \frac{1}{2\pi} \int_0^{2\pi} x_{sur}(\tau, \eta, \zeta, \varphi, \lambda) \cos m\varphi, \quad m = 0, 1, \dots, M_2, \quad (13)$$

$$x_{sur}(\tau, \eta, \zeta, \varphi, \lambda) = p_{sur}^0(\tau, \eta, \zeta, \varphi, \lambda) + 2 \sum_{m=1}^{M_2} p_{sur}^m(\tau, \eta, \zeta, \lambda) \cos m\varphi. \quad (14)$$

For foreseeing consideration it is needed to choose only one total number of Fourier azimuth's harmonics according to condition  $M = \max\{M_1, M_2\}$ . The final level of informational contents is determined by processes of computational summation

and successive compression for structurized elements  $p_{atm}^m$  and  $p_{sur}^m$  after appropriate calibration procedures. It should be noted, that referred above finding problem of coefficients  $p_{atm}^m$  and  $p_{sur}^m$  factually is the problem of angular spectra finding for given initially discretized functions  $x_{atm}(\gamma, \tau, \lambda)$ . Further it is needed to emphasize that desirable discretization levels can't be arbitrary. According to mentioned above Nyquist-Kotelnikov's theorem current discretization frequency (level)  $K$  has to twice exceed highest signal frequency  $M$ :

$$K \geq 2M. \quad (15)$$

Practically the numerical interpolation with some beforehand given accuracy  $\varepsilon$  and foreseeing calculations of phase functions moments are often used. The level of discretization for the given phase function  $x_{atm}$  and  $x_{sur}$  in the frame of such approach will be entirely determined by desirable approximation's accuracy  $\tilde{\varepsilon}$  making use of interpolated atmospheric and surface phase functions  $\tilde{x}_{atm}$  and  $\tilde{x}_{sur}$ .

**5. Direct and Inverse Problems Solutions of Applied Radiative Transfer Theory.** The analysis similar to one carried out above for input optical data is needed for the intensity  $I(\tau, \eta, \zeta, \varphi, \tau_0)$  of the invironment radiation fields. By using of Fourie transform let us represent the intensity  $I(\tau, \eta, \zeta, \varphi, \tau_0)$  follows as

$$I(I(\tau, \eta, \zeta, \varphi, \tau_0) = I^0(\tau, \eta, \zeta, \tau_0) + 2 \sum_{m=1}^M I^m(\tau, \eta, \zeta, \tau_0) \cos m\varphi, \quad (16)$$

$$m = 0, 1, \dots, M, \quad \eta \in [-1, 1], \quad \zeta \in [0, 1], \quad \tau \in [0, \tau_0], \quad \varphi \in [0, 2\pi] \quad (17)$$

In [5] for the case of vertically nonuniform atmosphere bounded from arbitrary reflecting bottom the system of exact Fredholm's integral equations had been received for azimuth's harmonics of downwelling  $I^m(\tau, \eta, \zeta, \tau_0)$  and upwelling  $I^m(\tau, -\eta, \zeta, \tau_0)$  intensities. Received in [5] exact integral equations system can be considered as general mathematical base for the investigation of mentioned above scientific reseaching trends with the help of direct modeling field operators  $L$  and multitude  $G$  for input optical data. However we have to remember that our aim consists in the providing of unified mathematical description of whole informational system according to mathematical transforms

$$G_{g,\varepsilon} \Rightarrow L_{g,\tilde{\varepsilon}}^{-1}(G) \Rightarrow L_{\tilde{g},\tilde{\varepsilon}}^{-1}(L[G]) \Rightarrow G_{g,\varepsilon}^*, \quad (18)$$

first of all on the basis of reciprocal connection between construction of direct field operators  $L$  and appropriate to them inverse field operators  $L^{-1}$ . In this context it is necessary to use mathematically strict reciprocal description of the connection  $L_{g,\tilde{\varepsilon}}(G) \Rightarrow L_{\tilde{g},\tilde{\varepsilon}}^{-1}[L(G)]$ . Appropriate inverse operator  $L^{-1}$  for representation (18) can be constructed for example by using analytical approach developped in [6]. The connection between current angular discretization frequency having been determined by total number of azimuth's harmonics can be established again on the basis of mentioned above Nyquist-Kotelnikov's theorem following to [7]. The comprehensive modeling of values  $\rho^m(\eta, \zeta, \tau_0)$  and  $\sigma^m(\eta, \zeta, \varphi, \tau_0)$  carried out for real input optical

models of "atmosphere-surface" system [7] show that optimal correlation between numbers  $M$  and  $K$  in the frame of accuracy  $\tilde{\varepsilon} \sim 10^{-4}$  follows as :

$$K \simeq 2M + 30 \quad (19)$$

for primary atmospheric light scattering and single underlying surface light reflection. For the multiple atmospheric light scattering and multiple bottom light reflection given above empirical estimation in the framework of accuracy  $\varepsilon = 10^{-4}$  can be weakened on the basis of significant computational compression for total azimuth's harmonics number  $M$  without essential loss of remarked above informational contents [8]. Taking into account the Nuquist-Kotelnikov's theorems [3] and possibility of remarked azimuth's harmonics number compression in the frame  $\tilde{\varepsilon} \sim 10^{-4}$  we can receive for the total "atmosphere-underlying surface" system the final empirical correlation between current discretization level  $K$  and azimuth's harmonics total number  $M$  associated with the upwelling and downwelling intensities according to following estimation [4]:

$$K \geq 2cN, \quad (20)$$

where compression multiplier  $c$  is equal  $\frac{1}{6}$  approximately. Accepting total number azimuth's harmonics  $N \sim 200 - 300$  for real atmospheric phase function  $x_{atm}(\cos \gamma)$  we have variation of grid points frequency up  $K \sim 400 - 600$  to  $K \sim 130 - 200$  for given beforehand approximation's accuracy  $\tilde{\varepsilon} \sim 10^{-4}$ . It should be noted that given above estimation (20) is conciliated quite with analogous results received early by M.King [9].

**Conclusion.** Modern environment remote sensing investigations are based on the whole complex of methods and tools for the receiving and thematic analysis of corresponding comprehensive multispectral information. Therefore appropriate mathematical imitative modeling and informational contents researching have actual and important significance. Unfortunately now we have only the fragmentary optimal methods and approaches which would be united quite with mentioned above complex scientific and technological problems. Factually this methods and tools have to help us in the forming of appropriate knowledge data bases and expert systems for the estimation, control, prediction and management of geospherical and biospherical processes in global and regional scales. Naturally in the full measure the realization of all remarked research trends and problems is possible only in a future on the basis of complex system approaches.

## References

1. Smokty O.I., G.Guseinov (1993) The Informational Contents and Optimal Planning for the Earth's Surface Spectrophotometric Remote Sensing. Prepr. No 157, St.Petersburg Institute for Informatics and Automation of RAS.
2. Guzzi R., O.I.Smokty (1995) The Application of the Radiative Transfer Theory to the Informational Contents modelling Connected with the Interpretation of a Multispectral Satellite Environment Data. IGARS'95, Florence.
3. Cox I., Y.Sheppard (1986) Informational Capacity and Resolution in Optical Systems. J. of Optical Soc. Amer. Vol.3, No 18, pp.1152-1164.

4. Smokty O.I. (1995) The Application of Multigrid Relaxation Methods to the Environment Radiation Fields Modelling. Techn. Rep. No 1-95, IMGA-CNR, Modena.
5. Smokty O.I. The radiative transfer in a vertically non-uniform atmosphere bounded by an arbitrary reflecting bottom (submitted to Transport Theory and Statistic Physics, 1997).
6. Smokty O.I. (1993) The Angle-Structure Method of Inverse Problems Solving of Radiative Transfer Theory. IGARS'93, Tokyo, pp.2155-2157.
7. Smokty O.I., A.S.Anikonov, A.E.Ilyin, N.V.Kobjakova (1992) The Calibration Models of the Optical Characteristics of the Earth's atmosphere. Prepr. No 158, St.Petersburg Institute for Informatics and Automation of RAS.
8. Smokty O.I.(1986) Modeling of Radiation Fields for the Problem of Space Spectrophotometry. Nauka, Leningrad.
9. King M. Number of Terms Required in the Fourier Expansion of the Reflection Function for Optically Thick Atmospheres. (1983) J.Quant, Spectrosc.Radiat.Transf., Vol.30. No 2, pp.143-161.

# AN AUTOMATED COMPLEX OF THE CONTROL AND FORECASTING THE EXTREME SITUATIONS IN AN ENVIRONMENTAL MONITORING SYSTEM

Sergey Lenchuk, Vladimir Kulganov, Yury Shvilin,  
Sergey Nesterov, Evgeny Mescheryakov and Leonid Sorokin

Military Engineering-Space Academy of St.Petersburg  
13, Jdanovskaya str., St.Petersburg, 197082, Russia  
penev@mail.iias.spb.su

**Abstract.** The problem of environmental monitoring and the prediction of an environmental development is of crucial importance for the modern society. The ways of its solution are referred to in the governmental resolution of the Russian Federation N 1085 of 2.11.95 and are considered as an essential part of the governmental programme: "Establishing the Integrated National Automated System for Monitoring the Radiation Situations in the Territory of the Russian Federation". Since many regions of our country have already been regarded to be ecologically disastrous by some parameters this problem needs an urgent solution for the further development of the country and society.

**Keywords.** Automated control, forecasting, environmental monitoring system.

**Monitoring system.** A mobile automated complex of the environmental control. An environmental monitoring system is aimed at solving the following important problems:

1. Environmental situation data acquisition.
2. The received data processing in order to evaluate the informative attributes and criteria for the environmental pollution estimation.
3. Data formation and representation in a real time scale.
4. Creating the data banks and situation models; establishing the numerical criteria of an extreme situation.

An environmental monitoring system is built by an hierarchical principle. The primary unit performing an environmental control is a mobile automated complex equipped with data gathering, preliminary processing and transmitting facilities. The missions of an operator's Automated Working Place (AWP) are to be as follows: the measurement data acquisition, their recording, storing, express-analyzing; the measurement results transmission via a communication channel to the central control post in order to make effective management decisions concerning the environmental pollution caused by an accident; as well as express-analyzing the psycho-physiological condition of mobile complex squads acting in the regions of extreme situations.

This complex is a measuring computing information system and is based on the measuring and monitoring mobile units. They provide the effective remote control of the various environmental characteristics in a polluted region during an operating patrol. Environmental parameters data acquisition is carried out by sensors or units of the control of different physical fields. There are some counters for indicating the radiation doses, the concentration of strongly acting harmful substances and the weather parameters on a mobile complex. These devices for the environmental parameters investigation are connected via an

interface unit to the PC of the Notebook type which operates the measurement modes of the devices and their operation time-table by means of a special program. It serves for:

- selecting the type of the measuring device and the range being measured;
- assigning the channel number;
- interpreting the value of the parameter being measured.

The results obtained are recorded in the protocol and are express-analyzed by the operator in charge of an environmental situation estimation. Through a built-in the results of monitoring are transmitted to the Central Control Post in a digital code. To locate a mobile complex an operator's AWP is equipped with a unit for measuring the current navigational parameters and is connected via an interface unit to the Notebook. A great deal of environmental parameters primary measurements carried out during a mobile complex patrol by each device are accumulated and processed preliminary on a PC. Each information measuring channel uses its own methods of data processing and the priority information assignment.

An APW of an environmental situation estimation operator of a mobile complex is a part of an information analytical centre and is a radio network element possessing an information compatibility with the Network data bases of the Centre of the Control and Forecasting the Extreme Situations.

***Anti-radiation protection of a mobile automated complex of an environmental control.***

An experience of an accident consequences liquidation shows that in some cases there arise serious difficulties in accomplishing the necessary works in the zones with high levels of radiation due to operators' overdosing. Under such conditions the problem of increasing the radiation stability of the ergonomic system "person-machine" up to an appropriate level is of great importance, one of the solutions being at the expense of improving the physical radiation shield of mobile units. Sites with very high density of pollution have as a rule a local character and produce essentially anisotropic radiation field.

It is impossible to predict the character of pollution and the mobile units position as to the main radiating sources. To improve the effectiveness of the antiradiation protection there has been developed a device of an Adaptive Anti-radiation Protection (AARP) used for mobile and stationary objects which is physically a number of communicating vessels of elastic material filled with the solutions of salts or water and placed immediately around an operator. Volume and thickness of a protective environment from each spatial direction depend on power distribution of the radiating field. For each protected point the AARP configuration and mass are determined separately by means of a PC on the basis of the previously performed experimental researches of the objects.

***An automated centre of the control and forecasting the extreme situations.*** In the Centre of the Control and Forecasting the Extreme Situations the control posts are realized as AWP's of environmental situation estimation operators according to the possible types of extreme situations and are integrated in an information network. It is based on the Digital Network of Integrated Services (DNIS) and is realized as multifunctional terminals (MFT) performing the "transparent" interaction, that is without the DNIS interference through the network of environmental parameters transmission among the various user's equipment.

The main tasks of the Centre of the Control and Forecasting the Extreme Situations in the environmental monitoring system are:

1. The tasks of information processing:
  - statistical processing of the information being received;
  - making the initial description of the environmental situation.
2. The tasks of decision making:
  - the current situation classification;
  - the extreme situation control and forecasting its development.
3. The tasks of information displaying and storing:
  - extreme situation displaying on electronic topographical cards;
  - visualizing the results obtained in a form suitable for an operator;
  - data archiving and their reduction.
4. The development of the preliminary resolution on organizing and realizing the measures of the protection of population and the liquidation of extreme situations.

The tasks mentioned above are realized in a special program of a AWP of an operator in the Centre of the Control in the following modes:

1. Measurement results processing and representation;
2. Extreme situation development simulation;
3. Working with maps;
4. Data bases management.

The mode "Measurement Results Processing and Representation" provides gathering and processing the results of measurements carried out by a mobile complex during a patrol.

In this mode one specifies:

- the region and route of a patrol;
- the time interval of the control;
- results representation form;
- measurement results.

The mode "Extreme Situation Development Simulation" is a data base of the situational models in the extreme situations classes. The data obtained can be used for planning the patrol route of a mobile complex. The simulation of an extreme situation development together with the characteristics of a source, meteorological and topographical conditions enables us to determine a necessary route of a mobile complex to establish real borders of a region of pollution. The mode "Data Bases" is aimed at forming the archive and providing the interaction with other systems in the Centre of the Control. The mode "Maps" provides the work with topographical maps and is a "natural substrate" for all other operation modes of a AWP of an environmental situation estimation operator.

***An automated express analyzing system of the psycho-physiological condition of mobile complex squads.*** To effectively evaluate the physical condition and the ability for work of mobile complex squads in accomplishing their tasks it is necessary to have a system of gathering and transmitting the appropriate information on the psycho-hysiological condition of operators at the Central Control Post. The main requirements for such system are: self-descriptiveness, adequacy of the measured parameters of the psycho-physiological condition of an operator organism for evaluating their physical condition and their ability for work, mobility, portability and reliability. The principle of the

variance pulse measurement permitting the main parameters of the physical condition to be determined by the character of pulses meets these requirements. The essence of this method is to construct a variance curve of a RR electrocardiogram interval distribution which gives the possibility to reveal the laws of the distribution of the parameter being investigated and to define its major characteristics. In addition, the following parameters are estimated: blood pressure, body temperature and breath frequency.

The mock-up of an automated complex realizing the above mentioned principle consists of the following devices:

1. Counter of a photoplethysmogram based on infrared light and photo diodes.
2. Amplifier- converter.
3. Multichannel controller of an analog-digital converter and multiplexer.
4. FM-radiomodem integrated on an input and output with the port RS232 IBM compatible with a PC.

This complex enables a physical condition and their ability for work of up to 30 men of mobile complex squads to be evaluated in a real time scale. The results obtained are transmitted via a communication channel to the Central Control Post for making decisions as to the psycho- physiological condition of a mobile complex squad.

## A U T H O R S   I N D E X

- |                          |                              |
|--------------------------|------------------------------|
| Acaccia G. M. 3,1244     | Bogdanov A. A. 2,600         |
| Afanasiev V. 1,71        | Bogomolov S. 1,204           |
| Agaponov S. 2,612        | Bolkhovitinov V. K. 1,27     |
| Aidu E. A. I. 3,874      | Bolotin Yu. V. 3,983         |
| Akhmedjanov F. M. 3,1032 | Boltunov G. 3,1267           |
| Akunov T.A. 3,1022       | Borodin V. V. 2,681          |
| Akunova A. 3,1022        | Boukanov I. P. 1,48          |
| Al-Noman A. A. 1,159     | Bowker P. 3,866              |
| Aleksejev A. A. 2,648    | Bukatova I. L. 2,433,580     |
| Alexandrov M. 1,181      | Bukov V. 2,687               |
| Alexeeva S. V. 1,335     | Bun R. 1,351                 |
| Allik R. 1,313           | Burdakov S. F. 3,1283        |
| Altbregin A. G. 3,1290   | Busch T. E. 1,215; 3,933     |
| Ampilova N. 3,922        | Butner E. G. 1,82            |
| Andreyev A. V. 2,720     | Callegari M. 3,1244          |
| Andreyevsky V. R. 1,118  | Canudas-de-Wit C. 3,1213     |
| Andrievsky B.R. 3,958    | Chernavsky A. V. 2,476       |
| Anikonov A. S. 1,165,173 | Chernousko F.L. 1,xlv        |
| Anokhin V. V. 1,221      | Chetvertakov M. M. 3,1194    |
| Arkhipov V. D. 3,1315    | Chudin E. E. 3,857           |
| Artemyeva O. 2,484       | Chugreyev O. 1,247           |
| Ashimov A. A. 1,295      | Dalakishvili G. 3,1047       |
| Azarenko S. A. 3,1194    | Daurov S. 3,1329             |
| Badmaeva S. V. 1,339     | Degtyarjov O. V. 3,1042      |
| Bagdouyev V. N. 1,344    | Demianov V. V. 1,139,142,150 |
| Bagljuk Yu. V. 1,27,31   | Depecker J.-C. 2,555         |
| Bagretsov S. 2,458       | Derin O. A. 2,632            |
| Bakhtadze N. N. 2,429    | Diduk G. A. 3,1059           |
| Baluev K. S. 2,629       | Domaratsky A. 2,785          |
| Barabanov N. E. 1,339    | Dorogov A. Yu. 2,648         |
| Baranoff S. 2,785,796    | Duke V. A. 2,663             |
| Baranov V. V. 3,1002     | Dvornikov V. E. 3,881        |
| Barinova N. E. 3,874     | Dymkin E.G. 3,958            |
| Baroni P. 3,1076         | Eleuov A. U. 3,881           |
| Barski A. 1,279          | Eperin A. P. 3,1142          |
| Baskakov L. V. 3,1059    | Erismann A. 1,271            |
| Berezovskaya F. S. 2,486 | Ershov B. A. 3,910           |
| Beskid P. P. 1, 104      | Eslami M. 3,1008             |
| Betourne A. 3,1275       | Evlampiev A. A. 3,1231       |
| Blatov I. V. 3,874       | Evnevitch E. L. 2,780        |
| Blekhman I. I. 3,972     | Evstifeyev V. V. 3,1042      |
| Blinkova N. V. 3,910     | Evtihiiev N. 2,484           |
| Bogdanov A. 2,621        | Fadeev A. V. 2,629           |

Fang Tinjian. 3,938  
 Fatikow S. 3,1223  
 Fedorov L. E. 1,53,56  
 Fefe A. 3,1275  
 Fishman M. N. 2,478  
 Flegontov A. 2,572  
 Fomin V. N. 3,997  
 Fradkov A. L. 1, lxvi; 3,958  
 Freidovich L. B. 3,1205  
 Fridman A. Ya. 2,441  
 Galka V. L. 3,1100,1122  
 Gassanenko M. 2,796  
 Gerasimova I. 2,500  
 Chernavsky A. V. 2,476  
 Globa V. G. 1,150  
 Golubtsov P V. 2,512  
 Gomareli N. 3,1047  
 Goosev S. A. 3,1052  
 Gorelov I. P. 1,211  
 Gorokhov A. 2,760  
 Goswami A. 3,1213  
 Grazdantseva E. Yu. 2,441  
 Greck B. V. 3,1094  
 Gribin V. P. 1,211  
 Gudz H. 1,351  
 Gugushvili A. 3,1047  
 Gusev Yu. M. 3,1032  
 Habibulin R. K. 3,1194  
 Hagin A. 1,181  
 Hagin N. 1,181  
 Heath G. H. 3,866  
 Hoeniger T. 3,1306  
 Hovanov N. V. 2,808  
 Hvostov A. 3,1198  
 Ignatiev M. 3,1134  
 Ignatiev V. 1,241  
 Ilyasov B. 2,606  
 Indeycev A. I. 1,88  
 Ismagilova L. 2,606  
 Ivanischev V. V. 2,448  
 Ivanov G. G. 3,881  
 Ivanov V. Y. 2,564  
 Ivanova E. G. 1,395  
 Ivanova S. E. 1,142  
 Kachitsin V. V. 1,142

Kamachkin A. M. 1,389  
 Kang Nansheng 3,938  
 Karev G. P. 2,486  
 Karp V. 2,484  
 Karp V. P. 2,480  
 Karpenko A. S. 2,494  
 Karplus W. J. 3,857  
 Karpov Yu. 1,181  
 Karpushkin V. N. 2,476  
 Kashtankin V. V. 1,53,56  
 Katretchko S. 2,506  
 Khatib O. 1, xxxiii  
 Khokhlovski V. 1,22  
 Khotimsky D. A. 1,191  
 Khoudiakov M. M. 3,1142  
 Khvorov I. A. 1,211  
 Kirillov V. J. 2,629  
 Kirin A. N. 1,966  
 Kirin N. E. 1,966  
 Kirshner A. L. 1,95  
 Kiselev V. G. 2,845  
 Kisselev V. B. 2,729,744  
 Kobrin V. P. 3,896  
 Kochenov D. A. 2,629  
 Kofanov Yu. 2,823,826  
 Kogan B. Y. 3,857  
 Kolbahev M. 1,241  
 Kolesov Yu. 2,704,712  
 Kolushev F. A. 2,600  
 Konjukhov A.P. 3,990  
 Konoplev V. A. 3,990  
 Kornikov V. V. 2,808  
 Korolev V. A. 3,1290  
 Korshunov A. 1,358  
 Korshunov A. 1,358  
 Korshunov G. I. 2,841  
 Korshunov I. G. 2,848  
 Kossovski N. K. 2,522  
 Kotlyarov V. 2,817  
 Kotomin A. 2,760  
 Kotrikadze T. 1,364  
 Kovalenko A. 2,623  
 Krasilnikov I. A. 3,853,926  
 Krasovskii A. A. 1, xxxi  
 Krenkel E. 2,509

Krenkel T. 2,509  
 Krymsky V. G. 3,1032  
 Kryukov V. 1,225  
 Kulabanov V. 2,687  
 Kulakov F. M. 3,1231,1297  
 Kulchitski O. Yu. 1,368  
 Kulganov V. 3,1345  
 Kulik B. 3,1087  
 Kurochkin A. I. 1,95  
 Kuteeva G. A. 3,885  
 Kuzichkin A. V. 1,211  
 Kuznetsov D. F. 1,368  
 Kuznetsov V.A. 1,251  
 Laengle T. 3,1255,1306  
 Lamperti G. 3,1076  
 Lastochkin N. 2,785  
 Lenchuk S. 3, 1345  
 Lickefett M. 1,287  
 Lipatova N. G. 1,330  
 Litskevich A. P. 1,150  
 Lototsky V. A. 2,429  
 Lueth T. C. 3,1255  
 Lukashevitch I.P. 2,478  
 Luschik V. L. 1,53,56  
 Lyubetsky V. A. 2,469  
 Maaseidvaag F. 1,3  
 Machinskaya R. I. 2,478  
 Madani K. 2,543,555  
 Makarova M. V. 1,377  
 Makarychev V. P. 3,1315  
 Makrusev V. V. 1,330  
 Makrusev V. V. 2,439  
 Malmberg H. 1,22  
 Marley V.E. 2,451  
 Maslennikov V. A. 2,696  
 Maslov V. G. 1,165,173  
 Mastin S. P. 2,772  
 Matveev P. I. 2,753  
 Maximenko I. 2,687  
 Maximov E. M. 2,429  
 Mercier G. 2,555  
 Mescheryakov E. 3,1345  
 Miasnikova G. 3,1149  
 Michalevski E. 1,279  
 Michelini R. C. 3,1244

Miheev S. E. 2,835  
 Miroshnik I. 3,1267  
 Misenov B. A. 1,382  
 Mishkin D. T. 2,580  
 Mit'ko V. B. 3,1198  
 Mit'ko A. 3,1198  
 Molfino R. M. 3,1244  
 Moraal P. 1,3  
 Morgunova S. N. 3,983  
 Morozov V. 2,785  
 Moskvina B. V. 2,669  
 Mozhaev A. S. 3,1169  
 Musijchuk Yu. I. 3,917,926  
 Nagibina O. L. 3,972  
 Naumov M. V. 1,95  
 Naumov V. B. 3,1231,1297  
 Nebylov A. V. 1,77  
 Nekhorosheva A. G. 2,480  
 Nelepin R. A. 1,34,389  
 Nemokaev Ju. S. 1,14  
 Neokesariyskiy V. N. 3,1231  
 Nesterov S. 3,1345  
 Nicholson D. J. 3,933  
 Nicosia S. 3,1275  
 Nikiforov V. O. 3,943  
 Nikitichev M. M. 1,82  
 Nikitin A. P. 2,480  
 Nikoltsev V. A. 1,118  
 Novikov V. N. 1,82  
 Novodjilov A. N. 1,41,59  
 Oleynik A. G. 2,441,753  
 Oleynik O. V. 2,441  
 Osipov G. S. 1,99  
 Ovsjannikov E. K. 3,1329  
 Ovsyannikov A. D. 1,382  
 Ovsyannikov D. A. 1,382  
 Pahomov A. A. 3,1178,1181  
 Panamarev G. E. 1,142  
 Pankova L. N. 2,780  
 Panteleev S. V. 3,1237  
 Papenko H. 3,1189  
 Paramonov L. V. 3,1283  
 Pasynkov V. E. 3,910  
 Pavlov S. 2,496  
 Penev G. D. 1,395; 2,592

Pereverzev-Orlov V. 2,518  
 Pervishko A. F. 3,1237  
 Pervozvanski A. A. 3,1205  
 Petrossov V. 1,22  
 Petrov Ju. P. 3,1118  
 Pinaev D. 2,817  
 Pjatnitskaj G. 2,826  
 Podgurski Yu. 1,231  
 Podoplekin Yu. F. 1,118  
 Pogliano P. 3,1076  
 Pogromsky A. Yu. 3,943  
 Poletykin A. G. 1,41  
 Polonnikov R. I. 3,890,896  
 Poluektov R. A. 2,739  
 Polushin I.G. 3,951  
 Ponomaryov V. 1,xl  
 Popov I. 2,621  
 Popov V. V. 1,142,150  
 Popovich V. V. 3,1159,1184  
 Popow O. S. 3,1052  
 Popyack L. J. 1,lxxvi  
 Prangishvili I. V. 1,lx  
 Priadko A. I. 3,1237  
 Ptitsina L. K. 2,681  
 Pudova N. 2,484  
 Putov V.V. 3,951  
 Radoushinsky M. 1,313  
 Rakhmanova I. O. 1,279  
 Rakitin I. D. 3, 1142  
 Razumovich V. V. 1,77  
 Razzoli R. P. 3,1244  
 Rembold U. 3,1233,1255,1306  
 Reshmin S. A. 1,xlv  
 Rodionov A. A. 3,1159,1189  
 Rogalev Y. N. 1,104  
 Roussel L. 3,1213  
 Ryabchenko V. 2,687  
 Ryabinin I. A. 3,1069  
 Salyga V. I. 3,1002  
 Sapegin V. 1,71  
 Sazonov A. 1,10  
 Sazonov A. E. 1,48  
 Skormin V. A. 1,lxxvi,215; 3,933  
 Segen A. 2,826  
 Sejdun Mefty 1,104

Semenov S. I. 3,1237  
 Semykina A 1,351  
 Senchenko V. G. 1,150  
 Senichenkov Yu. 2,704,712  
 Seregin I. A. 2,808  
 Sergeev A. G. 1,88  
 Shachgeldyan C. 1,225  
 Shaluha V. A. 3,1231  
 Shalyto A. A. 1,31  
 Shamberov V. N. 1,401  
 Shaposhnikov S. O. 1,109  
 Shemanin Yu. 1,231  
 Shiegin V. 3,1267  
 Shiriaev A. S. 3,980  
 Shishaev M. G. 2,753  
 Shpakov M. V. 1,298  
 Shpakov V. M. 1,298  
 Shvilin Yu. 3,1345  
 Sihn W. 1,287  
 Simakov I. I. 1,41,125  
 Simakov I. P. 1,14,41,125  
 Singh A. K. 2,780  
 Sizov V. G. 2,492  
 Skala L. Z. 2,480  
 Smirnov A. V. 1,279,320  
 Smirnov E. N. 3,1231  
 Smokty O. I. 3,1337  
 Smolentsev S.V. 1,10,66  
 Sokolov N. E. 1,77  
 Sokolova S. P. 1,295  
 Solojentssev E. D. 3,1109  
 Solovjev A. E. 3,1324  
 Solovyev A. A. 1,34  
 Sorokin L. 3,1345  
 Sova R. U. 2,632  
 Strobel G. 3,1329  
 Stebenev A.S. 3,1142  
 Stenina I. 2,518  
 Stepanov I. V. 1,109  
 Strakhovich E. V. 3,922  
 Suvorov A. I. 1,306  
 Suvorov N. B. 3,905  
 Taits B. M. 3,871  
 Tarasov V. V. 2,421  
 Tarnovsky S. 2,621

Tchaschnikova V. V. 1,408	Yampolskaya S.A. 1,251
Terekhov V. A. 2,584	Yankovsky A. 1,204
Timofeev A. V. 2,537,564,632	Yaworsky P. S. 2,650
Tishkov A. V. 2,522	Yegorov S. 1,231
Titomir L. I. 3,874,881	Yevin I. A. 2,502
Tomashevsky L. G. 3,1094	Yu Hongchuan 3,938
Tomchina O. P. 3,972	Yusupov L. N. 1,150
Torgashev V. 1,71	Yusupov R. M. 1,xl; 2,729
Tornielli G. 3,1076	Yusupov V. R. 2,672
Tretiakov A. G. 3,1184	Zaborovski V. 1,231
Tretyakov M. J. 2,629	Zanella M. 3,1076
Trunov V. G. 3,874	Zavalishin A. A. 1,53
Tsinman L. L. 2,492	Zhabko A. P. 1,382
Turusov S. N. 1,109	Zhivoglyadov V. P. 1,251
Ushakov A. V. 3,1022	Zigalenko E. N. 3,1059
Ushakov C. U. 1,165,173	Zlotnikov K. A. 2,672
Ustinov S. M. 2,696	Zubalevich E. 2,606
Uvaisov S. 2,826	Zuenkov M. A. 1,41
Vaccaro J.M. 2,650	Zvontzov V. V. 2,564
Valeeva R. 2,606	
Valigi P. 3,1275	
Vartanov V. L. 3,1315	
Varzhapetjan A. G. 2, 769,848	
Vashchenko E. 2,512	
Vassilyev S. 1,lxxxiii,411; 2,530	
Veremei E. I. 1,382	
Verzoun N. 1,247	
Viahhi I. E. 3,1237	
Vidomenko V. P. 1,259	
Vinogradov M. 2,638	
Vishnevsky E. P. 1,165,173	
Voinov E. A. 1,118	
Voinov V. 1,181	
Voitetsky V. V. 1,liii	
Volobueva O. P. 3,1064	
Voloshinova T. V. 3,910	
Volsky V. E. 1,95	
Vorobiev V. I. 2,780	
Vorobjev M. N. 1,56	
Vovk S. V. 1,27	
Wambo S. 3,881	
Woern H. 3,1255,1306	
Yakimova K. S. 3,972	
Yakovlev A. 2,458	
Yakovlev V. B. 2,584	